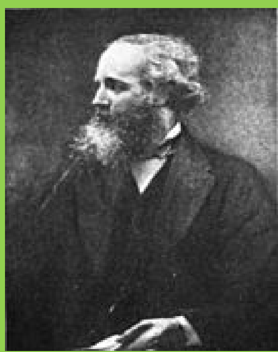
BSCPH-102

B. Sc. I YEAR

ELECTRICITY AND MAGNETISM



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UNIT 1 QUANTIZATION OF CHARGE, MILLIKAN'S OIL DROP EXPERIMENT AND COULOMB'S LAW

Structure

1.1 INTRODUCTION

Dear learners, in lower classes, you have seen that when we rub two bodies together, the both bodies begin to attract light bodies like cotton, straws, and feathers of birds or small pieces of paper. Such experienceswere studied byancient Unani philosopher Thales about 2500 years back. When he rubbed amber with woolen cloth, amber acquires the property of attracting light bodies. When a glass rod is rubbed with silk, it also acquires the same property of attracting light bodies. When a body acquires such type of property, it is said to be electrified or the body is said to be charged electrically. In Unani language amber is said to be electronand the energy due to which amber acquires the property of attracting light bodies is called electricity.When we rub any solid material with another material under suitable conditions, it gets charged electrically. The process of acquiring charges by bodies when they are rubbed with each other is known as frictional electrification.

It is found experimentally that there are of two types of charges- positive charge and negative charge. In fact, when we rub two bodies with each other, there is a transfer of electrons from one body to another. The body which loses its electrons becomes positive (positively charged) and the body which gains electrons from first body becomes negative (negatively charged). Thus, there are of two charges i.e. positive charge and negative charge. The magnitude of charges on each body depends on the number of transferred electrons. The names positive and negative charges were given by an American Scientist named Benjamin Franklin in 1750. The names of positive and negative charges are purely conventional.

In this unit, the learners shall study thevarious properties of charges, experiment showing the quantization of charge i.e. Millikan’s oil drop experiment, Coulomb’s law and its applications.

1.2 OBJECTIVES

After studying this unit, you should be able to-

- know about charges and their properties
- learn quantization of charge
- learn about Millikan’s oil drop experiment
- know about Coulomb’s law and their applications in daily life
- solve problems using the theory of Millikan’s oil drop experiment
- apply Coulomb’s law

1.3 PROPERTIES OF CHARGES

We know that charges have peculiar properties. Let us know about these properties of charges-

(a) Like charges repel and unlike charges attract.

(b) A charged body attracts to uncharged (neutral) bodies due to electrostatic induction.
(c) Charge on a body remains unaffected by motion i.e. the charge on a body or particle remains the same whether it is at rest or moving with any velocity.
(d) The electric chargeis additive. It means that the total charge on an extended body is the algebraic sum of the charges located at different points in the body. If a body has positive and negative charges both, then the net charge of the body is the algebraic sum of all the charges i.e. $Q = \Sigma q$. A neutral body has equal amount of positive and negative charges so that the charge on a neutral body is always zero.
(e) Charge is conserved i.e. it can neither be created nor destroyed but it may simply be transferred from one body to another body.
(f) Charge is quantized i.e. any physically existing charge is an integral multiple of the elementary charge (e).

Now, we shall discuss the properties of conservation of charge and quantization of charge in detail.

1.3.1 Conservation of Charge

Charge can neither be created nor destroyed but it may simply be transferred from one body to another body. This is known as conservation of charge or principle of conservation of charge. The principle of conservation of charge may also be stated as "The net charge of an isolated system remains constant." Charge is conserved in every physical and chemical process.

1.3.2 Quantization of Charge

"Charge is created by transfer of electrons; therefore the net charge on a body is always an integral multiple of magnitude of charge on an electron."

We know that the charge on a body is produced due to excess or deficiency of electrons. Electron cannot be divided into further smaller parts. Therefore, charge on a body is integral multiple of the amount of charge on electron. This smallest amount of charge is $1.6\text{x } 10^{-19}$ coulomb and is denoted by 'e'. The magnitude of charge on an electron is called the fundamental charge or elementary charge. Therefore, we can say that any physically existing charge is always an integral multiple of fundamental charge '±e' i.e. all existing charges are found to be 'ne' (where n is a positive) such as e, 2e, 3e,……………., -e, -2e, -3e,……….. Mathematically, we can write $q = \pm ne$, where 'n' is integer, n = 1, 2, 3,……… and 'e' is a positive quantity equal to $+ 1.6\text{x } 10^{-19}$ coulomb. 'e' is also known as the quantum of charge. No charge is found in the fraction of e (as 0.5 e or 0.7 e or 2.7 e…… etc.). It means that electric charge cannot be divided indefinitely. This property of charge is called 'quantization' or 'atomicity' of charge.

The charges of some natural elementary particles are as follows-

charge of electron: -e, charge of proton: +e, charge on α-particle: +2e

The value of the elementary charge is so small that we do not experience the quantization of charge in daily life. Millikan's oil drop experiment and many other experiments confirm the quantum nature of charge.

1.4 MILLIKAN'S OIL DROP EXPERIMENT

This is the experiment which confirms the quantum nature of charge. Let us discuss Millikan's oil drop experiment.

In 1909, Millikan performed a series of experiments to demonstrate the existence of elementary charge. He used tiny oil drops while latter on plastic spheres of known mass were used instead of oil drops.

The apparatus consists of two parallel metallic plates P_1 and P_2 connected to +ve and –ve terminals of a battery through potential divider as shown in figure 1. The observations are taken with the microscope; hence the spheres (plastic balls) are illuminated by intense light.

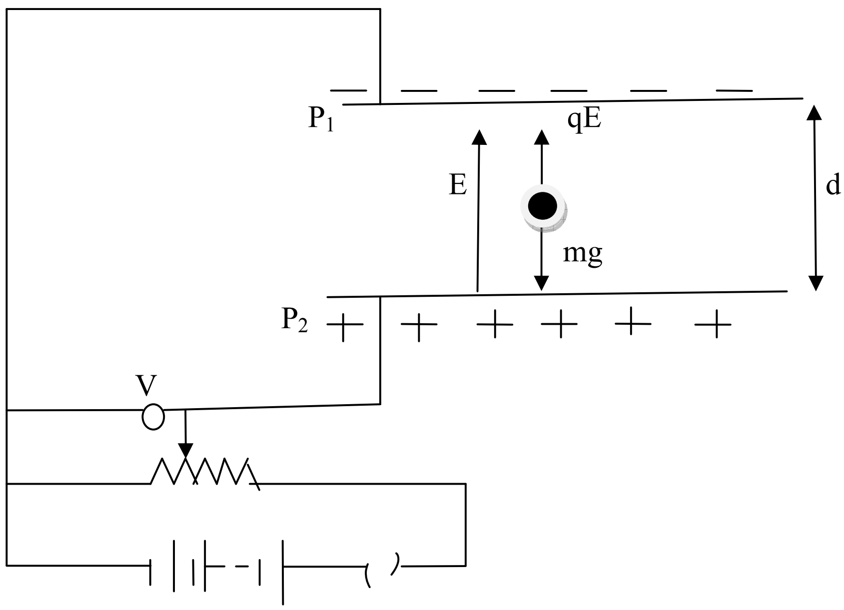


Figure 1

The plastic balls are charged by friction and are thrown between two plates through a tube by a blower. At first, when the key is open, these spheres start falling due to force of gravity. Since their masses are equal and an equal viscous force due to air acts on them, all spheres have the same constant velocity called terminal velocity. Now, the key is closed, the plates P_1 and P_2 are charged and an electric field between plates P_1 and P_2 is established. Now the charged spheres experience two forces-

1. Force of gravity
2. Electric force

If lower plate is connected to the +ve terminal of battery, the direction of electric field is upward and therefore an electric force due to this field acts upward. Since the charge on the spheres is different they experience different electric force.

We can adjust the electric potential by potential divider and establish equilibrium. Thus in the condition of equilibrium, the two forces are equal in magnitude but opposite in direction.

If the mass of the sphere is 'm' and charge 'q', then we have-

$qE = mg$ …..(1)

If the two plates P_1 and P_2 are separated by a distance d and the potential difference between them is V, then electric field $E = V/d$

From equation (1), $q\ V/d = mg$

or $q = mgd / V$ …..(2)

But mgd = constant, since the masses of all spheres are same

Therefore, $q \propto \frac{1}{V}$ …..(3)

or $qV = \text{constant}$ …..(4)

Various potentials are applied to balance the spheres of different charge. If spheres have charges $q_1, q_2, q_3, \ldots$, then corresponding potentials are $V_1, V_2, V_3, \ldots$, hence

$q_1V_1 = q_2V_2 = q_3V_3 \ldots = \text{constant}$

Since $\frac{1}{V}$ is proportional to charge q of the ball, the ratio $\frac{1}{V_1}, \frac{1}{V_2}, \frac{1}{V_3}$ ………… are in integral multiple ratio. Obviously, the charge on each ball is integral multiple of minimum value. That minimum value of charge is electronic charge 'e'.

Thus $q = ne$ …..(5)

or $\frac{q}{e} = n$ (an integer)

Thus Millikan's oil drop experiment confirms the quantum nature of charge.

Example 1: A plastic piece rubbed with wool is found to have a negative charge of 5×10^{-7} coulomb. Calculate the number of electrons transferred.

Solution: Given $q = 5 \times 10^{-7}$ coulomb

Using q = ne, we get-

$n = \frac{q}{e} = \frac{5\times10^{-7}}{1.6\times10^{-19}} = 3.125 \times 10^{12}$

Example 2: In Millikan's oil drop experiment the charge on any three drops was found to be $1.6\text{x}10^{-19}$, $4.8\text{x}10^{-19}$ and $9.6\text{x}10^{-19}$ coulomb. What is the conclusion of these results?

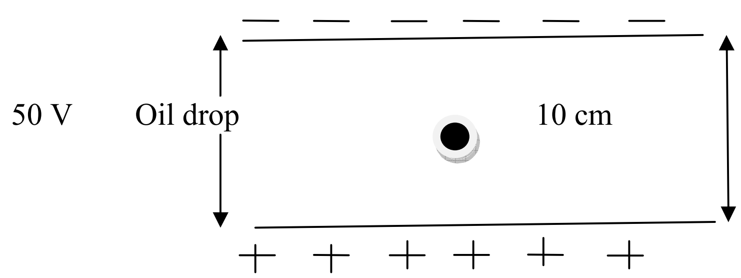
Solution: The given charges on the drops are- q_1= $1.6\text{x}10^{-19}$coulomb, q_2 = $4.8\text{x}10^{-19}$ coulomb= 3x $1.6\text{x}10^{-19}$coulomb, q_3= 6x $1.6\text{x}10^{-19}$ coulomb

Obviously, the maximum common factor among the given charges is $1.6\text{x}10^{-19}$coulomb and this is the minimum possible charge. Therefore, the elementary charge = $1.6\text{x}10^{-19}$coulomb.

Also, all given charges are integral multiples (i.e. 1 times, 3 times and 6 times) of elementary charge. This confirms the quantum nature of charge. The conclusion of these results is that elementary charge is $1.6\text{x}10^{-19}$coulomb and charge is quantized.

Self Assessment Question (SAQ) 1: Wool rubbed with a polythene piece is found to have a +ve charge. State from which to which the transfer of electrons took place. Is there a transfer of mass from wool to polythene or vice versa?

Self Assessment Question (SAQ) 2:A oil drop of mass 5 gm is hanging in equilibrium between two charged plates as shown in figure. Calculate the magnitude and nature of charge on the drop.



1.5 COULOMB'S LAW

You have read in the previous sections that two like charges repel each other and two unlike charges attract each other. Thus, we can say that a force acts between two charges. This force is known as 'electric force'. The electric force between like charges is repulsive and that between unlike charges is attractive.

In 1785, Coulomb, on the basis of experiments, stated a law regarding the force acting between two charges. According to this law, "The force of attraction or repulsion between two point charges is directly proportional to the product of the charges and inversely proportional to the square of distance between them. The direction of this force is along the line joining the two charges". This law is called Coulomb's inverse square law.

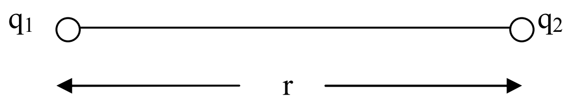


Figure 2

If two point charges q_1 and q_2 are separated by a distance r, then the force F acting between them is given by-

$F \propto \frac{q_1 q_2}{r^2}$

$$F = k\frac{q_1 q_2}{r^2} \qquad \text{.....(6)}$$

Where, k is proportionality constant, whose value is given by $\frac{1}{4\pi\varepsilon_0}$, if the charges are placed in vacuum (or air). If the charges, distance and the force are measured in coulomb(C), meter (m) and Newton(N) respectively, then $\frac{1}{4\pi\varepsilon_0}$ = 9 x 10^9 N-m^2/C^2. The constant ε_0 is read as epsilon zero and called 'permittivity of free space'. Its value is 8.85x 10^{-12} C^2/N-m^2.

If $q_1 = q_2 = 1$ coulomb and r = 1 meter, then from equation (6), we get-

$F = 9 \text{ x } 10^9 \text{x } \frac{1\times1}{1^2} = 9 \text{ x } 10^9$ Newton

Hence, 1 coulomb is that charge which, when placed at a distance of 1 meter from an equal and similar charge in vacuum (or air), repels it with a force of 9 x 10^9 Newton.

If charges are placed in a medium like glass, wax, paper etc., then the force between the charges is given by-

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} \qquad \text{.....(7)}$$

where ε is called the absolute permittivity of the material medium and is equal to $K\varepsilon_0$ i.e. $\varepsilon = K\varepsilon_0$, where K is a dimensionless constant known as the dielectric constant or relative permittivity or specific inductive capacity of the material and the material is called dielectric.

We can write the equation (7) as-

$$F = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{r^2} \qquad \text{.....(8)}$$

For all dielectrics the value of K is greater than 1. Obviously, we can see that if there is a dielectric between the charges, then the electric force between the charges decreases. For metals K is infinite and for water K= 81

In vector form, we can write –

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r} \;.....(9)$$

where $\hat{r}$ is the unit vector along $\vec{r}$.

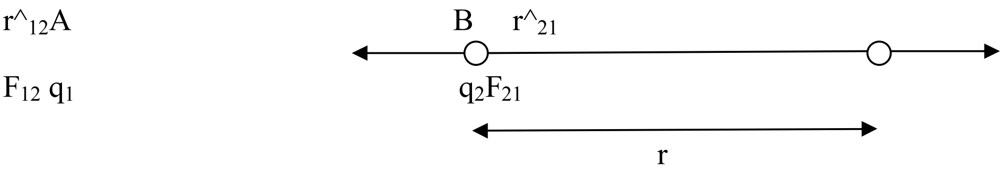


Figure 3

Let us consider two point charges q_1 and q_2 are placed at points A and B respectively and the distance between them is r.

The force exerted on charge q_1 due to charge q_2 can be written as-

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12} \qquad .....(10)$$

(Since $\hat{r}_{12} = \frac{\vec{r}_{12}}{r}$, the unit vector along B to A i.e. along position vector $\vec{r}_{12}$)

Similarly, the force exerted on charge q_2 due to charge q_1 can be written as-

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{21}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^3}\vec{r}_{21} \qquad .....(11)$$

But $\vec{r}_{12} = -\vec{r}_{21}$, therefore equation (11) can be written as-

$$\vec{F}_{21} = -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^3}\vec{r}_{12} \qquad .....(12)$$

Comparing equations (10) and (12) we get-

$$\vec{F}_{12} = -\vec{F}_{21}$$

It means that Coulomb's force exerted on q_1 by q_2 is equal and opposite to the Coulomb's force exerted on q_2 by q_1 ; in accordance with Newton's third law. Thus, Newton's third law also holds for electrical forces.

1.5.1 Conditions of Validity of Coulomb's Law

In the previous section, you have seen that Coulomb's law between two point charges is an inverse square distance law. It holds only for point charges and spherical charges at sufficient separation, assuming the charge to be concentrated at their centres, however, it may be applied to extended objects provided the distance between them is much larger than their dimensions. Both charges must be point charges i.e. the extension of charges should be much smaller than the separation between the charges. The separation between the charges must be greater than nuclear distance (10^{-15} m) because for distances less than 10^{-15} m, the nuclear attractive forces become dominant over all other forces.

1.5.2 Importance of Coulomb's Law

Dear learners, as you know that Coulomb's law is true for point charges separated by from very large distances to very small distances such as atomic distances ($\approx 10^{-11}$m) and nuclear distances ($\approx 10^{-15}$ m). Therefore, it is not only gives us the force acting between charged bodies but also helps in explaining the forces which bind electrons with nucleus in an atom, two or more atoms in a molecule and many atoms or molecules in solids and liquids. In our daily life, we experience many forces which are not gravitational but are electrical. The particles in the nucleus (protons and neutrons) of an atom are bound together by a very strong attractive force named as the nuclear force. This force neither depends upon whether a particle is charged or uncharged nor it has any relation with the Coulomb's law. But it does not mean that the protons in the nucleus do not have Coulomb's electrical repulsive force between them. The electrical repulsive force is there, although it is negligible in comparison to the nuclear attractive force, and plays a vital role inside the nucleus. If this force would not have been there, the heavy nuclei would not have been radioactive and the heavy elements beyond uranium (which are unstable) would have been stable.

1.5.3 Comparison of Coulomb's Force and Gravitational Force

In addition to Coulomb's force, gravitational force also acts between two charged bodies. The comparison between Coulomb's force and Gravitational force is tabulated below-

S. No.	Coulomb's force	Gravitational force
1.	The Coulomb's force (electrical force) between two charged bodies of charges q_1 and q_2 at separation r is given as- $F_e = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{r^2}$	The gravitational force acting between two bodies of masses m_1 and m_2 at separation r is given as- $F_g = \frac{Gm_1m_2}{r^2}$, where G is known as Universal Gravitational Constant and G = 6.67 x 10^{-11} N-m^2/Kg^2
2.	The Coulomb's force may be attractive or repulsive in nature.	The gravitational force is always attractive.
3.	The Coulomb's force (electrical force) depends upon the medium between the charges.	The gravitational force is independent of medium between the masses.
4.	The Coulomb's force is much stronger.	The gravitational force is much weaker than the Coulomb's force.

Example 3: Calculate the Coulombian force between two protons when the distance between them is 4 × 10^{-15} meter. Also give the nature of this force.

Solution: Given r = 4 × 10^{-15} meter

We know, the charge on proton = 1.6 × 10^{-19} C (positive), therefore, $q_1 = q_2 = + 1.6 \times 10^{-19}$ C

Applying Coulomb's law-

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

or $F = 9\times 10^{9} \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19} /(4\times10^{-15})^2$

= 14.4 Newton (repulsive)

Example 4: Show that the gravitational force is negligible in comparison to electric force in hydrogen atom in which the electron and proton are about 5.3 × 10^{-11} metre apart.

Solution: The gravitational force between electron and proton is given by-

$F_g = \frac{Gm_1m_2}{r^2}$, Here m_1= mass of electron= 9.1×10^{-31} Kg, m_2 = mass of proton = 1.6 ×10^{-27} Kg, r = 5.3 × 10^{-11} m and G = 6.67 x 10^{-11} N-m^2/Kg^2

Therefore, $F_g = \frac{6.67\times10^{-11}\times9.1\times10^{-31}\times1.6\times10^{-27}}{(5.3\times10^{-11})^2}$

$$= 3.69\times10^{-47} \text{ N}$$

Now electric force $F_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$

Here $\frac{1}{4\pi\varepsilon_0} = 9 \text{ x } 10^9 \text{ N-m}^2/\text{C}^2$, $q_1 = q_2 = 1.6 \times10^{-19}$ C

Therefore, $F_e = 9 \text{ x } 10^9\times \frac{1.6\times10^{-19}\times1.6\times10^{-19}}{(5.3\times10^{-11})^2} = 8.2\times10^{-8}$ N

Obviously, gravitational force is negligible in comparison to electric force in hydrogen atom in which the electron and proton are about 5.3×10^{-11} metre apart.

Example 5:A charge Q is divided into two parts such that they repel each other with a maximum force when placed at a certain distance apart. Find the distribution of charge.

Solution: Let the two parts of charge Q be Q' and Q-Q'. The force between two parts is given as-

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q'(Q-Q')}{r^2}$$

For maximum value of F, $\frac{dF}{dQ'} = 0$ (r is constant)

Therefore, $$\frac{1}{4\pi\varepsilon_0}\frac{Q-2Q'}{r^2} = 0$$

or $$Q-2Q' = 0$$

or $Q' = Q/2$

Therefore, the charge Q should be divided into two equal parts.

Self Assessment Question (SAQ) 3: Two identical metallic spheres, having unequal opposite charges are placed at a distance of 0.30 metre apart in air. After bringing them in contact with each other, they are again placed at the same distance apart. Now the force of repulsion between them is 0.183 N. Calculate the final charge on each of them.

Self Assessment Question (SAQ) 4: Two point charges +4Q and +Q are fixed at a distance r apart. Where a third point charge q should be placed on the line joining the two charges so that it is in equilibrium? In which condition the equilibrium will be stable and in which unstable?

Self Assessment Question (SAQ) 5: Calculate absolute permittivity of water if dielectric constant of water is 81.

Self Assessment Question (SAQ) 6: Two positively charged particles, each of mass 1.7×10^{-27} Kg and carrying a charge of 1.6×10^{-19} coulomb, are placed at a distance *l* apart. If each one experiences a repulsive force equal to its weight, find *l*.

1.6 SUMMARY

In the present unit, you have studied about electric charge, how it was discovered and its properties. You have studied that there is no effect of motion on the charge of a body i.e. the charge on a body or particle remains the same whether it is at rest or moving with any velocity. Charge is conserved i.e. it can neither be created nor destroyed but it may simply be transferred from one body to another body. You have also studied about the quantization of charge i.e. electric charge cannot be divided indefinitely. Millikan's oil drop experiment has been discussed which confirms the quantum nature of charge. You have also studied Coulomb's law, its conditions of validity and importance. According to Coulomb's law, "The force of attraction or repulsion between two point charges is directly proportional to the product of the charges and inversely proportional to the square of distance between them. The direction of this force is along the line joining the two charges". This law is called Coulomb's inverse square law.You have studied that Coulomb's law holds only for point charges and spherical charges at sufficient separation, assuming the charge to be concentrated at their centres, however, it may be applied to extended objects provided the distance between them is much larger than their dimensions. This law is true for atomic and nuclear distances. You have studied the comparison between Coulomb's law and Gravitational force. It is clear that Coulomb's law between two charged bodies is much stronger than the Gravitational force acting between them. Many solved examples are given in the unit to make the concepts clear. To check your progress, self assessment questions (SAQs) are given place to place.

1.7 GLOSSARY

Transfer- shift, transmit

Conserved- preserved

Performed- carried out or completed an action or function

Demonstrate- show, display

Intense- concentrated, powerful

Illuminate- light up

Viscous- thick, sticky

Vacuum- void, vacuity

Dielectric- that does not conduct electricity, insulating

Validity- legality, legitimacy

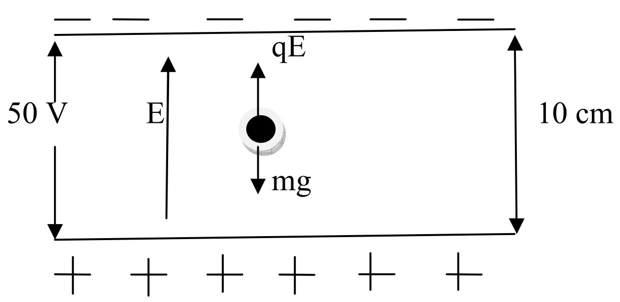
1.8 TERMINAL QUESTIONS

1. Explain quantization of charge. Hence define elementary charge.
2. How many electrons must be removed from a piece of metal to give it + 1 $\times 10^{-7}$ C of charge?
3. State and explain the principle of conservation of charge.
4. Discuss Millikan's oil drop experiment to verify the quantum nature of electric charge.
5. In Millikan's experiment, an oil drop of radius 10^{-4} cm remains suspended between the plates which are 1 cm apart. If the drop has a charge of 5e over it, calculate the potential difference between the plates. The density of oil may be taken as 1.5 g/cc.
6. State Coulomb's law in electrostatics. Mention two similarities and two dissimilarities between electrostatic and gravitational interactions.
7. Does Coulomb's law of electric force obey Newton's third law of motion?
8. Give the importance of Coulomb's law.
9. Give comparison of Coulomb's force and Gravitational force.

1.9 ANSWERS

Self Assessment Questions (SAQs):

1. When two neutral bodies are rubbed together, electrons of one body are transferred to the other. The body which gains electrons is negatively charged and the body which loses electrons is positively charged. When wool is rubbed with a piece of polythene, the wool becomes positively charged and polythene becomes negatively charged. It means that electrons are transferred from wool to polythene.
   As we know that electrons have finite mass, therefore mass is transferred from wool to polythene.
   The transferred mass = number of electrons transferredx mass of one electron
2. Given mass of drop m = 5 gm = $5\text{x}10^{-3}$ Kg, d= 10 cm= 0.10 m, V = 50 Volt
   and g = 9.8 m/sec^2



The electric field between the plates $E = \frac{V}{d} = \frac{50}{0.10} = 500$ volt/m (vertically upward from positive plate to negative plate)

Weight of the drop $W = mg = 5\times10^{-3} \times 9.8 = 49 \times 10^{-3}$ Newton (vertically downward)

Electric force acting on the drop F = qE = q x 500 Newton

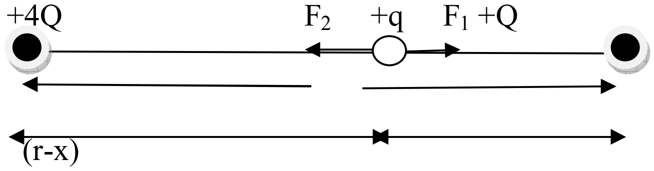
For the equilibrium, the two forces i.e. weight of drop and electric force acting on the drop should be equal and opposite in direction. The weight of the drop will act vertically downward, therefore the electric force on drop should act vertically upward. Therefore, F = W

or $q \times 500 = 49 \times 10^{-3}$ or $q = 49 \times 10^{-3}/500 = 0.000098$ coulomb (Positive charge)

3. When identical metallic spheres are brought in contact, then after separation they carry equal charges.
   Let Q be the charge on each sphere, then force of repulsion between them will be-
   $$F = \frac{1}{4\pi\varepsilon_0}\frac{Q \times Q}{r^2}$$
   Here F = 0.183 N, r = 0.30 metre, $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N-m^2/C^2
   Therefore, $0.183 = 9 \times 10^9 \times \frac{Q^2}{(0.30)^2}$
   or $Q^2 = 183 \times 10^{-14}$
   or $Q = \sqrt{183 \times 10^{-14}} = 13.527 \times 10^{-7} = 1.35 \times 10^{-6}$ Coulomb = 1.35 μC
4. Let the third point charge q is placed between the charges + 4Q and + Q at a distance x from +4Q.
   The distance of third charge from + Q = r-x
   Let third point charge q be positive.

r

x



The electric force on the charge +q due to the charge +4Q is-

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{4Q \times q}{x^2} \quad \text{(repulsive)}$$

Similarly, the electric force on the charge +q due to the +Q is-

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q \times q}{(r-x)^2} \quad \text{(repulsive)}$$

For the equilibrium of charge +q, the above two forces should be equal and opposite. Therefore, $F_1 = F_2$

$$\frac{1}{4\pi\varepsilon_0}\frac{4Q \times q}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q \times q}{(r-x)^2}$$

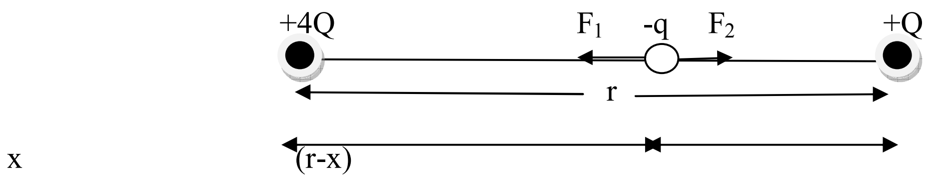
or $4(r-x)^2 = x^2$

or $2(r-x) = \pm x$

or $x = 2r/3$ or $2r$

Only x= 2r/3 is possible because the charge + q is in between +4Q and +Q. Hence, for equilibrium, the charge +q will be placed at a distance 2r/3 from the charge + 4Q in between the two charges. If you displace the charge +q slightly from its equilibrium position(suppose towards right) then F_1 will decrease and F_2 will increase. Hence a net force $(F_2 - F_1)$ will act on the charge +q towards left, due to which the charge will return to its equilibrium position. Thus the equilibrium of the charge +q is stable.

Let third point charge q be negative.



In this case, the force F_1 and F_2 will be of attraction and their directions will be according to the adjoining diagram. The charge –q will still be in equilibrium. If you displace this charge slightly towards right (say) then F_1 will decrease and F_2 will increase. Hence a net force $(F_2 - F_1)$ will act on the charge –q but now its direction will be towards right. Hence the charge will go on moving towards right. Thus the equilibrium of the charge –q is unstable.

5. Given K = 81, $\varepsilon_0 = 8.85 \times 10^{-12}$ $C^2/N\text{-}m^2$
   $\varepsilon = K\varepsilon_0 = 81 \times 8.85 \times 10^{-12} = 7.16 \times 10^{-10}$ $C^2/N\text{-}m^2$
6. The repulsive force $F = \frac{1}{4\pi\varepsilon_0}\frac{Q \times Q}{l^2}$

Here, $Q = 1.6 \times 10^{-19}$ coulomb

Therefore, $F = 9 \times 10^9 \times \frac{1.6 \times 10^{-19} \times 1.6\times10^{-19}}{l^2}$

But $F = mg = 1.7\times10^{-27}\times9.8 = 1.66\times10^{-26}$ N

Therefore, $9 \times 10^9 \times \frac{1.6 \times 10^{-19} \times 1.6\times10^{-19}}{l^2} = 1.66\times10^{-26}$

or $l = 0.117$ metre

Terminal Questions:

2. Given, $q = +1\times10^{-7}$ C

Using $q = ne$ or $n = q/e = 1\times10^{-7}/1.6\times10^{-19} = 6.25\times10^{11}$

5. Given $r = 10^{-4}$ cm $= 10^{-6}$ m, $d = 1$ cm $= 0.01$ m, $q = 5e = 5\times1.6\times10^{-19} = 8\times10^{-19}$ C,

$\rho = 1.5$ g/cc $= 1.5\times10^{-3}/10^{-6} = 1.5\times10^{3}$ Kg/m^3

Volume of drop $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\times3.14\times(10^{-6})^3 = 4.18\times10^{-18}$ m^3

Mass of drop m = density× volume = $\rho\times V = 1.5\times10^{3}\times4.18\times10^{-18} = 6.27\times10^{-15}$ Kg

For equilibrium, $qE = mg$

or $q\times V/d = mg$ or $V = mgd/q = 6.27\times10^{-15}\times9.8\times0.01/(8\times10^{-19})$

$= 768.075$ volt

7. Yes

1.10 REFERENCES

1. Electricity and Magnetism, Sehgal, Chopra, Sehgal, Sultan Chand & Sons, New Delhi

2.Electricity and Magnetism with Electronics, K.K. Tewari, S. Chand & Company Pvt. Ltd., New Delhi

3. Electricity and Magnetism, Satya Prakash, Pragati Prakashan, Meerut

4. Nootan Physics, Kumar, Mittal, Nageen Prakashan Pvt. Ltd., Meerut

1.11 SUGGESTED READINGS

1. Concepts of Physics, Part II, HC Verma, Bharati Bhawan, Patna

2.Fundamentals of Physics, David Halliday, Robert Resnick, Jearl Walker, John Wiley & Sons

UNIT 2 INTENSITY AND POTENTIAL, GAUSS'S THEOREM, APPLICATIONS

Structure

2.1 INTRODUCTION

In the previous unit, you have learnt about charges, their properties, quantization and conservation of electric charge and Coulomb's law. We know from our early studies that the mutual interaction between charged bodies can be interpreted as due to the force which each exert on the other, even though there is no material connection between them. This action at a distance view was considered to be inconvenient and troublesome. Faraday in 19^{th} century introduced the so-called field concept to explain the mutual interactions between two charged bodies. This concept was subsequently developed by Maxwell. In this unit, you will study and learn about electric field, electric field intensity (strength) in different cases, electric potential and its calculation in different cases. In the unit, you will also study electric flux, Gauss's law and the applications of Gauss's law. The various concepts have been presented in a simple and clear manner.

2.2 OBJECTIVES

After studying this unit, you should be able to-

- learn about electric field and electric potential
- learn about electric lines of force
- compute electric field intensity and electric potential in various cases
- understand electric flux
- understand Gauss's law and its applications
- solve problems based on electric field, electric potential and Gauss's law

2.3 CONCEPT OF ELECTRIC FIELD

Let us consider an electric charge q located in space. If you bring another charge q_0 near the charge q, then the charge q_0 experiences a force of attraction or repulsion due to the charge q. The force experienced by q_0 is said due to the electric field created by the charge q. Thus, **"The space surrounding an electric charge in which another charge experiences a force (attractive or repulsive), is called the electric field of the electric charge".** We can say that" The region in which a charge experiences a force is called the electric field".

If a charge q_0 experiences a force in the space surrounding the charge q, then charge q is called the 'source charge' and the charge q_0 is called the 'test charge'. The source-charge may be a point-charge, a group of point-charges or a continuous distribution of charges. Further, the test charge must be vanishingly small so that it does not modify the electric field of the source charge.

2.4 INTENSITY OF ELECTRIC FIELD

In order to determine the intensity (strength) of electric field at a point in the electric field, let us place an infinitesimal positive test charge q_0 at that point. The force acting on this test charge is measured and this force divided by the test charge gives electric field strength. The test charge is assumed so small that it does not cause any change in initial electric field. Accordingly the electric field strength (or intensity) is defined as follows-

"The intensity of electric field at a point in an electric field is the ratio of the force acting on the test charge placed at that point to the magnitude of the test charge". It is a vector quantity and its direction is along the direction of force.

Thus, if $\vec{F}$ be the force acting on a test charge q_0 placed at a point in an electric field, then the intensity of electric field $\vec{E}$ of the field at that point is given by-

$$\vec{E} = \frac{\vec{F}}{q_0} \quad .....(1)$$

Here, we have assumed that test charge q_0 is infinitesimal, therefore the definition of intensity of electric field may be expressed as-

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} \quad .....(2)$$

Force $\vec{F}$ is a vector quantity and test charge q_0 is a scalar quantity. Hence intensity of electric field$\vec{E}$ will also be a vector quantity and its direction will be the same as the direction of the force $\vec{F}$ i.e. the direction in which the positive charge placed in the electric field tends to move. If test charge be negative, then the direction of electric field $\vec{E}$ will be opposite to the direction of the force acting on the negative charge.

Obviously, the unit of intensity(strength) of electric field is Newton/metre.

If the intensity of electric field $\vec{E}$ at a point in an electric field be known, then we can determine the force $\vec{F}$ acting on a charge q placed at that point by the following equation-

$$\vec{F} = q\vec{E} \quad .....(3)$$

2.5 ELECTRIC LINES OF FORCE

In the previous sections, we have studied that a charge placed in an electric field experiences an electrostatic force. If the charge be free, then it will move in the direction of the force. If the direction of the force continuously changes then the direction of motion of the charge also continuously changes i.e. it moves along a curved path. The path of a free positive charge in an electric field is called 'electric line of force'. Hence, **"an electric line of force is that imaginary smooth curve drawn in an electric field along which a free, isolated unit positive charge moves. The tangent drawn at any point on the electric line of force gives the direction of the force acting on a positive charge placed at that point".** We can represent an electric field by lines of force.

Now we can define the intensity (strength) of electric field in terms of electric lines of force as follows-

"The intensity of electric field at any point is defined as a vector quantity whose magnitude is measured by the number of electric lines of force passing normally through per unit small area around that point and whose direction is along the tangent on line of force drawn at that point".

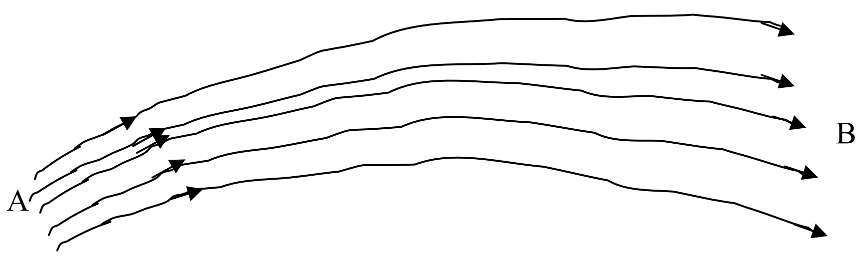
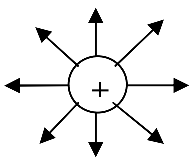


Figure 1

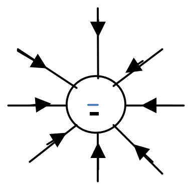
Accordingly, nearer are the electric lines of force, stronger is the electric field and if farther are the electric lines of force, weaker is the electric field. In the figure 1, the electric field strength at A is greater than that at B.

2.5.1 Properties of Electric Lines of Force

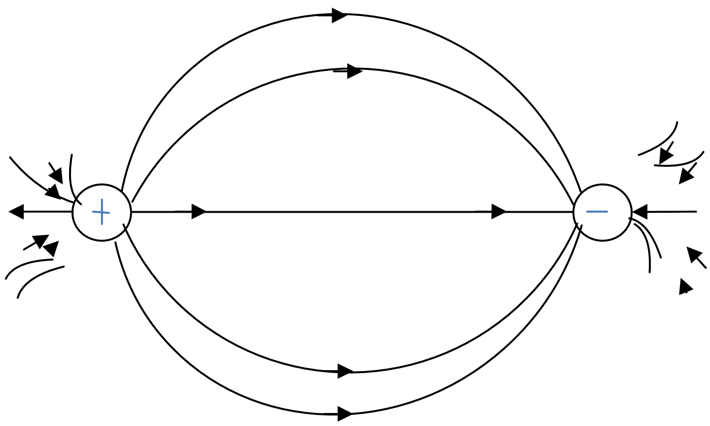
(i) The electric lines of force appear to start from positive charge and to end on a negative charge. If there is a single charge, they may start or end at infinity.



(a)



(b)



(c)

Figure 2

(ii) The tangent drawn at any point on the line of force gives the direction of the force acting on a positive charge at that point.

(iii) No two electric lines of forces can intersect each other because if they do so, then two tangents can be drawn at the point of intersection which would mean two directions of electric field intensity at one point which is impossible. In figure 3, two directions of electric field at point of intersection P have been shown which is not possible.

1P E

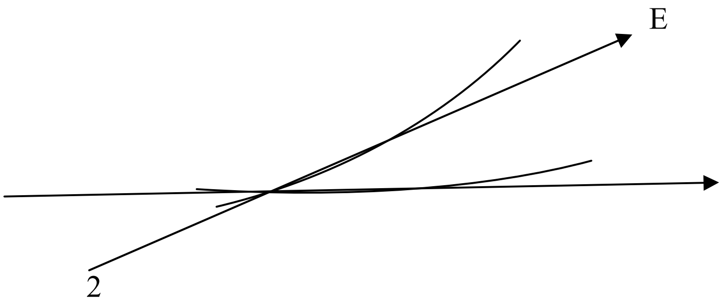
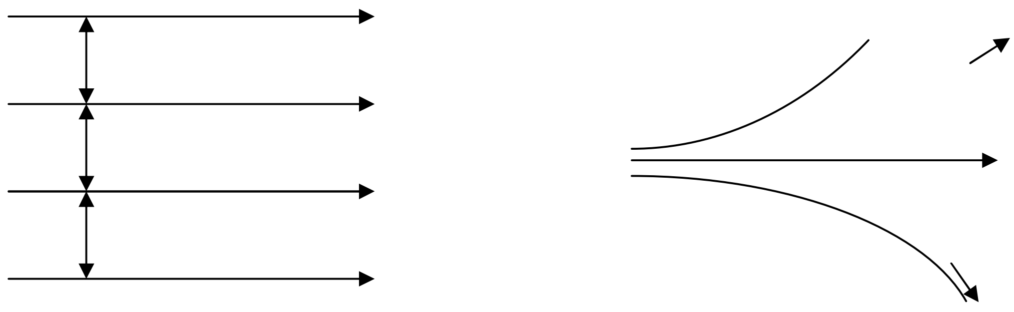


Figure 3

(iv) The electric lines of force do not pass through a conductor because electric field inside a conductor is zero.

(v) The equidistant electric lines of force represent uniform electric field while electric lines of forces at different separations represent non-uniform electric field. The relative closeness of lines of force in different regions of space expresses the relative strength of the electric field in different regions. In regions, where lines of force are closer, the electric field is stronger whereas in regions where lines of force are farther apart, the field is weaker.



(a) (b)

Figure 4

(vi) The electric lines of force have a tendency to contract in length like a stretched elastic string and separate from each other laterally. The reason is that opposite charges attract and similar charges repel.

(vii) The electric lines of force are always in the form of open curves, they do not form closed loops.

(viii) The electric lines of force are imaginary but the electric field they represent is real.

2.6 CALCULATION OF ELECTRIC FIELD INTENSITY

In this section, we shall calculate electric field intensity in various cases viz. due to a point charge, due to a system of point charges and due to a continuous charge distribution.

2.6.1 Due to a Point-charge

Let us consider an isolated point charge +q coulomb placed at a point M in air (or vacuum). In the electric field produced by the charge +q there is a point P, distant r meter from M, at which the intensity of electric field is to be calculated.

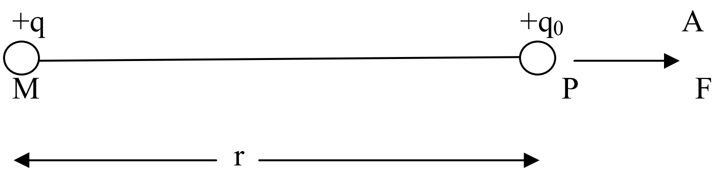


Figure 5

Let us assume that a test charge q_0 is placed at the point P. According to Coulomb's law, the electric force acting on q_0, $F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}$ Newton

The intensity of electric field at point P, $E = \frac{F}{q_0}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \text{ N/C} \quad \text{(along direction from P to A)} \quad \text{.....(4)}$$

If the system is placed in a medium of dielectric constant K, then

$$E = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{r^2} \text{ N/C} \quad \text{(along direction from P to A)} \quad \text{.....(5)}$$

In vector form, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$ .....(6)

Where $\hat{r}$ is a unit vector pointing from the source charge towards the test charge.

Equation (6) can be written as-

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r} \quad \left(\text{since } \hat{r} = \frac{\vec{r}}{r}\right) \quad \text{.....(7)}$$

If the source charge at M is –q, then the direction of the electric field E at point P would have been along PM (i.e. towards the charge –q).

2.6.2 Due to a System of Point Charges

If there are n point charges q_1, q_2, q_3,…………q_n then each of them will produce the same intensity at any point which it would have produced in the absence of other point charge. Hence the intensity of the field $\vec{E}$ at a point P due to all the n charges will be equal to the vector sum of the intensities $\vec{E}_1, \vec{E}_2, \vec{E}_3$..... …………produced by the separate charges as P-

or $\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_i^n \frac{q_i}{r_i^2}\hat{r}_i$ …..(8)

Where r_i is the distance of P from the charge q_i.

2.6.3 Due to a Continuous Charge Distribution

If there is a continuous distribution of charge, then the summation in the above expression will be replaced by integration.

If the charge is distributed on a line, then electric field intensity at a point P is-

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_l \frac{\lambda dl}{r^2}\hat{r} \quad \text{….. (9)}$$

Where λ is the linear charge density (or charge per unit length) and dl is the length of small element.

If the charge is distributed on a surface, then the electric field intensity at a point P is –

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\sigma dS}{r^2}\hat{r} \quad \text{…..(10)}$$

Where r is the distance of the point P from a surface element dS and σ is the surface charge density (i.e. charge per unit surface area)

Similarly, if the charge is distributed in a volume, then

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho dV}{r^2}\hat{r} \quad \text{…..(11)}$$

Where ρ is the volume charge density i.e. charge per unit volume.

2.6.4 Physical Significance of Electric Field

The electric field is a vector quantity which may vary from point to point in magnitude and direction. The magnitude of electric field at any point is a measure of electric force on a unit positive test charge, assuming that the test charge does not perturb the field of the system and its direction is that of electrostatic force on the test charge. This implies that the electric field is the characteristic of the charges of system and is independent of the test charge. The test charge is simply introduced for measurement of electric field in a suitable manner.

The true physical significance of electric field appears only when we keep in view that electrostatic interaction is only a part of general fundamental force known as electromagnetic interaction. When two charges q_1 and q_2 are in accelerated motion, then either accelerated charge (say q_1) produces electromagnetic wave which propagates with speed of light; reaches on another charge (say q_2) and causes a force on it.

Thus, the force between two distant charges is not instantaneous but appears with a time delay. Thus electric field (as well as magnetic field) is detected by their interaction forces; but they are not simply mathematical terms but are regarded as physical quantities which may be measured by the forces exerted by them on single charges or diploes.

2.7 ELECTRIC POTENTIAL

The electric field produced by a charge can be described in two ways-

(i) by the intensity of electric field $\vec{E}$ at a point in the field and
(ii) by the electric potential V

The intensity of electric field $\vec{E}$ is a vector quantity while electric potential V is scalar. Both these quantities are inter-related. In the study of electric field, the electric potential is an extremely important quantity. Both of them are the characteristic properties of a point in the field.

We know that in an electric field, a free positive charge tends to move along the direction of the electric field. When a positive test charge is brought opposite to the direction of electric field, work is done against the Coulomb's force of repulsion. To define absolute potential at any point, the potential at infinity is assumed to be zero.

"The electric potential at any point in an electric field is defined as the work done by external force in carrying unit positive test charge from infinity to that point, without any acceleration".

Let W is the work done in bringing positive test charge q_0 from infinity to any point in electric field, then electric potential at that point is-

$$V = \frac{W}{q_0} \qquad \text{.....(12)}$$

The electric potential is a scalar quantity. Its S.I. unit is Joule/Coulomb. It's another unit is volt.

If q_0 =1 coulomb, W= 1 Joule, then

$V = \frac{1\ \text{Joule}}{1\ \text{Coulomb}} = 1$ volt

i.e. 1 volt is the electric potential at a point in an electric field if the work done in bringing one coulomb of electric charge from infinity to that point is 1 joule, provided the charge of 1 coulomb does not affect the original electric field.

2.7.1 Potential Difference

Let us define electric potential difference between two points in an electric field. **"The ratio of work done by external force in carrying a positive test charge from one point to another in an electric field is called the potential difference between those points".**

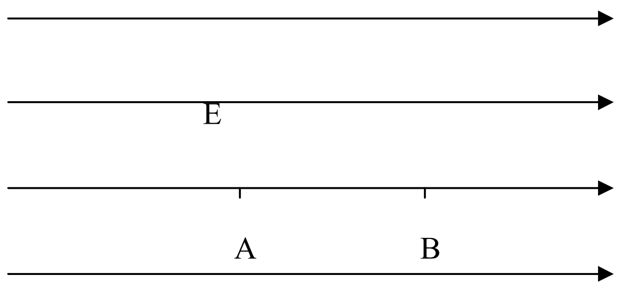


Figure 6

If W_{BA} is the amount of work done in moving the test charge q_0 from B to A against the direction of electric field , then the potential difference between points A and B is given by-

$$V_A - V_B = \frac{W_{BA}}{q_0} \quad \text{.....(13)}$$

or simply $\Delta V = \frac{W}{q}$ .....(14)

Both the work W_{BA} and the charge q_0 are scalars, therefore potential difference $V_A - V_B$ will also be a scalar quantity. If in carrying a positive test charge from the point B to the point A, work is done by an external agent against the electric force, then the potential of point A is said to be higher than the potential of point B. In figure 6, the electric potential of point A is higher than the potential of point B. This also means that in an electric field a free positive charge moves from a region of higher potential to a region of lower potential. Conversely, a free negative charge moves from lower potential to higher potential.

If source charge (charge producing the electric field) is –q, then in taking the positive test charge q_0 from point B to point A work would have been done by the electric force itself. In that case the electric potential of the point A would have been lower than the potential of the point B.

The unit of work done W_{BA} is Joule and the unit of charge q_0 is coulomb. Therefore, the unit of potential difference is Joule/Coulomb.

Now we can define 1 volt potential difference. If W_{BA}= 1 Joule, q_0 = 1 Coulomb then

$$V_A - V_B = \frac{1\text{ Joule}}{1\text{ Coulomb}} = 1\text{ volt}$$

i.e. if 1 joule of work is done in carrying a test charge of 1 Coulomb from one point to the other in an electric field, then the potential difference between those points will be 1 volt.

2.7.2 Physical Significance of Electric Potential

Positive charge always flows from higher potential to lower potential just as a liquid always flows from higher pressure (or higher level) to lower pressure (or lower level) or heat always flows from higher temperature to lower temperature. There is no relation of direction of flow of charge with the quantity of charge as in the case of liquid flow or heat flow. Thus, the electric potential is that physical quantity which determines the direction of flow of positive charge. When we put two conducting bodies of unequal potentials in contact, the charge continues to flow from one body to another until their potentials become equal. The positive charge always flows from higher potential to lower potential, while negative charge always flows from lower to higher potential. When two conductors are kept in contact, the electrons flow from lower potential to higher potential until their potentials become equal.

Example 1: Compute the electric field intensity at a point 20 cm away in vacuum from an electric charge of 4×10^{-9} C.

Solution: Given r = 20 cm= 0.20 m, q= 4×10^{-9} C
The intensity of electric field E is given as-

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$= 9\times10^9\times\frac{4\times10^{-9}}{(0.20)^2} = 900\text{N/C}$$

Example 2: An electron covers a distance of 60 mm when accelerated from rest by an electric field of intensity 2×10^4 N/C. Calculate the time of travel. (The mass of electron= 9×10^{-31} Kg, Charge on electron = 1.6×10^{-19} C respectively.

Solution: Given s= 60 mm= 0.06 m, E = 2×10^4 N/C, m= 9×10^{-31} Kg, q= 1.6×10^{-19} C

Electric force on electron F = qE = $1.6\times10^{-19}\times2\times10^4 = 3.2\times10^{-15}$ N

Acceleration experienced by electron $a = \frac{F}{m} = \frac{3.2\times10^{-15}}{9\times10^{-31}} = 3.5\times10^{15}$ m/sec^2

Now, using second equation of motion $s = ut + \frac{1}{2}at^2$

$0.06 = 0\times t + \frac{1}{2}\times 3.5\times 10^{15}\times t^2$

or $0.06 = \frac{1}{2}\times 3.5\times 10^{15}\times t^2$

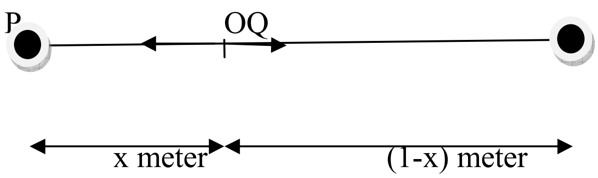
or $t^2 = 0.03\times 10^{-15} = 0.3\times 10^{-16}$

or $t = 0.54\times 10^{-8}$ sec

Example 3: Two point charges of 5×10^{-19} C and 20×10^{-19} C are separated by a distance 1 meter. At which point on the line joining them, the electric field is zero? If a charge 12×10^{-19} C is placed at this point then what will be the force acting on it?

Solution: Let the two charges are placed at point P and point Q and the electric field at a point O between them is zero. Let the distance of point O from point P is x meter then the distance of O from point Q will be (1-x) meter.

5×10^{-19} C$E_2E_1$$20\times10^{-19}$



The electric field at point O due to charge at P, $E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

$= 9\times10^9\times\frac{5\times10^{-19}}{x^2}$ (along PO)

Similarly, the electric field at point O due to charge at Q, $E_2 = 9\times10^9\times\frac{20\times10^{-19}}{(1-x)^2}$ (along QO)

Obviously, both electric fields are oppositely directed. If the resultant electric field at O is zero, then $E_1 = E_2$

$9\times10^9\times\frac{5\times10^{-19}}{x^2} = 9\times10^9\times\frac{20\times10^{-19}}{(1-x)^2}$

$4x^2 = (1-x)^2$

or $2x = \pm(1-x)$

or $2x = (1-x)$ or $-(1-x)$

or $x = 1/3$ meter or $x = -1$ meter

$x = -1$ meter is not possible because the point O is between point P and point Q . Therefore $x = 1/3$ meter is the distance of point O from point P where the resultant electric field will be zero.

Since at point O, resultant electric field E = 0, therefore force on charge 12×10^{-19} C at point O is-

$F = qE = 12\times10^{-19} \times 0 = 0$

i.e. the net force acting on the charge 12×10^{-19} C at point O is zero.

Example 4: If 40 Joule work is done in bringing a charge 4×10^{-19} C from infinity to a point in electric field, what is the potential at that point?

Solution: Given W = 40 Joule, $q = 4\times10^{-19}$ C

Using $V = \frac{W}{q} = \frac{40}{4\times10^{-19}} = 10^{20}$ volt

Example 5: How much work is done in bringing a charge of 2.5×10^{-6} C from one point to another, if the potential difference between the two points is 4 volt?

Solution: Given $q = 2.5\times10^{-6}$ C, $\Delta V = 4$ volt

Using, $\Delta V = \frac{W}{q}$

or $W = \Delta V\times q = 4\times2.5\times10^{-6} = 10^{-5}$ Joule

Self Assessment Question (SAQ) 1: Calculate the electric field intensity at a point where a charge of 5×10^{-4} C experiences a force of 2.25 N.

Self Assessment Question (SAQ) 2: Anα-particle is kept in an electric field of 1.5×10^{5}N/C. Calculate the force on the particle.

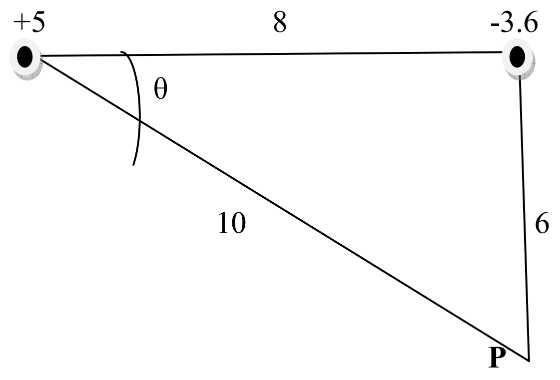
Self Assessment Question (SAQ) 3: What is the intensity of electric field due to a helium nucleus at a distance of 1 A^0 from the nucleus?

Self Assessment Question (SAQ) 4: A point charge of 6×10^{-8} C is situated at the coordinate origin. How much work will be done in taking an electron from the point $x_1$meter to x_2 meter where potential difference is 50 volt?

Self Assessment Question (SAQ) 5: The electric field intensity at a point on the line joining two point charges is zero. What conclusion can you draw about the charges?

Self Assessment Question (SAQ) 6: Calculate the acceleration of an electron in an electric field of 9×10^{5} N/C. The charge on an electron is 1.6×10^{-19} C and its mass is 9.1×10^{-31} Kg.

Self Assessment Question (SAQ) 7: In the given diagram, calculate the resultant intensity of electric field at the point P due to all charges. The charges are in μC and the distances in cm. If a charge 1 μC is placed at point P, what will the force on this charge? Also give the direction of force acting on the charge.



2.8 ELECTRIC POTENTIAL AS LINE INTEGRAL OF ELECTRIC FIELD

Let us consider a region in electric field. The intensity of electric field at any point is specified by$\vec{E}$. Let a positive test charge q_0 be displaced from point P to point Q, opposite to the direction of electric field. Then the external force on test charge $\vec{F} = - q_0\vec{E}$

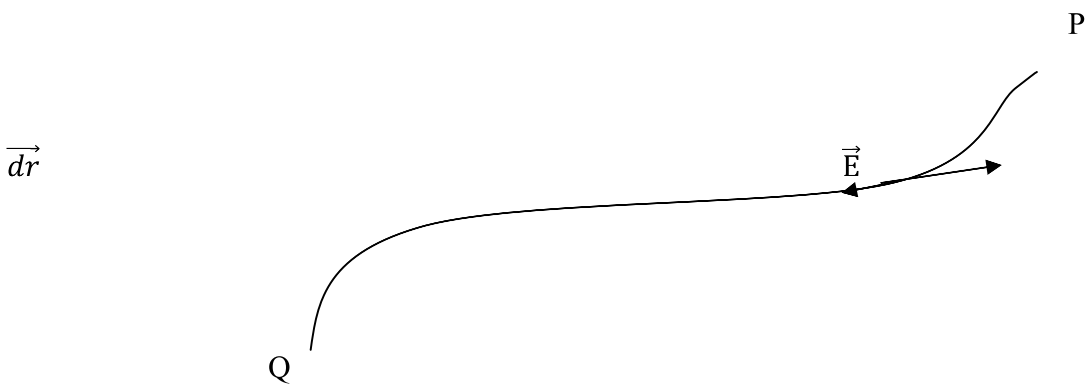


Figure 7

Therefore, the work done in displacing the test charge through a small displacement $\overrightarrow{dr}$ will be-

$$dW = \vec{F}.\overrightarrow{dr} = - q_0\vec{E}.\overrightarrow{dr}$$

The total work done in displacing the charge from point P to point Q is

$$W_{PQ} = -q_0\int_P^Q \vec{E}.\overrightarrow{dr}$$

where the integral extends along the path from P to Q.

Therefore, the potential difference between two points P to Q will be-

$V_Q - V_P = \frac{W_{PQ}}{q_0} = -\int_P^Q \vec{E}.\overrightarrow{dr}$.....(15)

If the point P is taken at infinity, the reference level for zero potential, i.e. V_P = 0, then the potential at point Q, $V_Q = -\int_{\infty}^{Q} \vec{E}.\overrightarrow{dr}$ …..(16)

Thus, the electric potential at any point in an electric field is defined as the negative of line integral of electric field from infinity to given point.

2.9 ELECTRIC FIELD AS NEGATIVE GRADIENT OF POTENTIAL

P δr Q

(x, y, z) (x+δx, y+δy, z+δz)

Figure 8

Let us consider that V and V + δV are the electric potential at two neighbouring points P and Q having coordinates (x, y, z) and (x+δx, y+δy, z+δz) respectively.

Since electric potential V is a function of (x, y, z) i.e., V = V (x, y, z), then the potential difference between points P and Q may be written in the following general form-

$\Delta V = \frac{\partial V}{\partial x}\delta x + \frac{\partial V}{\partial y}\delta y + \frac{\partial V}{\partial z}\delta z$

Or, $(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}).(\hat{i}\delta x + \hat{j}\delta y + \hat{k}\delta z) = \vec{\nabla} V.d\vec{r}$ …..(17)

If $\vec{E}$ is the electric field intensity in the region of points P and Q, then by definition, the potential difference between two points P and Q separated by distance $\delta\vec{r} = (\hat{i}\delta x + \hat{j}\delta y + \hat{k}\delta z)$ is given by-

$\Delta V = -\vec{E}\,.\,\overrightarrow{\delta r}$ …..(18)

Comparing equation (17) and equation (18), we get-

$-\vec{E}\,.\,\overrightarrow{\delta r} = \nabla V.\overrightarrow{\delta r}$

or $(\vec{E} + \nabla V).\overrightarrow{\delta r} = 0$

Since $\overrightarrow{\delta r}$ is arbitrary, we must have-

$\vec{E} + \vec{\nabla} V = 0$

or $\vec{E} = -\vec{\nabla} V$ = - grad V …..(19)

Thus, the electric field intensity at any point is equal to the negative gradient of the potential at that point.

Equation (19) can be written in terms of components as-

$$\vec{E} = \hat{i}E_x + \hat{j}E_y + \hat{k}E_z = -(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}) \quad .....(20)$$

Comparing coefficients, we get-

$$E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z} \quad .....(21)$$

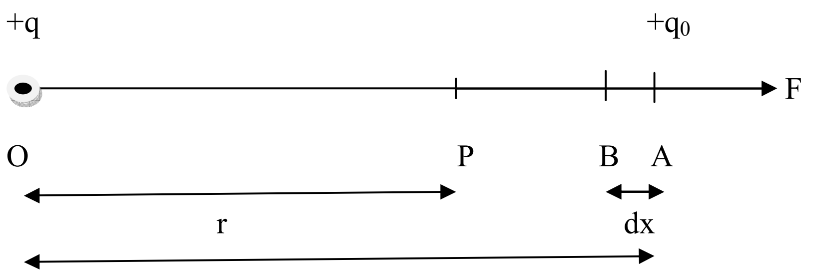
In general, $E = -\frac{dV}{dr}$

2.10 CALCULATION OF ELECTRIC POTENTIAL

In this section, we shall calculate the electric potential in different cases. Let us discuss one by one.

2.10.1 Due to a Point-charge

Let us consider a charge of +q coulomb is placed at a point O (as shown in figure) in air (or vacuum). Let P is the point at which the electric potential is to be calculated. The distance of point P from O is r.



x

Figure 9

Let a test charge $+q_0$ is placed at point A, distant x from point O and away from point P.

By Coulomb's law, the electric force acting on q_0 is given by-

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{x^2} \quad \text{(along OA)}$$

Let us consider another point B at a distance dx from point A towards O (i.e. at a distance –dx from A). Then the work done in bringing the test charge $+q_0$ from point A to point B against the force F is-

$$dW = \vec{F}.\overrightarrow{dx} = F\,dx\cos 180^0$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{x^2}(dx) = -\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{x^2}dx$$

Therefore, the total work done in bringing the test charge $+q_0$ from infinity to P is-

$$W = \int_{\infty}^{r}\left(-\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{x^2}\,dx\right) = -\frac{1}{4\pi\varepsilon_0}q\,q_0\int_{\infty}^{r}\left(-\frac{dx}{x^2}\right)$$

$$= -\frac{1}{4\pi\varepsilon_0}q\,q_0\left[\frac{1}{x}\right]_{\infty}^{r} = -\frac{1}{4\pi\varepsilon_0}q\,q_0\left[\frac{1}{r}-\frac{1}{\infty}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r}$$

By definition, the electric potential at point P,

$$V = \frac{W}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

or

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad \text{.....(22)}$$

If the system is in a medium of dielectric constant K, then

$$V = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{r} \qquad \text{.....(23)}$$

Similarly, the electric potential at point P due to a charge –q is given as-

$$V = -\frac{1}{4\pi\varepsilon_0 K}\frac{q}{r} \qquad \text{.....(24)}$$

2.10.2 Due to a system of point charges

Electric potential, being a scalar quantity, has no direction. Therefore, the electric potential at any point due to a group of point charges is found by calculating the potential due to each charge and then adding algebraically the quantities so obtained.

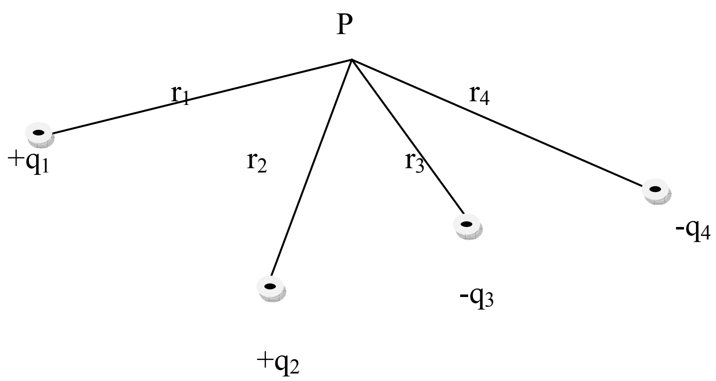


Figure 10

If a point P is at distances r_1, r_2, r_3, r_4 ……… from the point charges $+q_1$, $+q_2$, $-q_3$, $-q_4$,………respectively, then the resultant electric potential at that point will be-

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} - \frac{q_3}{r_3} - \frac{q_4}{r_4} \dots\dots\dots\dots\right] \quad .....(25)$$

If there are n point charges, then the electric potential due to them at a point P will be-

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i}^{n}\frac{q_i}{r_i} \quad .....(26)$$

Where r_i is the distance of the point from the charge q_i.

2.10.3 Due to a continuous charge Distribution

If the charge distribution be continuous, then the summation in the above expressions will be replaced by integration i.e.

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} \quad .....(27)$$

Where dq is a differential element of the charge distribution and r is its distance from the point at which V is to be calculated.

If the charge distribution is linear charge of charge per unit length (λ), then the charge on length element dl is dq = λ dl, then

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{\lambda dl}{r} \quad .....(28)$$

If the charge is distributed continuously over an area S, then dq = σ dS, where σ is surface density of charge. Then, we have $V = \frac{1}{4\pi\varepsilon_0}\int\int\frac{\sigma dS}{r}$ .....(29)

Where integral is surface integral.

Similarly, if the charge is distributed continuously within a volume V, then

$$V = \frac{1}{4\pi\varepsilon_0}\int\int\int\frac{\rho dV}{r} \quad .....(30)$$

Where ρ is the volume charge density and integral is volume integral.

Example 6:Calculate the electric potential due to pint charge $+1.1\times10^{-9}$ C at a distance of 100 mm.

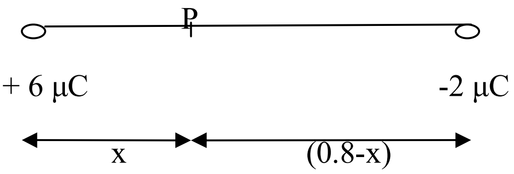
Solution: Given, $q = +1.1\times10^{-9}$ C, r = 100 mm = 0.1 m

Using $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$, the electric potential $V = 9\times10^{9}\times\frac{1.1\times10^{-9}}{0.1}$

$$= +99 \text{ V}$$

Example 7: Two point charges + 6 μC and -2 μC are 0.8 m apart. Locate the point at which the electric potential is zero.

Solution: Let us suppose that at point P, the electric potential is zero. Let the distance of point P from charge + 6 μC is x. Obviously, the distance of point P from charge will be (0.8-x) m.



The electric potential at point P due to charge + 6 μC is, $V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

$$= 9\times10^9 \times\frac{6\times10^{-6}}{x}\ \text{V}$$

Similarly, the electric potential at point P due to charge -2 μC, $V_2 = 9\times10^9 \times\frac{(-2)\times10^{-6}}{(0.8-X)}$ V

$$= -\ 9\times10^9 \times\frac{2\times10^{-6}}{(0.8-X)}$$

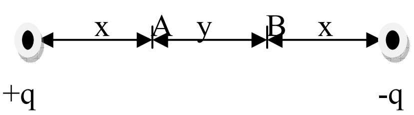
Since the electric potential at P is zero, it means the algebraic sum of V_1 and V_2 should be zero i.e, $V_1 + V_2 = 0$

or $$9\times10^9 \times\frac{6\times10^{-6}}{x} - 9\times10^9 \times\frac{2\times10^{-6}}{(0.8-X)} = 0$$

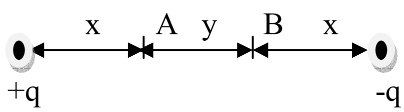
or $$\frac{3}{x} = \frac{1}{(0.8-x)}$$

or $$x = 0.6 \text{ m.}$$

Example 8: Determine the value of $V_A - V_B$ in the given arrangement.



Solution:



The electric potential at point A due to charge +q, $V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = 9\times10^9 \times\frac{q}{X}$

Similarly, the electric potential at A due to charge –q, $V_2 = 9\times10^9 \times\frac{(-q)}{(x+y)} = -\ 9\times10^9 \times\frac{q}{(x+y)}$

Total electric potential at point A, $V_A = V_1 + V_2 = 9\times10^9 \times\frac{q}{x} - 9\times10^9 \times\frac{q}{(x+y)} = 9\times10^9 \times[\frac{q}{x} - \frac{q}{(x+y)}]$

Similarly, the electric potential at point B due to charges +q and –q are $9\times10^9 \times\frac{q}{(x+y)}$ and $-\ 9\times10^9 \times\frac{q}{x}$ respectively.

The total electric potential at point B, $V_B = 9\times10^9 \times\frac{q}{(x+y)} - 9\times10^9 \times\frac{q}{x} = 9\times10^9 \times[\frac{q}{(x+y)} - \frac{q}{x}]$

Therefore, $V_A - V_B = 9\times10^9 \times[\frac{q}{x} - \frac{q}{(x+y)}] - 9\times10^9 \times[\frac{q}{(x+y)} - \frac{q}{x}]$

$$= 9\times10^9 \times[\frac{q}{x} - \frac{q}{(x+y)} - \frac{q}{(x+y)} + \frac{q}{x}] = 9\times10^9 \times\frac{2qy}{x(x+y)}$$

or $$V_A - V_B = \frac{1}{4\pi\varepsilon_0}\frac{2qy}{x(x+y)}$$

Self Assessment Question (SAQ) 8: The electric field intensity is zero at a point. Will the electric potential be necessarily zero at that point?

Self Assessment Question (SAQ) 9: The electric potential is constant throughout a given region of space. What is the electric field intensity in that region?

2.11 THE ELECTRIC FLUX

The electric flux through a surface is defined as the total number of electric lines of force passing through that surface normally.

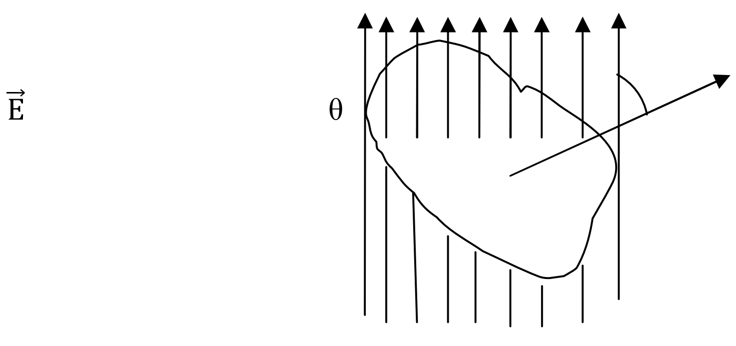


Figure 11

The electric flux through an elementary area dS is defined as the dot product (or scalar product) of electric field and the surface area i.e.

Electric flux $d\emptyset = \vec{E}\,.\,\overrightarrow{dS}$ …..(31)

Let θ be the angle between the direction of electric field $\vec{E}$ and the direction of surface area $\overrightarrow{dS}$ then, $d\emptyset = E\,dS\,\cos\theta$ …..(32)

The total electric flux through entire surface $\vec{S}$ is obtained by adding up the scalar quantity $\vec{E}\,.\,\overrightarrow{dS}$ for all elements of area into which the surface has been divided.

Thus the total electric flux $\emptyset = \sum\vec{E}\,.\,\overrightarrow{dS}$ …..(33)

If the surface is continuous and electric field is different at different surface elements, then summation in equation (33) is replaced by integration, therefore the total electric flux through the entire surface, $\phi = \int_S \vec{E}.d\vec{S}$ …..(34)

Electric flux is a scalar quantity. Its unit is Newton-metre^{2}Coulomb^{-1}.

2.12 THE GAUSS'S THEOREM

Karl Friedrich Gauss gave a theorem that relates total outward electric flux through a hypothetical closed surface. This theorem is known after his name Gauss's theorem. The hypothetical closed surface is called Gaussian surface.

Gauss's theorem states that the net outward normal electric flux through a closed surface of any shape is equal to $1/\varepsilon_0$ times the total charge contained within that surface, i.e.

$$\oint_S \vec{E}.d\vec{S} = \frac{1}{\varepsilon_0}\sum q \quad …..(35)$$

Where $\oint_S$ indicates the surface integral over whole of the closed surface, $\sum q$ is the algebraic sum of all the charges (i.e. net charge in coulombs) enclosed by the surface S.

Proof: Let us first proof Gauss's theorem for internal point.

Direction of normal

$d\omega \overrightarrow{dS}$ θ

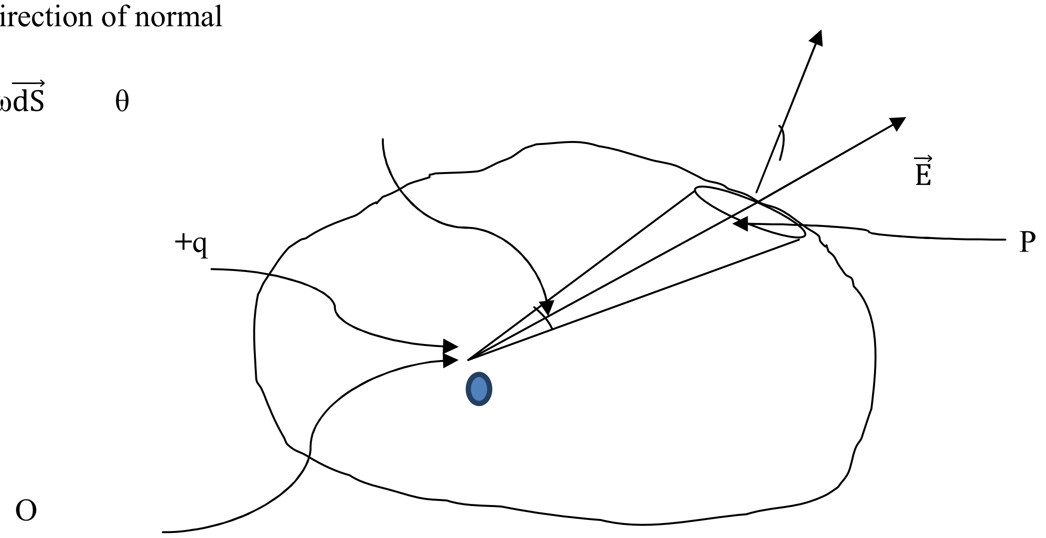


Figure 12

Let a point charge +q coulomb be placed at point O within the closed surface. Let $\vec{E}$ be the electric field intensity at point P. Let OP = r and the permittivity of the free space or vacuum be ε_0.

Let us consider a small area $\overrightarrow{dS}$ of the surface surrounding the point P. The electric flux through $\overrightarrow{dS}$ is-

$$d\emptyset = \vec{E} \cdot \overrightarrow{dS} \qquad \text{.....(i)}$$

Electric field intensity at point P, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{q\vec{r}}{r^3}$

Therefore, from equation (i), $d\emptyset = \frac{1}{4\pi\varepsilon_0} q\frac{\vec{r}.\overrightarrow{dS}}{r^3}$ .....(ii)

But $\frac{\vec{r}.\overrightarrow{dS}}{r^3} = \frac{dSCos\theta}{r^2}$ = solid angle subtended by area dS at point O.

θ = angle between $\vec{E}$ and $\overrightarrow{dS}$

From equation (ii), we get-

$$d\emptyset = \frac{1}{4\pi\varepsilon_0} q \;\; d\omega = \frac{q}{4\pi\varepsilon_0} \; d\omega$$

Hence electric flux through entire closed surface-

$$\emptyset = \oint_S \vec{E} \cdot \overrightarrow{dS} = \frac{q}{4\pi\varepsilon_0}\oint d\omega \qquad \text{.....(iii)}$$

But $\oint d\omega$ is the solid angle due to the entire closed surface S at an internal point O = 4π

Therefore, from equation (iv), $\emptyset = \frac{q}{4\pi\varepsilon_0} 4\pi = \frac{1}{\varepsilon_0} q$

If there are many charges $+q_1$, $+q_2$, $+q_3$, ...$-q_1'$, $-q_2'$, $-q_3'$,..........inside the closed surface, each charge will contribute to the electric flux. For positive charges, the flux will be outward and hence positive; for negative charges, the flux will be inward and negative. Therefore, the total electric flux in such a case is-

$$\emptyset = \frac{1}{\varepsilon_0} q_1 + \frac{1}{\varepsilon_0} q_2 + \frac{1}{\varepsilon_0} q_3 + \ldots\ldots\ldots - \frac{1}{\varepsilon_0} q_1' - \frac{1}{\varepsilon_0} q_2' - \frac{1}{\varepsilon_0} q_3' - \ldots\ldots\ldots.$$

$$= \frac{1}{\varepsilon_0}(q_1 + q_2 + q_3 + \ldots\ldots\ldots - q_1' - q_2' - q_3' - \ldots\ldots\ldots.)$$

$$= \frac{1}{\varepsilon_0}\sum q$$

Thus $\phi = \frac{1}{\varepsilon_0}\sum q$

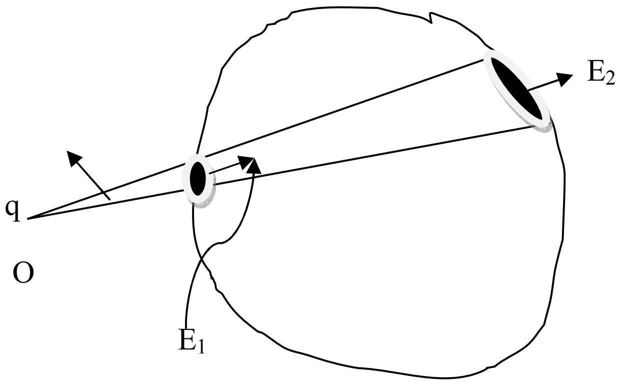
Where $\sum q$ is the algebraic sum of the charges within the closed surface.

Hence, net electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times the total charge (in coulomb) enclosed within the surface which is Gauss's theorem.

Now let us proof Gauss's theorem for external charge.

Let us consider a closed surface S enclosing no charge, charge q is placed at the external point O. We construct a cone of lines of force from charge q to cut the surface. The surface of any shape is intersected in an even number of patches (here 2); the contribution to electric flux due to these intersecting surfaces are $-\frac{q}{4\pi\varepsilon_0}$ dω and $+\frac{q}{4\pi\varepsilon_0}$ dω, therefore that flux through the surface is zero.Hence, charges external to Gaussian surface do not contribute to electric flux. Thus, Gauss's theorem $\oint_S \vec{E} \cdot \vec{dS} = \frac{1}{\varepsilon_0} \times$ charge enclosed by surface, is true whether external charges are present or not.

dω



S

Figure 13

If the system is in a medium of dielectric constant K, then Gauss's theorem can be written as-

$\int_S \vec{E} \cdot \vec{dS} = \frac{1}{\varepsilon}\sum q$ O

$$= \frac{1}{\varepsilon_0 K}\sum q \qquad .....(36)$$

It is to be noted that Gauss's theorem remains valid as such even for charges in motion. Moreover it is applicable to any field obeying inverse square law.

2.13 APPLICATIONS OF GAUSS'S THEOREM

It is very interesting that that Gauss's theorem provides a convenient method for determination of electric field intensity in symmetrical cases. Here, we consider a imaginary Gaussian surface symmetrical to given charge, compute electric flux through this Gaussian surface and equate this flux to $\frac{1}{\varepsilon_0}$ × charge enclosed by the surface. Now let us discuss some important applications of Gauss's theorem.

2.13.1 Electric Field due to a Point-charge

Let us consider a point charge q coulomb placed at point O. We have to find out the electric field intensity due to this charge at a point P distant r from it. Let us consider a closed spherical surface with centre at O and the point P lying on it. By symmetry the electric field $\vec{E}$ has the same magnitude all over the surface and points everywhere normally outwards.

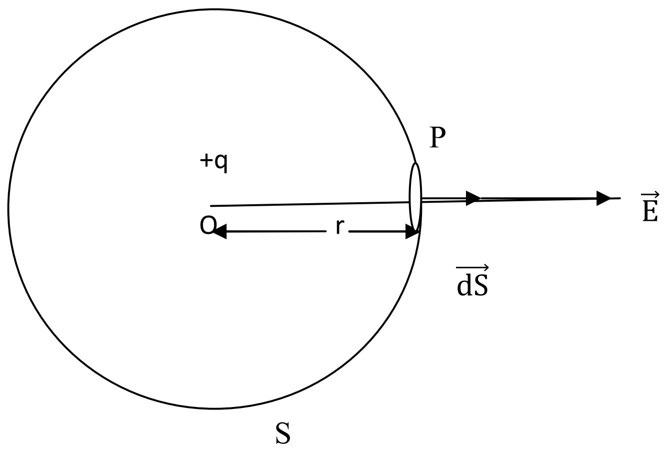


Figure 14

Electric flux through the spherical surface, $\emptyset = \oint_S \vec{E} \cdot \vec{dS} = \oint_S E\, dS \cos\theta$

$$= E \oint_S dS = E\,(4\pi\, r^2)$$

(Since $\oint_S dS = 4\pi\, r^2$, total surface area of sphere)

Charge enclosed by the surface = q

According to Gauss's theorem,

$$\oint_S \vec{E} \cdot \vec{dS} = 1/\varepsilon_0 \sum q$$

$$E\,(4\pi\, r^2) = \frac{1}{\varepsilon_0}\, q$$

or $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

In vector form, we can write-

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\,\vec{r}$$

This is the expression for electric field intensity due to a point charge using Gauss's theorem.

2.13.2 Electric Field due to a charged spherical shell

Let us consider a thin spherical shell of radius R and carrying charge Q with centre O. Let us first calculate the electric field outside the charged spherical shell.

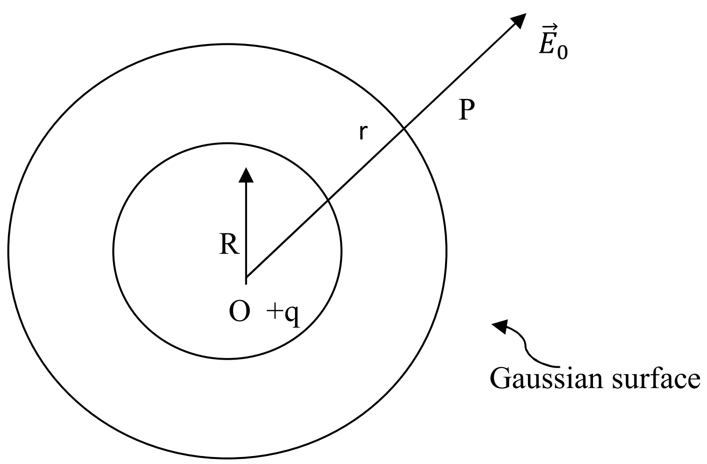


Figure 15

Let us consider a point P at a distance r outside the shell. Let us draw an imaginary spherical surface of radius r = OP, concentric with the shell. By symmetry the electric field E_0 at each point of surface is same and is directed radially outward. Let the value of electric field at the surface be E_0.

The net electric flux through the entire surface, $\varnothing = \oint_S \vec{E}_0 \cdot \vec{dS} = \oint_S E_0\, dS\, \text{Cos}0$

$= \oint_S E_0 \, dS = E_0 \oint_S dS = E_0 4\pi r^2$ (Since $\oint_S dS = 4\pi r^2$, total surface area of sphere)

Total charge enclosed by the surface = + q

Using Gauss's theorem, $\emptyset = \oint_S \vec{E} \cdot \vec{dS} = 1/\varepsilon_0 \sum q$

$$E_0 \, 4\pi r^2 = 1/\varepsilon_0 \sum q$$

or $$E_0 = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$$

Which is same if the charge Q was kept at the centre O. Hence the electric field intensity at a point outside a charged spherical shell is same as though the charge was kept at the centre O.

Now, let us calculate the electric field inside the charged spherical shell. For this, let us consider a point P' inside the shell at a distance of r i.e. r < R. Let us draw an imaginary Gaussian surface of radius r(r = OP'), concentric with the shell. If $\vec{E_i}$ is the electric field inside the shell, then by symmetry $\vec{E_i}$ is same at each point of spherical surface and is directed radially outward.

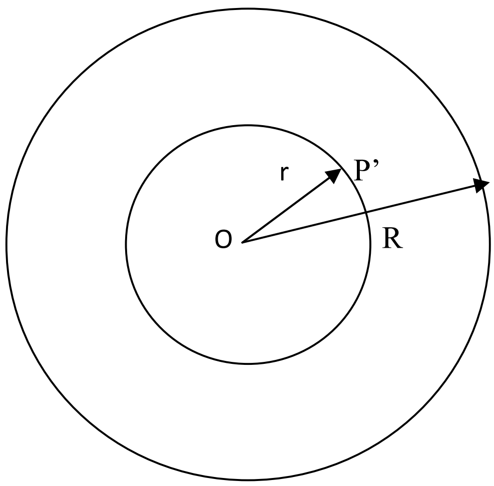


Figure 16

Net electric flux through the surface, $\emptyset = \oint_S \vec{E_i} \cdot \vec{dS} = \oint_S E_i \, dS \, Cos0$

$= \oint_S E_i \, dS = E_i \oint_S dS = Ei \, 4\pi r^2$ (Since $\oint_S dS = 4\pi r^2$, total surface area of sphere)

Total charge enclosed by the surface $\sum q = 0$

Therefore, using Gauss's theorem, we have-

$$\emptyset = \oint_S \vec{E} \cdot \overrightarrow{dS} = 1/\varepsilon_0 \sum q$$

$$E_i \, 4\pi \, r^2 \quad = 0$$

or $$E_i = 0$$

Thus electric field intensity at each point within the shell is zero.

Example 9: How much electric flux will come out through surface $\overrightarrow{dS}$ = 5 k^ kept in an electric field $\vec{E}$ = 3i^ + 7j^ + 4k^ ?

Solution: Here $\overrightarrow{dS}$ = 5 k^, $\vec{E}$ = 3i^ + 7j^ + 4k^

Electric flux , $\emptyset = \vec{E} \cdot \overrightarrow{dS}$ = (3i^ + 7j^ + 4k^). 5 k^

= 20 units

Example 10: 1 coulomb charge is kept at the centre of a cube of side 5 cm. Find out the electric flux coming out of any face of the cube.

Solution: Given q = 1 coulomb (the charge enclosed by the surface)

Net flux through the cube, $\emptyset = 1/\varepsilon_0 \times$ total charge enclosed by the surface

$$= 1/\varepsilon_0 \times 1 = 1/\varepsilon_0$$

Cube has six faces. By symmetry the electric flux through each of cube face will be same. Hence the electric flux through a face of cube= $\frac{1}{6} \times 1/\varepsilon_0 = \frac{1}{6\varepsilon_0} = 1.884 \times 10^{10}$ N-m^2/C^2

Self Assessment Question (SAQ) 10: A charge q is kept at the centre of a cube of side 'a'. What is the electric flux through any one face of cube?

Self Assessment Question (SAQ) 11: Choose the correct option-

The electric field intensity inside a spherical shell is-

(a) Always zero (b) sometimes zero (c) infinite (d) none of these

Self Assessment Question (SAQ) 12: State True or False-

Gauss's law is basically equivalent to Coulomb's law.

2.14 SUMMARY

In the present unit, we have studied the concept of electric field, electric lines of force and their properties. We learnt that an electric line of force is that imaginary smooth curve drawn in an electric field along which a free, isolated unit positive charge moves. The tangent drawn at any point on the electric line of force gives the direction of the force acting on a positive charge placed at that point. We have also learnt that no two electric lines of forces can intersect each other. We have established the expressions for electric field intensity and electric potential due to a point charge, a system of point charges and continuous charge distribution. We have studied and analysized the physical significance of electric field intensity. In the present unit, we have learnt about electric potential and electric potential difference. We have learnt that the electric field intensity at any point is equal to the negative gradient of the potential at that point. In the present unit we have learnt that the electric flux through a surface is defined as the total number of electric lines of force passing through that surface normally. We have studied and proved Gauss's theorem in electrostatics. We have derived some expressions for electric field intensity using Gauss's theorem. Several solved examples are given in the unit to make the concepts clear. To check your progress, self assessment questions (SAQs) are given place to place.

2.15 GLOSSARY

Experience- occurrence

Set-up- arrangement

Perturb- disturb, agitate

Characteristic- properties

Independent- autonomous, free

Significance- implication, importance

Exert- apply, put forth, bring to bear

2.16 TERMINAL QUESTIONS

1. Give the concept of electric field.

2. Draw electric lines of force due to an isolated negative charge.

3. Define electric lines of force and discuss their important properties.

4. Two electric lines of force never intersect each other. Why?

5. Establish the expression for electric field intensity at a point due to a point charge.

6. Explain the physical significance of electric field intensity.

7. Define potential difference between two points. Hence define electric potential at a point.

8. A charge $+7 \times 10^{-19}$ C is moved between two points. The potential difference between those points is zero. Estimate the work done in this process.

9. What is the physical significance of electric potential?

10. Prove that the electric potential is the negative of line integral of electric field.

11. Give the derivation of the electric field from electric potential.

12. Prove, $\vec{E}$ = - grad V

13. How does the electric potential due to a point charge vary with distance?

14. Establish an expression for electric potential due to a point charge.

15. Calculate the electric potential at the centre of a square of side 'a' which carries at its four corners charges q_1, q_2, q_3 and q_4.

16. What is electric flux? What is its unit? Give its significance.

17. State and proof Gauss's theorem in electrostatics.

18. Using Gauss' theorem, prove that the electric field intensity due to a charged spherical shell at a point outside the shell is given by-

$E_0 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$, where Q is the charge on shell and r is the distance of a point outside from the centre of shell.

19. Establish the expression for electric field intensity due to a point charge at a distance r as an application of Gauss's theorem.

2.17 ANSWERS

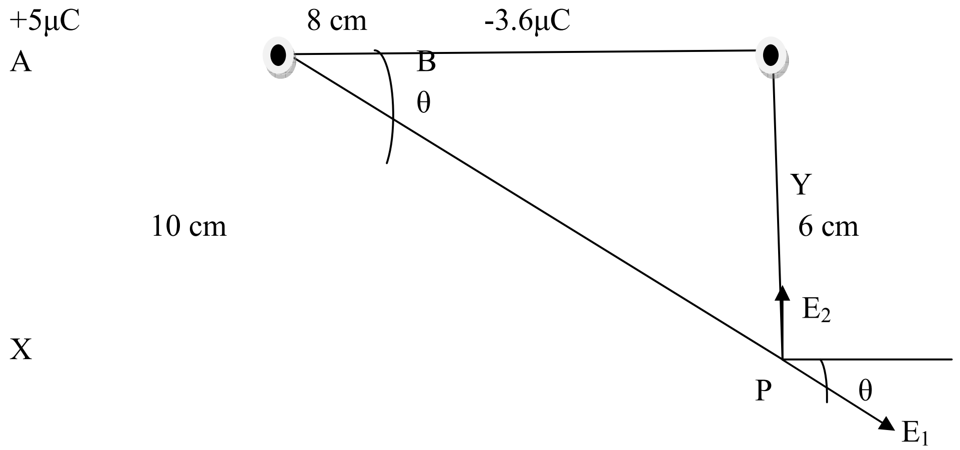
Self Assessment Questions (SAQs):

1. Given $q = 5\times 10^{-4}$ C, F = 2.25 N
   Using F = qE, the intensity of electric field $E = F/q = 2.25/5\times 10^{-4} = 4.5\times10^{3}$ N/C
2. Given $E = 1.5\times10^{5}$ N/C, we know that the charge on α-particle $q = +3.2\times10^{-19}$ C
   Using F = qE, the force on α-particle $F = 3.2\times10^{-19}\times1.5\times10^{5} = 4.8\times10^{-14}$ N
3. The helium nucleus has a positive charge equal to that on an α-particle i.e. the charge on helium nucleus $q = + 3.2\times10^{-19}$ C, here $r = 1\ A^0 = 10^{-10}$ meter

We know that $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

$$= 9\times10^{9}\times\frac{3.2\times10^{-19}}{(10)^{-10}} = 2.88\times10^{11} \text{ N/C}$$

4. Here, $q = 6\times10^{-8}$ C, $\Delta V = 50$ volt
   Using $\Delta V = \frac{W}{q}$, the necessary work $w = q\,\Delta V = 6\times10^{-8}\times50 = 3\times10^{-6}$ Joule
5. We can conclude that the charges are similar.
6. Here $q = 1.6\times10^{-19}$ C, $m = 9.1\times10^{-31}$ Kg, $E = 9\times10^{5}$ N/C
   Electric force on electron $F = qE = 1.6\times10^{-19}\times9\times10^{5} = 1.44\times10^{-13}$ N
   Now $F = ma$ or $a = F/m = 1.44\times10^{-13}/9.1\times10^{-31} = 1.58\times10^{17}$ m/sec^2

7. 

The electric field intensity at point P due to charge +5μC, $E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

$$= 9\times10^{9}\times\frac{5\times10^{-6}}{(10\times10^{-2})^2} = 4.5\times10^{6} \text{ N/C (along AP)}$$

Similarly, the electric field intensity at point P due charge -3.6μC, $E_2 = 9\times10^{9}\times\frac{3.6\times10^{-6}}{(6\times10^{-2})^2}$

$$= 9\times10^{6} \text{ N/C (along PB)}$$

Let us resolve E_1 and E_2 into its components considering origin at P and X-axis and Y-axis as shown in figure.

Resultant electric field intensity along X-axis, $E_x = E_{1x} + E_{2x}$

$$= (4.5\times10^{6}\cos\theta) + 0$$

$$= (4.5\times10^{6}\times\frac{8}{10}) = 3.6\times10^{6} \text{ N/C}$$

Similarly, total electric field intensity along Y-axis, $E_y = E_{1y} + E_{2y}$

$= (-4.5\times10^6 \sin\theta) + (9\times10^6)$

$= (-4.5\times10^6 \times \frac{6}{10}) + (9\times10^6)$

$= 6.3\times10^6$ N/C

Resultant electric field intensity at point P, $E = \sqrt{E_x^2 + E_y^2}$

$= \sqrt{(3.6\times10^6)^2 + (6.3\times10^6)^2} = 7.3\times10^6$ N/C

If the resultant electric field intensity at P makes an angle θ with +X-axis, then $\theta = \tan^{-1}\frac{E_y}{E_x}$

$= \tan^{-1}(\frac{6.3\times10^6}{3.6\times10^6}) = \tan^{-1}(1.75)$

The force on 1μC placed at point P, $F = qE = 1\times10^{-6}\times7.3\times10^6 = 7.3$N (in the direction of E)

8. The electric field intensity is zero at a point exactly midway between two equal and similar charges, but the electric potential at that point is twice that due to a single charge. Therefore, the electric potential will not be necessarily zero at that point.
9. We know that $E = -\frac{dV}{dr}$

   But V is constant throughout a given region space i.e. V is constant with r. Therefore, $\frac{dV}{dr} = 0$ and hence E is zero.
10. Total charge enclosed by the surface = q

    Net flux through the cube, $\emptyset = 1/\varepsilon_0 \times$ total charge enclosed by the surface

    $= 1/\varepsilon_0 \times q = q/\varepsilon_0$

    Cube has six faces. By symmetry the electric flux through each of cube face will be same. Hence the electric flux through a face of cube $= \frac{1}{6}\times q/\varepsilon_0 = \frac{q}{6\varepsilon_0}$

11. (a)
12. True

Terminal Questions:

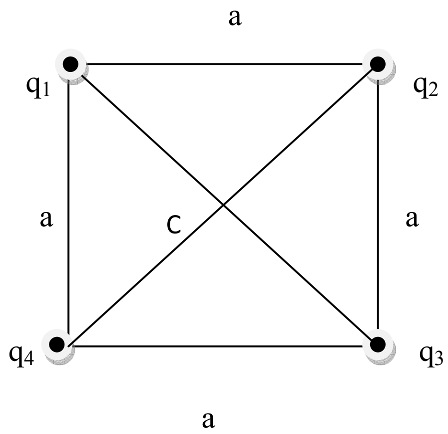
8. The potential difference between two points is zero i.e. $\Delta V = 0$

   The work done in the process $W = q\,\Delta V = q \times 0 = 0$

13. Since $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ i.e. $V \alpha \frac{1}{r}$

Obviously, the electric potential is inversely proportional to distance. The magnitude of electric potential increases with decrease in distance.

15.



The length of diagonal of square = $a\sqrt{2}$

The half of the length of diagonal = $a\sqrt{2}/2 = a/\sqrt{2}$

The electric potential at the centre C due to charge q_1, $V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{\left(\frac{a}{\sqrt{2}}\right)}$

The electric potential at the centre C due to charge q_2, $V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left(\frac{a}{\sqrt{2}}\right)}$

The electric potential at the centre C due to charge q_3, $V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{\left(\frac{a}{\sqrt{2}}\right)}$

The electric potential at the centre C due to charge q_4, $V_4 = \frac{1}{4\pi\varepsilon_0}\frac{q_4}{\left(\frac{a}{\sqrt{2}}\right)}$

The total electric field at the centre of square $V = V_1 + V_2 + V_3 + V_4$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{\left(\frac{a}{\sqrt{2}}\right)} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{\left(\frac{a}{\sqrt{2}}\right)} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{\left(\frac{a}{\sqrt{2}}\right)} + \frac{1}{4\pi\varepsilon_0}\frac{q_4}{\left(\frac{a}{\sqrt{2}}\right)}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}}{a}[q_1 + q_2 + q_3 + q_4]$$

2.18 REFERENCES

1. Engineering Physics, Satya Prakash, Pragati Prakashan, Meerut

2. Nootan Physics, Kumar, Mittal, Nageen Prakashan Pvt. Ltd., Meerut

3. Fundamentals of Physics, David Halliday, Robert Resnick, Jearl Walker, John Wiley & Sons

4. Electricity and Magnetism with Electronics, K.K. Tewari, S. Chand & Company Pvt. Ltd., New Delhi

2.19 SUGGESTED READINGS

1. Concepts of Physics, Part II, HC Verma, Bharati Bhawan, Patna

2. University Physics, Sears, Zemansky, Young, Narosa Publishing House, New Delhi

3. Introduction to Engineering Physics, A.S. Vasudeva, S. Chand & Company Ltd., New Delhi.

UNIT 3 POTENTIAL AND FIELD DUE TO LONG CHARGED WIRE, SPHERE, DISC, ELECTRIC DIPOLE AND ENERGY STORED IN AN ELECTRICFIELD

Structure

3.1 INTRODUCTION

In the previous unit, you have learnt about electric field, electric lines of force and their properties. You have calculated electric field intensity and potential due to a point charge, a system of point charges and a continuous charge distribution. You have also studied electric flux, Gauss's theorem and applications. In the present unit, you will study, calculate, learn and analyze the electric potential and electric field due to an arbitrary charge, long charged wire, sphere and disc. You will also study about electric dipole and calculate the electric field intensity and potential in different cases of electric dipole. In this unit, you will learn also about energy stored in an electric field.

3.2 OBJECTIVES

After studying this unit, you should be able to-

- learn about electric potential and electric field due to various bodies
- learn about electric dipole
- compute electric field intensity and electric potential in various cases
- solve problems based on electric field, electric potential and electric dipole

3.3 ELECTRIC FIELD INTENSITY AND POTENTIAL DUE TO AN INFINITELY LONG CHARGED WIRE

Let us consider a section of an infinitely long straight wire charged uniformly. Let the linear charge density (i.e. charge per unit length) of wire be λ C/m.

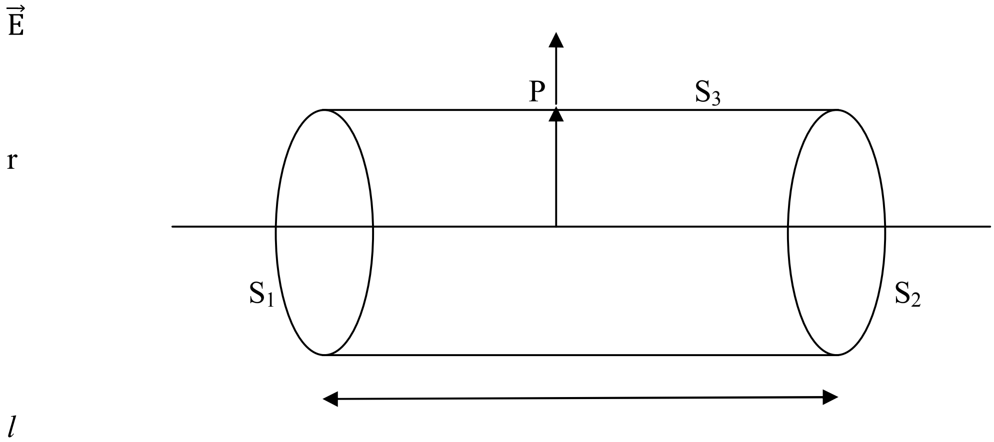


Figure 1

Let us consider an imaginary cylindrical surface (Gaussian surface) of radius r and length l co-axial with the line charge and enclosed by two flat circular surfaces perpendicular to the line charge. By symmetry the electric field intensity $\vec{E}$ is equal in magnitude and is directed normally at every point of the curved cylindrical surface. Obviously, there are three closed surfaces- two flat surfaces S_1 and S_2; one curved cylindrical surface S_3.

Applying Gauss's theorem-

$$\int_S \vec{E} \cdot \vec{dS} = 1/\varepsilon_0 \sum q$$

or $$\oint_{S1} \vec{E} \cdot \vec{dS} + \oint_{S2} \vec{E} \cdot \vec{dS} + \oint_{S3} \vec{E} \cdot \vec{dS} = 1/\varepsilon_0 \sum q$$

or $$\int_{S1} (E\, dS \cos 90^0) + \int_{S2} (E\, dS \cos 90^0) + \int_{S3} (E\, dS \cos 0^0) = 1/\varepsilon_0 \sum q$$

or $$0 + 0 + E \int_{S3} dS = 1/\varepsilon_0 (\lambda \times l) \quad [\text{ since total charge } \sum q = \lambda \times l]$$

or $$E\,(2\pi r l) = 1/\varepsilon_0 \times \lambda l \quad [\text{ since} \oint_{S3} dS = 2\pi r l = \text{total curved surface area}]$$

or $$E = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda}{r} \qquad \text{.....(1)}$$

The equation (1) gives the electric field intensity due to an infinitely long charged wire at a distance r.

Now let us calculate electric potential due to an infinitely long charged wire. In the previous unit, we have learnt that the electric potential at a distance r from the axis is given as-

$$V_r = -\int_\infty^r \vec{E} \cdot \vec{dr} \qquad \text{.....(2)}$$

Here at infinity (reference level), the potential is taken as zero. But in this case, reference distance cannot be taken as infinity since the wire itself extends to infinity. Hence in this case, we shall find the potential difference between two points distant r_1 and r_2 from the wire. We know that Potential difference $V_B - V_A = \frac{W_{PQ}}{q_0} = -\int_P^Q \vec{E} \cdot \vec{dr}$.....(3)

Using above relation, we have the electric potential difference $\Delta V = -\int_{r_2}^{r_1} \vec{E} \cdot \vec{dr}$

$$= -\int_{r_2}^{r_1} E\, dr \cos 0^0 = -\int_{r_2}^{r_1} E\, dr$$

$$= -\int_{r_2}^{r_1} \frac{1}{4\pi\varepsilon_0} \frac{2\lambda}{r} dr = \frac{1}{4\pi\varepsilon_0} 2\lambda \log_e \frac{r_2}{r_1}$$

$$= \frac{\lambda}{2\pi\varepsilon_0} \log_e \frac{r_2}{r_1}$$

Or potential difference $\Delta V = \frac{\lambda}{2\pi\varepsilon_0} log_e \frac{r_2}{r_1}$ …..(4)

The above expression gives the potential difference ΔV (or V_{r1}- V_{r2}) between two points distant r_1 and r_2 .

3.4 ELECTRIC FIELD INTENSITY DUE TO CHARGED SPHERE

Let us consider a non-conducting sphere of radius R. The charge Q is uniformly distributed over it. The charge density of sphere $\rho = \frac{Q}{\frac{4}{3}\pi R^3}$ …..(5)

P is the point at a distance r from the centre of sphere at which electric field intensity is to be determined. Now let us discuss different cases as follows-

Case (i) Point P lies outside the charge distribution, at external point (r >R)

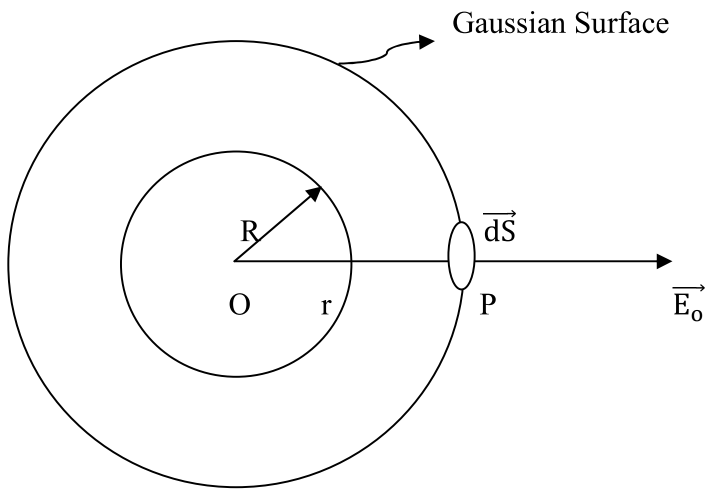


Figure 2

Obviously, OP = r. Let us draw a spherical surface (Gaussian surface) of radius OP = r concentric with the spherical surface. As the electric charge is uniformly distributed, by symmetry the electric field intensity E_O at every point of this spherical surface has the same magnitude and is directed along the outward drawn normal to the entire surface.

Total electric flux through the entire surface = $\int_S \overrightarrow{E_O} . \overrightarrow{dS} = \int_S E_O \, dS \, Cos \, 0^0$

$= \int_S E_O \, dS = E_O \int_S dS$

$= E_O\,(4\pi r^2)$ [since $\int_S dS$ = total surface area of spherical surface = $4\pi r^2$]

Total charge enclosed by the Gaussian surface = Q

Using Gauss's theorem-

$\int_S \vec{E} \cdot \vec{dS} = 1/\varepsilon_0 \sum q$ 0

$$E_O\,(4\pi r^2) = 1/\varepsilon_0 \times Q$$

$$E_O = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \quad .....(6)$$

This is the same if the charge Q was placed at the centre of sphere O. Hence, the electric field intensity at any point outside a spherical charge distribution is the same as through the whole charge were concentrated at the centre.

Case (ii) Point P lies on the surface of spherical charge distribution (r= R)

If point P is on the surface of spherical charge distribution, then r = R i.e. the distance of point P from the centre of sphere is equal to the radius of sphere. In this case, the electric field intensity on the surface of the spherical charge distribution is given as-

$$E_S = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2} \quad .....(7)$$

Case (iii) Point P lies inside the charge distribution, at internal point(r<R)

Let point P is inside the spherical charge distribution. The distance of point P from the centre of sphere is r.

$\vec{E_1}$

R P d

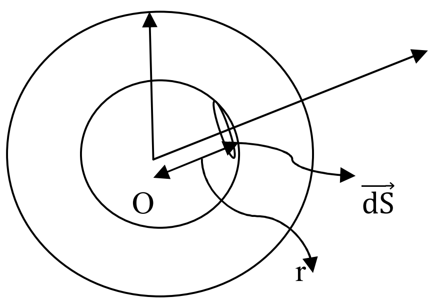


Figure 3

Let us consider a sphere of radius r concentric with spherical charge. Let the whole surface be divided into thin concentric shells. The electric field intensity at point P is the combined effect of shells outside P as well as those inside P. But we know that the electric field intensity due to outer shells is zero. Thus, the electric field intensity at point P is due to inner shells only.

By symmetry the electric field intensity E_i at every point of the spherical surface of radius r has the same magnitude and directed along the outward drawn normal to the surface.

The total electric flux through the entire surface = $\int_S \vec{E_i}\,.\vec{dS} = \int_S E_i\, dS \cos 0^0$

$$= \int_S E_i\, dS = E_i \int_S dS$$

$$= E_i\,(4\pi r^2) \qquad [\text{ since } \int_S dS = \text{total surface area of spherical surface} = 4\pi r^2]$$

Total charge enclosed by Gaussian surface, Q' = charge enclosed by a spherical core of radius r

=Volume of spherical core × volume charge density

$$= \frac{4}{3}\pi r^3 \rho$$

Using Gauss's theorem-

$$\int_S \vec{E_i}\,.\,\vec{dS} = 1/\varepsilon_0 \times Q'$$

0

$$E_i\,(4\pi r^2) = 1/\varepsilon_0 \times \left(\frac{4}{3}\pi r^3 \rho\right)$$

$$E_i = \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi r^3 \rho}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi r^3}{r^2} \times \frac{Q}{\frac{4}{3}\pi R^3} \qquad \left[\text{Since } \rho = \frac{Q}{\frac{4}{3}\pi R^3} \quad \text{equation (5)}\right]$$

or

$$E_i = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3} \qquad \text{.....(8)}$$

Therefore, the electric field intensity at internal point of a spherically symmetric charge distribution is directly proportional to the distance of the point from the centre of spherical charge.

We have observed that the electric field intensity outside the charge distribution is inversely proportional to the square of the distance of the point from the centre of spherical charge. In this way, the electric field intensity is maximum at the surface of the spherical charge equal to $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$. The variation of electric field intensity due to a uniformly charged non-conducting sphere is shown in the following figure 4

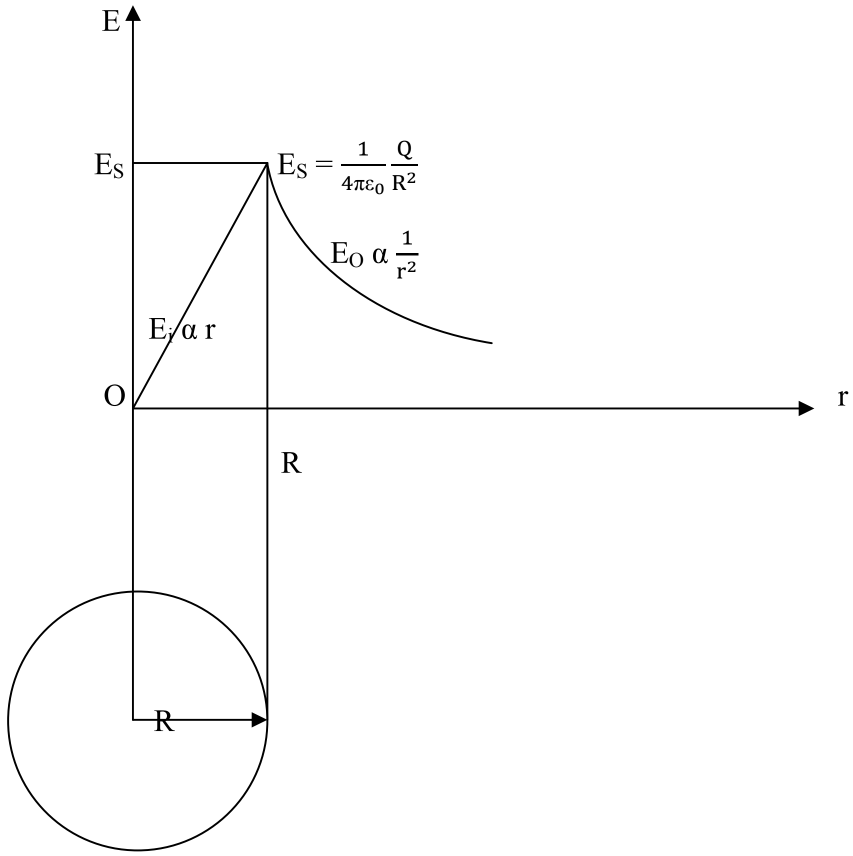


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Figure 4

3.5 ELECTRIC POTENTIAL DUE TO A CHARGED SPHERE

Let us consider a uniformly charged spherical volume of radius R containing charge Q. The volume charge density, $\rho = \frac{Q}{\frac{4}{3}\pi R^3}$. Let us learn and discuss the electric potential due to a charged sphere in various cases.

Case (i) At external point of spherical volume (r >R)

Let us consider a point P outside the spherical volume at a distance r > R.

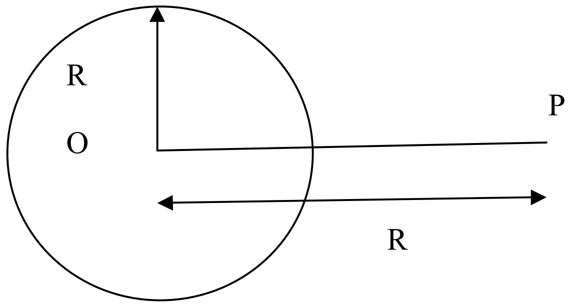


Figure 5

The electric field intensity at point P is given

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

The electric potential at point P,

$$V = -\int_{\infty}^{r} \overrightarrow{E_1}.\overrightarrow{dr} = -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}.\overrightarrow{dr}$$

$$= -\frac{Q}{4\pi\varepsilon_0}\int_{\infty}^{r} r^{-2}dr = -\frac{Q}{4\pi\varepsilon_0}\left[\frac{r^{-1}}{-1}\right]_{\infty}^{r} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} \qquad .....(9)$$

This expression is same as that of electric potential due to a charge placed at the centre O. In this way, for external points the spherical charge behaves as if the entire charge were concentrated at the centre of the spherical charge.

Case (ii) Inside the spherical charge i.e. at internal point (r < R)

Let us consider a point P' inside the spherical charge at a distance r from the centre O at which electric potential is to be determined.

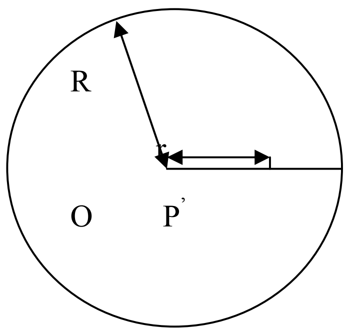


Figure 6

We have calculated the electric field intensity due to a uniformly charged non-conducting sphere at external and internal points as-

$\overrightarrow{E_O} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$ and $\overrightarrow{E_\iota} = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}\hat{r}$

The electric potential at point P' at a distance r < R from the centre is given by-

$$V = -\int_\infty^r \vec{E}.\overrightarrow{dr} = -\left[\int_\infty^R \overrightarrow{E_O}.\overrightarrow{dr} + \int_R^r \overrightarrow{E_\iota}.\overrightarrow{dr}\right]$$

Putting for $\overrightarrow{E_O}$ and $\overrightarrow{E_\iota}$ in the above expression, we get-

$$V = -\left[\int_\infty^R \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}.\overrightarrow{dr} + \int_R^r \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}\hat{r}.\overrightarrow{dr}\right]$$

$$= -\frac{1}{4\pi\varepsilon_0}\left[\int_\infty^R \frac{Q}{r^2}\hat{r}.\overrightarrow{dr} + \int_R^r \frac{Qr}{R^3}\hat{r}.\overrightarrow{dr}\right]$$

$$= -\frac{1}{4\pi\varepsilon_0}\left[\int_\infty^R \frac{Q}{r^2}dr + \int_R^r \frac{Qr}{R^3}dr\right] \quad [\text{Since } \hat{r}.\ \overrightarrow{dr} = 1\times dr \times \cos 0^0 = dr]$$

$$= -\frac{1}{4\pi\varepsilon_0}Q\left[\int_\infty^R \frac{1}{r^2}dr + \frac{1}{R^3}\int_R^r r\,dr\right] = -\frac{1}{4\pi\varepsilon_0}Q\left[-\left(\frac{1}{r}\right)_\infty^R + \frac{1}{R^3}\frac{1}{2}(r^2)_R^r\right]$$

$V = \frac{1}{4\pi\varepsilon_0}Q\left[\frac{3R^2 - r^2}{2R^3}\right]$…..(10)

This is the expression for electric potential at internal point.

3.6 ELECTRIC POTENTIAL DUE TO A CHARGED DISC

Let us consider a flat insulating disc of a radius 'a' carrying a positive charge Q spread uniformly over its surface. Let σ be the surface charge density of the disc. The disc considered here is non-conducting because if it is conducting, it would become a surface of constant potential and not that of uniform charge as the charge on the conducting disc would redistribute itself, crowding more towards the rim of the disc.

Let us calculate electric potential due to a charged disc.

Case (i) At point on the axis of symmetry

Let us consider a point P on the axis of symmetry at a distance x from the centre of the disc. Let us suppose that the disc is formed of a large number of thin concentric ring shaped elements. Let us consider one such ring shaped element of radius y and thickness dy.

The area of the ring element = circumference × thickness of the ring element = 2π y dy

The charge on the ring element dq = surface charge density × area of the ring element

$= \sigma\,(2\pi\, y\, dy)$

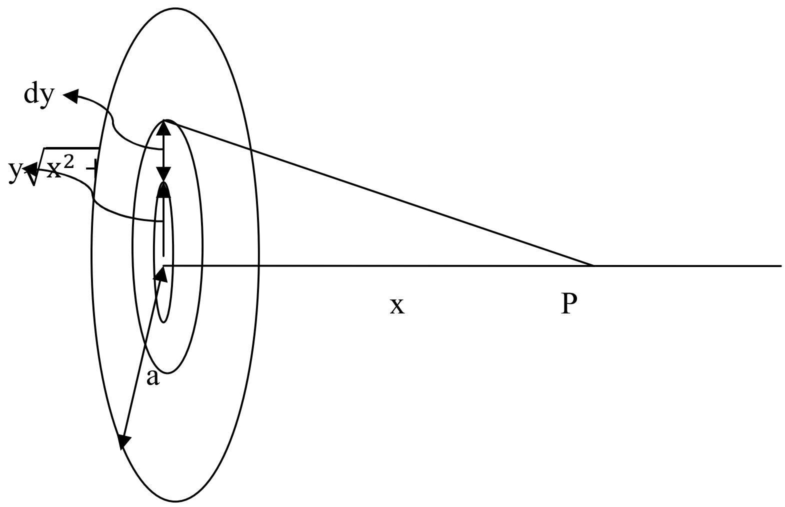


Figure 7

Electric potential at point P due to this ring element-

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{x^2+y^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\sigma\,(2\pi\, y\, dy)}{\sqrt{x^2+y^2}}$$

The electric potential at point P due to the entire disc, $V = \int_0^a \frac{1}{4\pi\varepsilon_0}\frac{\sigma\,(2\pi\, y\, dy)}{\sqrt{x^2+y^2}}$

$$= \frac{1}{4\pi\varepsilon_0}\sigma\, 2\pi \int_0^a \frac{y\, dy}{\sqrt{x^2+y^2}} = \frac{\sigma}{2\varepsilon_0}\int_0^a y\,(x^2+y^2)^{\frac{-1}{2}}\, dy$$

$$= \frac{\sigma}{2\varepsilon_0}\left[(x^2+y^2)^{\frac{1}{2}}\right]_0^a = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{x^2+a^2} - x\right]$$

or $\quad V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{x^2+a^2} - x\right]$, for $x > 0$ …..(11)

If x >> a, then $V = \frac{\sigma}{2\varepsilon_0}\left[x\left(1+\frac{a^2}{x^2}\right)^{\frac{1}{2}} - x\right] = \frac{\sigma}{2\varepsilon_0}\left[x + \frac{1}{2}\frac{a^2}{x^2} - x\right]$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\sigma\pi a^2}{x} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{x}$$

or $V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{x}$, for x >>a …..(12)

Thus for axial points distant x >>a, the disc behaves like a point charge.

Case (ii) At the centre of the disc

At the centre, x = 0, therefore from equation (11), we get-

$V_C = \frac{\sigma}{2\varepsilon_0} \left[\sqrt{0^2 + a^2} - 0\right] = \frac{\sigma a}{2\varepsilon_0}$ …..(13)

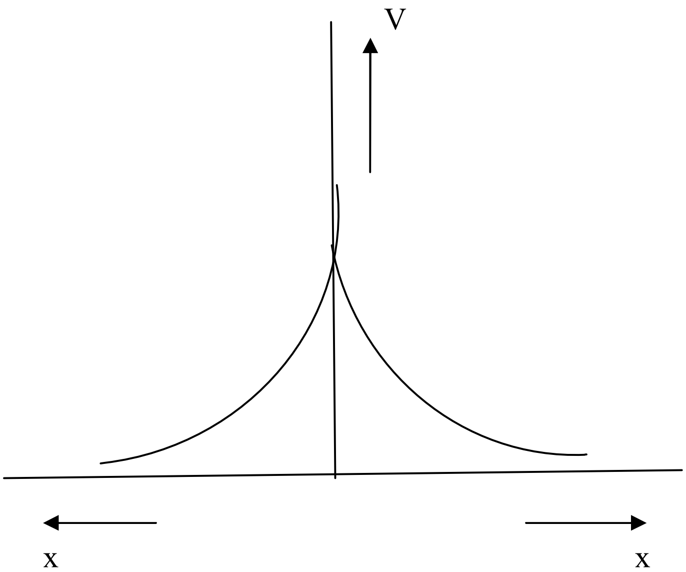


Figure 8

The figure 8 shows the variation of electrical potential along the axis of a uniformly charged disc.

Case (iii) At the rim or an edge of the disc

Let A be the point on the edge of the disc. Let us consider a segment CD of a ring centred at A of radius r and thickness dr.

Area of the segment = 2 r θ dr

Electric charge on this segment dq = 2 r θ dr σ

The electric potential at point A due to this segment, $dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{2\,r\,\theta\,dr\,\sigma}{r}$

$$= \frac{1}{4\pi\varepsilon_0}(2\sigma\theta)\,dr \qquad \text{.....(14)}$$

The electric potential at point A due to entire charge on the disc is given as-

$$V = \int_{\theta=\pi/2}^{\theta=0} \frac{1}{4\pi\varepsilon_0}(2\sigma\theta)\,dr \qquad \text{.....(15)}$$

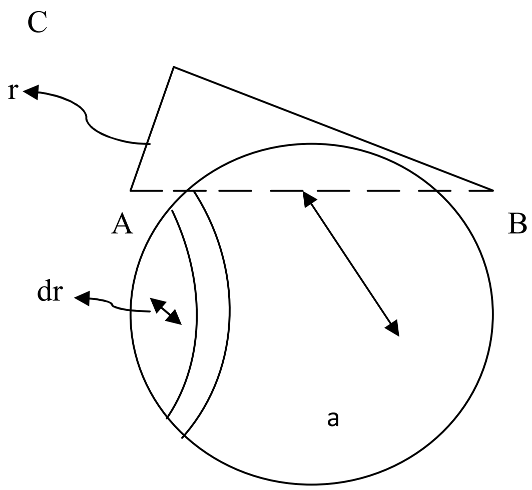


Figure 9

From figure 9, $r = 2a\cos\theta$

or $dr = -2a\sin\theta\, d\theta$

From equation (15), we get-

$$V = \frac{2\sigma}{4\pi\varepsilon_0}\int_{\theta=\pi/2}^{\theta=0}\theta(-2a\sin\theta\, d\theta) = \frac{\sigma a}{\pi\varepsilon_0}\int_{\theta=\pi/2}^{\theta=0}\theta\sin\theta\, d\theta$$

$$= \frac{\sigma a}{\pi\varepsilon_0}[\sin\theta - \theta\cos\theta]_0^{\pi/2} = \frac{\sigma a}{\pi\varepsilon_0} \qquad \text{.....(16)}$$

Accordingly, the electric potential falls from the centre of the disc to the edge or rim. This indicates that a uniformly charged disc is not an equipotential surface.

3.7 ELECTRIC FIELD DUE TO A CHARGED DISC

You have learnt that electric potential due to a charged disc on axial point is given as-

$$V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{x^2 + a^2} - x\right]$$

Electric field intensity at axial point P at axial point P (Figure 7) at a distance x from the centre of the disc, $E = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left[\frac{\sigma}{2\varepsilon_0}\left\{\sqrt{a^2 + x^2} - x\right\}\right]$

or $$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{a^2+x^2}}\right] \qquad .....(17)$$

At the centre of the disc, x = 0, therefore electric field intensity $E = \frac{\sigma}{2\varepsilon_0}$ .....(18)

At axial points x >>a, the electric field intensity, $E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{a^2+x^2}}\right]$

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - x(a^2 + x^2)^{-\frac{1}{2}}\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - \left(1 + \frac{a^2}{x^2}\right)^{-\frac{1}{2}}\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \left(1 - \frac{a^2}{2x^2}\right)\right], \quad \text{for } x >> a$$

or $$E = \frac{\sigma a^2}{4\varepsilon_0 x^2} = \frac{1}{4\pi\varepsilon_0}\frac{\pi\sigma a^2}{4\pi\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2}, \quad \text{for } x >> a \qquad .....(19)$$

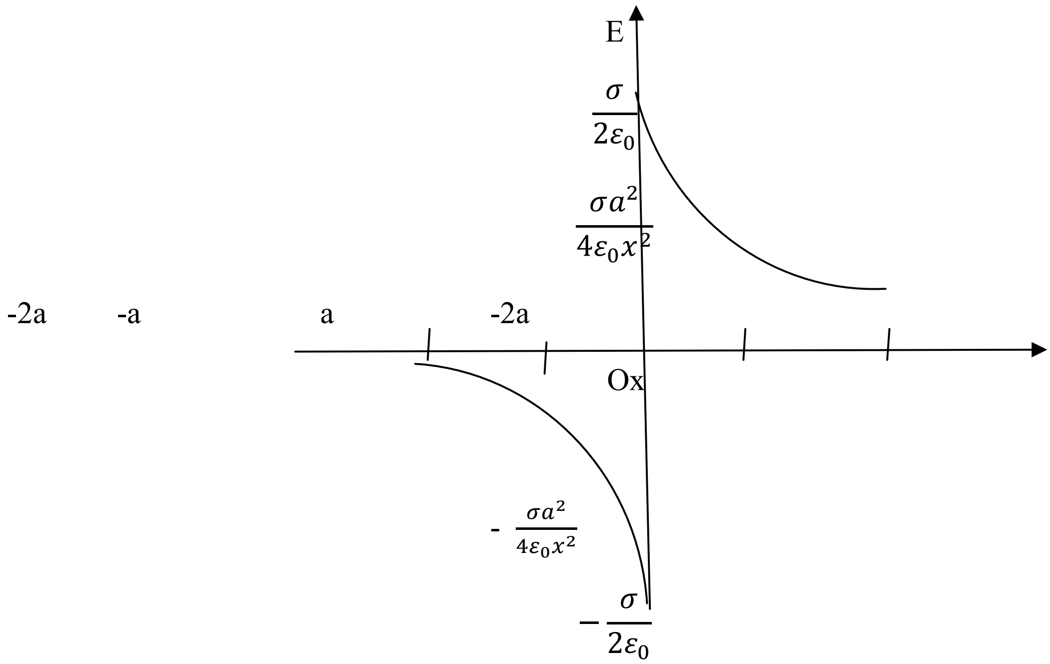


Figure 10

Figure 10 shows the variation of electric field intensity along the axis of a uniformly charged disc.

Example 1: The electric potential at the centre of an uniformly charged disc is 200 volt and the radius of the disc is 30 cm. Determine the charge on its surface?

Solution: Given, V_C = 200 volt, Radius of the disc, a = 30 cm = 0.30 m

We know, $V_C = \frac{\sigma a}{2\varepsilon_0}$

or $\sigma = V_C\,(2\varepsilon_0)/a = 200(2\times 8.85\times 10^{-12})/\,0.30 = 1.18\times 10^{-8}$ C/m^2

Now $\sigma = \frac{q}{\pi a^2}$ or $q = \sigma\,\pi\,a^2 = 1.18\times 10^{-8}\times 3.14\times (0.30)^2 = 3.33\times 10^{-10}$ C

Example 2: An infinite long conducting wire is stretched horizontally 3 metres above the surface of the earth. The wire has a charge of 1 C per m of its length. Determine the electric field intensity at a point on the earth vertically below the wire.

Solution: We know that electric field intensity due to an infinitely long wire at any point distant r is given by-

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}$$

Here, λ = 1 C per m, r = 3 m; therefore $E = 9\times 10^9\times\frac{2\times 1}{3} = 6\times 10^9$ N/C

Self Assessment Question (SAQ) 1: Estimate the electric potential difference between the centre and the surface of a sphere of radius 'R' with uniform charge density ρ within it.

Self Assessment Question (SAQ) 2: An infinite line charge generates an electric field intensity of 3×10^5 N/C at a distance of 2 cm. Calculate the value of linear charge density.

3.8 ELECTRIC DIPOLE

"If two equal and opposite charges are placed at a short distance apart, then this system is known as an electric dipole." The product of magnitude of one charge and the distance between the charges is called 'electric dipole moment' and it is denoted by 'p'.

$\vec{p}$

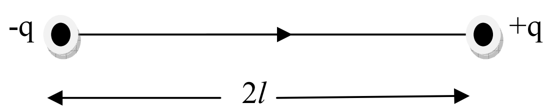


Figure 11

Let two charges –q and + q coulomb are placed at a distance 2l metre, then the electric dipole moment is-

$$p = q\times 2l = 2ql \qquad \text{…..(20)}$$

The electric dipole moment is a vector quantity whose direction is along the axis of the dipole pointing from the negative charge to the positive charge. The unit of electric dipole moment is coulomb-metre. Let us calculate the couple on an electric dipole in an uniform electric field.

3.8.1 Couple on an Electric Dipole in a Uniform Electric Field

Let us learn that what does happen with an electric dipole in an electric field. When an electric dipole is placed in a uniform electric field, a couple acts upon the dipole. This couple tends to align the electric dipole in the direction of the electric field. This is known as the 'restoring couple'.

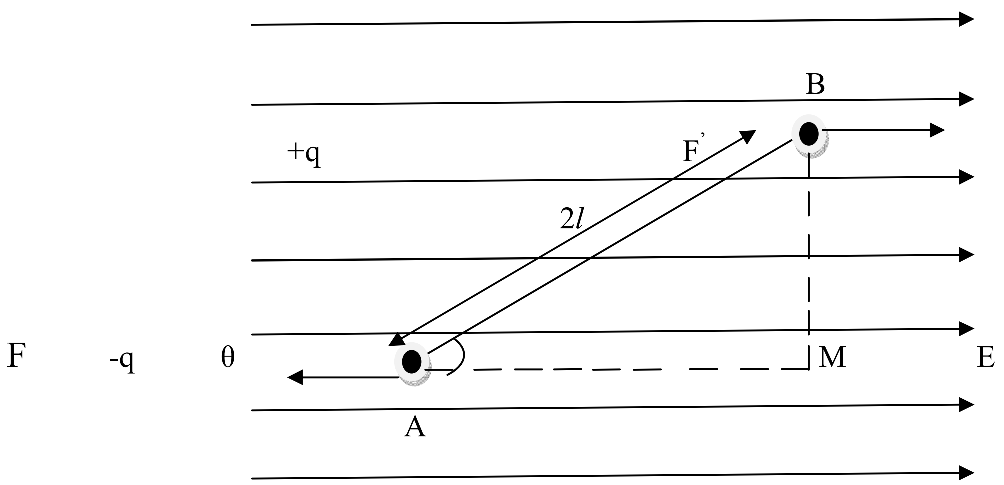


Figure 12

Let us consider an electric dipole AB placed in a uniform electric field E at an angle θ with the direction of electric field. –q and +q be the charges of electric dipole at a distance 2l from each other.

Due to electric field E, the electric force on charge –q of dipole, F = qE (in the opposite direction of E)

Similarly, the electric force on charge +q due to E, $F' = qE$ (in the direction of electric field E)

Obviously, the both forces are equal in magnitude but opposite in direction, due to this the net translator force on the electric dipole is zero, but these forces F and F (=F') constitute a couple which tends to align the dipole in the direction of the electric field E. This couple is restoring couple (τ).

The moment of this restoring couple

τ = magnitude of force × perpendicular distance between the lines of action of force

$$= F \times (BM) = qE \times 2l \sin\theta$$

$$= 2qlE \sin\theta = pE \sin\theta \qquad \text{(since } 2ql = p\text{)}$$

Therefore, $\tau = pE \sin\theta$ .....(21)

The unit of couple τ is Newton-metre

In vector form, $\vec{\tau} = \vec{p} \times \vec{E}$ .....(22)

Where $\vec{p}$ is a vector from the charge –q to +q.

If $\theta = 90^0$, i.e. electric dipole is placed perpendicular to electric field, then the couple acting on it is-

$$\tau = pE \sin 90^0 = pE$$

In this case, the couple acting on dipole will be maximum, therefore-

$$\tau_{max} = pE \qquad .....(23)$$

or

$$p = \frac{\tau_{max}}{E}$$

If E = 1N/C, then $p = \tau_{max}$ C-m, i.e., the moment of an electric dipole is equal to the couple acting on the dipole placed perpendicular to the direction of a uniform electric field intensity of 1 N/C.

If $\theta = 0^0$, i.e. dipole is placed parallel to electric field, then the couple acting on dipole-

$$\tau = pE \sin 0^0 = 0 \qquad .....(24)$$

i.e. if the dipole is placed parallel to the field, then no couple will act on dipole.

3.8.2 Work Done in Rotating an Electric Dipole in an Electric Field

Let us consider a dipole placed in a uniform electric field. If it is rotated from its equilibrium position, work has to be done.

Let us suppose that an electric dipole placed in electric field, is rotated through an angle θ from its equilibrium position. During rotation, the couple acting on the dipole changes. Let us suppose that at any instant, the dipole makes an angle α with the direction of electric field E.

The instantaneous couple acting on the dipole is-

$$\tau = pE \sin\alpha$$

Amount of work done in rotating the dipole from this position through an infinitesimally small angle dα is-

$$dW = \text{couple} \times \text{angular displacement}$$

$$= (pE \sin\alpha)\, d\alpha$$

Amount of work done in rotating the dipole through the angle θ from its equilibrium position is-

$$W = \int_0^\theta dW = \int_0^\theta (pE \sin\alpha)\, d\alpha$$

$$= pE \int_0^\theta \sin\alpha\, d\alpha = pE\left[-\cos\alpha\right]_0^\theta = -\, pE\,[\cos\alpha]_0^\theta$$

$$= -pE\,[\cos\theta - \cos 0] = -pE\,[\cos\theta - 1]$$

or $$W = pE\,(1 - \cos\theta) \qquad .....(25)$$

The above expression represents the work done in rotating an electric dipole in a uniform electric field through an angle θ from the direction of the electric field (i.e. equilibrium position).

If $\theta = 90^0$, i.e. the dipole is rotated through an angle 90^0 from its equilibrium position, then work done-

$$W = pE\,(1 - \cos 90^0)$$

$$= pE\,(1 - 0) = pE \qquad .....(26)$$

If the dipole is rotated through 180^0 from the direction of the electric field, then the work done-

$$W = pE\,(1 - \cos 180^0)$$

$$= pE\,(1+1) = 2pE \qquad .....(27)$$

This is the maximum work done for rotating a dipole.

3.8.3 Potential Energy of an Electric Dipole in an Electric Field

"The potential energy of an electric dipole in an electric field is defined as the work done in bringing the dipole from infinity to inside the electric field."

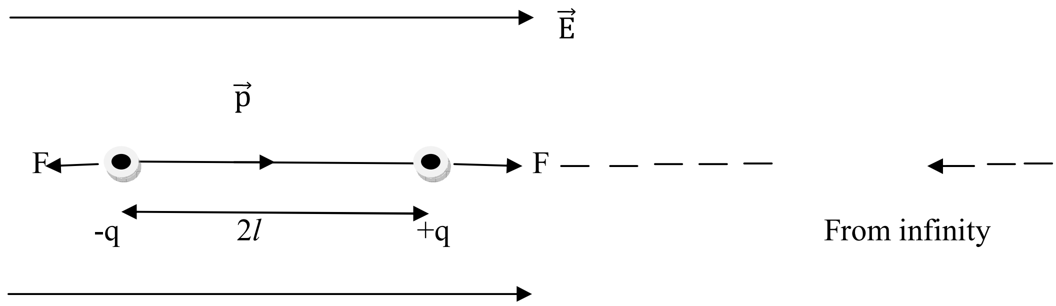


Figure 13

Let an electric dipole is brought from infinity to a uniform electric field E in such a way that the electric dipole moment p is always in the direction of electric field. Due to electric field E, a force F (= qE) acts on the charge +q in the direction of the electric field and an equal force F (=qE) on the charge –q in the opposite direction. Therefore, in bringing the electric dipole in the electric field from infinity, work will be done on the charge +q by an external agent, while work will be done by the electric field itself on the charge –q.

Obviously, when the dipole is brought from infinity into the electric field, the charge –q covers 2l distance more than the charge +q. Hence, the work done on –q charge will be greater. Therefore, the net work done in bringing the electric dipole from infinity into the electric field = force on charge (–q) × additional distance moved

$= (-qE)\times 2l = -(2ql)E = -pE$ [since $2ql = p$]

This work is the potential energy U_0 of the electric dipole placed in the electric field parallel to it i.e.

$$U_0 = -pE \quad \text{.....(28)}$$

In this position, the electric dipole is in stable equilibrium inside the field.

If we rotate the electric dipole in the electric field through an angle θ, then work will have to be done on electric dipole. This work is-

$$W = pE\,(1-\cos\theta) \quad \text{.....(29)}$$

This will result in an increase in the potential energy of the electric dipole. Hence, the potential energy of the dipole in the position θ will be given by-

$$U_\theta = U_0 + W$$

$$= -pE + pE\,(1-\cos\theta)$$

$= -pE + pE - pE\cos\theta = -pE\cos\theta$

or $U_\theta = -pE\cos\theta$ .....(30)

The above equation (30) represents the potential energy of the electric dipole.

In vector form, equation (30) can be written as-

$\vec{U} = -\vec{p}.\vec{E}$ .....(31)

If $\theta = 90^0$ i.e. the electric dipole is placed perpendicular to the electric field, then

$$U_{90} = -pE\cos 90^0 = 0$$

i.e. if we keep the electric dipole perpendicular to the electric field while bringing it from infinity into the electric field, then the work done on the charge +q by the external agent will be equal to the work on the charge –q by the electric field. In this way, the net work done on the dipole will be zero and hence the potential energy of the dipole will also be zero.

If $\theta = 180^0$, i.e. if we rotate the electric dipole through 180^0 from the position of stable equilibrium, then the potential energy, $U_{180} = -pE\cos 180^0 = +PE$

In this position, the electric dipole will be in unstable equilibrium.

3.8.4 Electric Field due to an Electric Dipole

In this subsection, you shall learn about electric field due to an electric dipole. You will calculate the electric field intensity on the axis of a dipole (i.e. end-on position) and equatorial line of a dipole (i.e. broad-side-on position).

(i) Electric field intensity at a point on the axis of a dipole (end-on position)

Let us consider an electric dipole situated in a medium of dielectric constant K. Let P be a point on the axis at a distance 'r' metre from the midpoint 'O' of the dipole at which electric field intensity is to be determined.

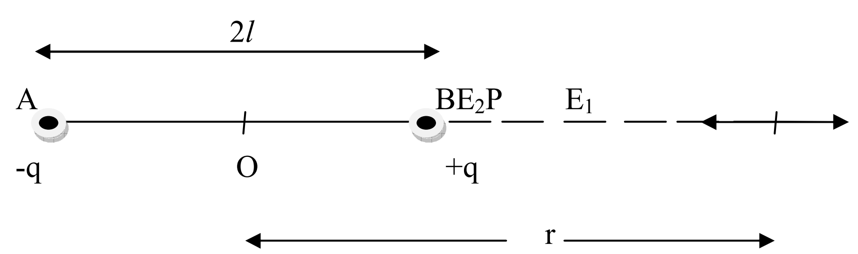


Figure 14

The distance of point P from charge –q = (r+l)

The distance of point P from charge +q = (r-l)

Therefore, electric field intensity at point P due to charge +q, $E_1 = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r-l)^2}$, (along BP)

Similarly, electric field intensity at point P due to charge -q, $E_2 = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r+l)^2}$, (along PA)

Obviously, both intensities are in opposite directions, therefore net electric field intensity at point P, $E = E_1 - E_2$ (since E_1-E_2)

$$= \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r-l)^2} - \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r+l)^2}$$

$$= \frac{q}{4\pi\varepsilon_0 K}\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right] = \frac{q}{4\pi\varepsilon_0 K}\left[\frac{(r+l)^2-(r-l)^2}{(r^2-l^2)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 K}\left[\frac{4lr}{(r^2-l^2)^2}\right] = \frac{1}{4\pi\varepsilon_0 K}\left[\frac{2(2ql)r}{(r^2-l^2)^2}\right]$$

or $$E = \frac{1}{4\pi\varepsilon_0 K}\left[\frac{2pr}{(r^2-l^2)^2}\right] \quad .....(32)$$

[since 2ql = p, electric dipole moment]

The direction of E is along BP i.e. along the axis of the dipole from the negative charge towards the positive charge.

If l<< r, then l^2 may be neglected in comparison to r^2. Then electric field intensity at point P,

$$E = \frac{1}{4\pi\varepsilon_0 K}\left[\frac{2pr}{r^4}\right] = \frac{1}{4\pi\varepsilon_0 K}\left[\frac{2p}{r^3}\right], \text{ N/C} \quad .....(33)$$

For air or vacuum, K =1, then $E = \frac{1}{4\pi\varepsilon_0}\left[\frac{2p}{r^3}\right]$ N/C .....(34)

(ii) Electric field intensity at a point on the equatorial line of a dipole (broad-side-on position)

Now let us calculate the electric field intensity at a point on the equatorial line. Let us suppose that the point P is situated on the right-bisector of the electric dipole AB at a distance 'r' metre from its mid-point 'O'.

Electric field intensity at point P due to charge +q, $E_1 = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(PB)^2}$

$$= \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)}, \quad \text{(along BP)}$$

Similarly, electric field intensity at point P due to charge –q, $E_2 = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(PA)^2}$

$$= \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)}, \qquad \text{(along PA)}$$

Obviously, the magnitudes of E_1 and E_2 are equal but directions are different.

Resolving E_1 and E_2 into their components-

Horizontal component of $E_1 = E_1 \cos\theta$ (parallel to BA)

Vertical component of $E_1 = E_1 \sin\theta$ (perpendicular to BA)

Similarly, horizontal component of $E_2 = E_2 \cos\theta$ (parallel to BA)

Vertical component of $E_2 = E_2 \sin\theta$ (perpendicular to BA)

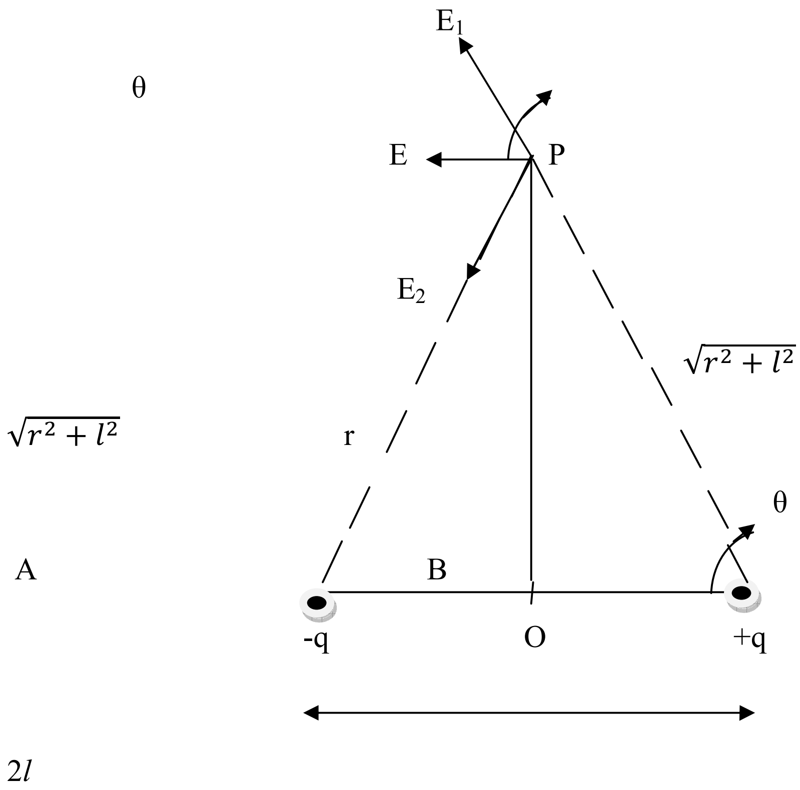


Figure 15

Vertical components of E_1 and E_2 ($E_1 \sin\theta$ and $E_2 \sin\theta$) are equal in magnitudes but opposite in direction, hence they cancel to each other. But horizontal components ($E_1 \cos\theta$ and $E_2 \cos\theta$) are in same direction. Hence the resultant electric field intensity at point P is-

$$E = E_1 \cos\theta + E_2 \cos\theta$$

$$= 2E_1 \cos\theta \qquad \text{(since } E_1 = E_2\text{)}$$

$$= 2\frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)} \cos\theta$$

But in right angled triangle POB, $\cos\theta = \frac{OB}{PB} = \frac{l}{\sqrt{r^2+l^2}}$

Therefore, $E = 2\frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r^2+l^2)}\frac{l}{\sqrt{r^2+l^2}}$

$$= \frac{1}{4\pi\varepsilon_0 K}\frac{2ql}{(r^2+l^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0 K}\frac{p}{(r^2+l^2)^{3/2}} \qquad [\text{since } 2ql = p]$$

Thus, $$E = \frac{1}{4\pi\varepsilon_0 K}\frac{p}{(r^2+l^2)^{3/2}} \qquad \text{.....(35)}$$

The direction of E is horizontal along BA i.e. parallel to the axis of dipole from positive charge to negative charge.

If l<< r, i.e. l is very small in comparison of r, then l^2 can be neglected in comparison to r^2; then

$$E = \frac{1}{4\pi\varepsilon_0 K}\frac{p}{(r^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0 K}\frac{p}{r^3} \text{ N/C} \qquad \text{.....(36)}$$

For air or vacuum, K =1 then $E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$ N/C .....(37)

From equations (33) and (36), it is clear that for a short dipole the electric field intensity on an axial point is twice the intensity at the same distance on the equatorial line.

3.8.5 Electric Potential due to an Electric Dipole

In this subsection, you will calculate electric potential at a point on the axis and equatorial line of a dipole.

(i) Electric potential at a point on the axis of the dipole(end-on position)

Let us consider an electric dipole AB placed in a medium of dielectric constant K. P is the point on the axis (end-on position) at a distance 'r' from the midpoint 'O' of the dipole at which electric potential is to be calculated.

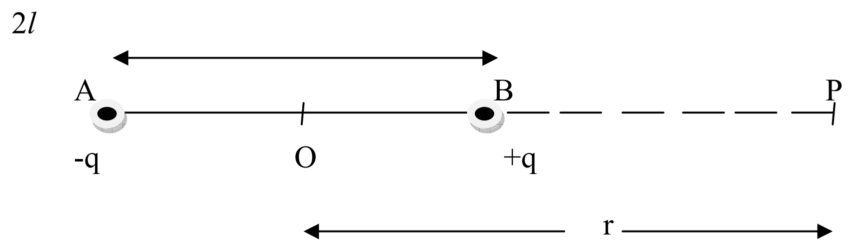


Figure 16

Electric potential at point P due to charge –q, $V_1 = \frac{1}{4\pi\varepsilon_0 K}\frac{(-q)}{AP} = -\frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r+l)}$

Similarly, electric potential at point P due to charge +q, $V_2 = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{BP} = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r-l)}$

Resultant electric potential at point P, $V = V_1 + V_2 = -\frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r+l)} + \frac{1}{4\pi\varepsilon_0 K}\frac{q}{(r-l)}$

$= -\frac{q}{4\pi\varepsilon_0 K}\left[\frac{1}{(r+l)} - \frac{1}{(r-l)}\right] = \frac{q}{4\pi\varepsilon_0 K}\frac{2l}{(r^2-l^2)} = \frac{1}{4\pi\varepsilon_0 K}\frac{2ql}{(r^2-l^2)} = \frac{1}{4\pi\varepsilon_0 K}\frac{p}{(r^2-l^2)}$, [since $2ql = p$]

Thus, $$V = \frac{1}{4\pi\varepsilon_0 K}\frac{p}{(r^2-l^2)} \quad \text{.....(38)}$$

If dipole is short i.e. $l \ll r$, then l^2 may be neglected in comparison to r^2, then-

$$V = \frac{1}{4\pi\varepsilon_0 K}\frac{p}{r^2} \text{ volt} \quad \text{.....(39)}$$

For air or vacuum, K =1 then $$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2} \text{ volt} \quad \text{.....(40)}$$

(ii) Electric potential at a point on the equatorial line of the dipole (broad-side-on position)

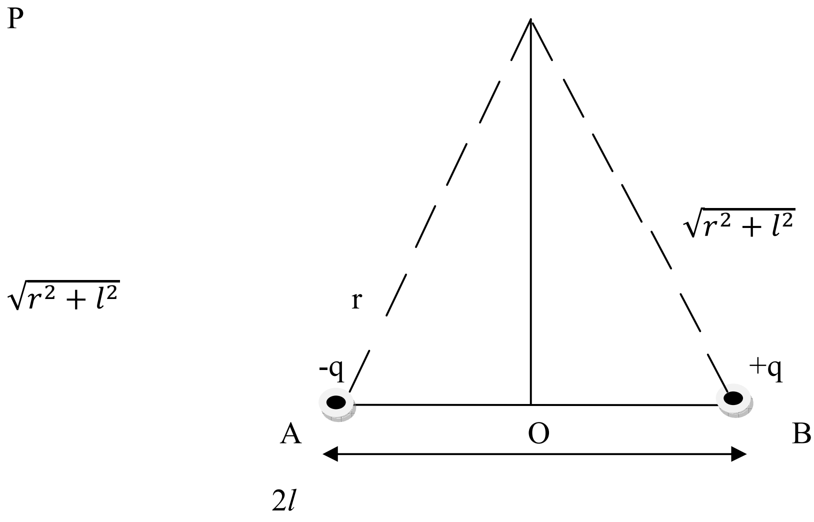


Figure 17

Now let us consider a point P on the equatorial line of a dipole at a distance ‘r’ from the midpoint ‘O’ of the dipole at which electric potential is to be calculated.

The electric potential at point P due to charge -q, $V_1 = \frac{1}{4\pi\varepsilon_0 K}\frac{(-q)}{AP} = -\frac{1}{4\pi\varepsilon_0 K}\frac{q}{\sqrt{r^2+l^2}}$

Similarly, the electric potential at point P due to charge +q, $V_2 = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{BP} = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{\sqrt{r^2+l^2}}$

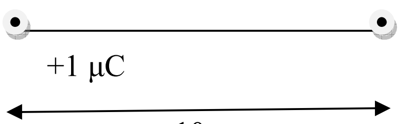
Resultant electric potential at point P, $V = V_1 + V_2$

$$= -\frac{1}{4\pi\varepsilon_0 K}\frac{q}{\sqrt{r^2+l^2}} + \frac{1}{4\pi\varepsilon_0 K}\frac{q}{\sqrt{r^2+l^2}}$$

$$= 0$$

Thus, the electric potential at an equatorial point of an electric dipole is zero.

Example 3: Calculate electric dipole momentof the following dipole-



-1μC +1 μC

10 cm

Solution: Here, $q = 1\ \mu C = 1 \times 10^{-6}$ C, $2l = 10$ cm $= 0.1$ m

Electric dipole moment $p = q \times 2l = 1 \times 10^{-6} \times 0.1 = 1\times10^{-7}$ C-m

Self Assessment Question (SAQ) 3: Two short electric dipoles of electric dipole moments p_1 and p_2 are in a straight line. Prove that the potential energy of each in the presence of the other is $-\frac{1}{2\pi\varepsilon_0}\frac{p_1 p_2}{r^3}$, where r is the distance between the dipoles.

3.9 SUMMARY

In the present unit, you have calculated electric potential and electric field intensity due to long charged wire, charged sphere and charged disc. You have studied thatthe electric potential falls from the centre of the disc to the edge or rim which indicates that a uniformly charged disc is not an equipotential surface. In this unit, you have learnt about electric dipole and electric dipole moment. If two equal and opposite charges are placed at a short distance apart, then this system is known as an electric dipole. The product of magnitude of one charge and the distance between the charges is called ‘electric dipole moment’. You have also study about the torque acting on an electric dipole in a uniform electric field which is given as $\tau = pE \sin\theta$, where p is the dipole moment, E, the intensity of electric field and θ is the angle that dipole makes with the direction of electric field.You have calculated the electric field intensity and potential due to dipole in end-on position and broad-side-on position.You have learntthat for a short dipole the electric field intensity on an axial point is twice the intensity at the same distance on the equatorial line. You have also studied thatthe electric potential at an equatorial point of an electric dipole is zero.

3.10 GLOSSARY

Uniformly- homogeneously

Non-conducting- in which there is no flow of current

Align- line up, ally

Constitute- make up, compose, comprise

Rotation- turning round, revolution

3.11 TERMINAL QUESTIONS

1. Establish an expression for electric field intensity due to a long charged wire.

2. Prove that the electric potential difference due to a long charged wire between two points distant r_1 and r_2 is $\Delta V = \frac{\lambda}{2\pi\varepsilon_0} log_e \frac{r_2}{r_1}$

3. A conducting sphere of radius 1 cm has an unknown charge. The electric field intensity at a point distant 2 cm from the centre of sphere is 2.7×10^4 N/C and points radially inward. Calculate the net charge on sphere.

4. Show that electric field intensity due to a charged sphere at an external point is given as-

   $E_O = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$, where symbols have their usual meanings. Show the variation of electric field due to a uniformly charged non-conducting sphere.

5. Establish the formula for electric potential due to a charged sphere.

6. Derive the formula for electric field due to a charged disc at a distance x from its centre. Also show that at the centre of disc, the electric potential is $\frac{\sigma a}{2\varepsilon_0}$, where 'σ' and 'a' are surface charge density and radius of disc.

7. Establish an expression for electric field due to a charged disc at an external point and hence show that the electric field at the centre of the disc is given as $E = \frac{\sigma}{2\varepsilon_0}$, where symbols have their usual meaning.

8. What do you mean by an electric dipole? Show that an electric dipole, in a uniform electric field, experiences only a torque and no net force.

9. Establish an expression for the torque acting on dipole in a uniform electric field.

10. Derive an expression for work done in rotating an electric dipole through an angle θ in an electric field.

11. Obtain the expression for potential energy of an electric dipole in an electric field.

12. Show that the electric field intensity due to an electric dipole at a point on end-on position is given by $E = \frac{1}{4\pi\varepsilon_0 K}\left[\frac{2pr}{(r^2-l^2)^2}\right]$, where symbols have their usual meaning.

13. Prove that in air, the electric field intensity due to an electric dipole at a point on the equatorial line of a dipole is $E = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2+l^2)^{3/2}}$, where symbols have their usual meaning.

14. Prove that at a point in the broad-side-on position of an electric dipole the electric potential is zero.

15. Explain, how is the electric potential due to a short electric dipole at a point r distant on the axis of the dipole is $\frac{1}{4\pi\varepsilon_0 K}\frac{p}{r^2}$? Here p and K are the electric dipole moment of dipole and dielectric constant of medium.

16. Two point charges of -3 μC and +3 μC are at a distance 0.2 cm apart from each other. Calculate-

(i) electric dipole moment of the dipole

(ii) electric field intensity at a distance of 60 cm from the dipole in broad-side-on position

(iii) electric potential at a distance of 60 cm from the dipole in broad-side-on position

(iv) electric field intensity at a distance of 60 cm from the dipole in end-side-on position

(v) electric potential at a distance of 60 cm from the dipole in end-side-on position

3.12 ANSWERS

Self Assessment Questions (SAQs):

1.We know that electric field intensity at a point distant r from the centre is given by-

$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}\hat{r}$, Q = Total charge on the sphere

The potential difference between the centre (r= 0) and the surface (r = R) is given by-

$V_0 - V_R = -\int_R^0 \vec{E}.\overrightarrow{dr}$

$$= -\int_R^0 \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}\hat{r}.\overrightarrow{dr} = -\int_R^0 \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}(1\times\ dr \times \cos 0^0)$$

$$= -\int_R^0 \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}\ dr = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}\int_R^0 rdr$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R}$$

But $\rho = \frac{Q}{\frac{4}{3}\pi R^3}$ or $Q = \frac{4}{3}\pi R^3 \rho$

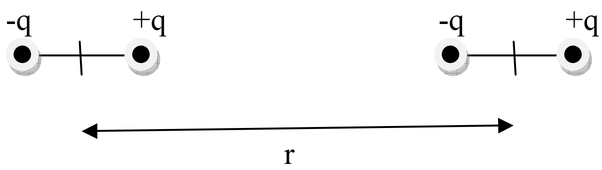
Therefore, $V_0 - V_R = \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi R^3 \rho}{2R} = \frac{\rho R^2}{6\varepsilon_0}$

2. Given, $E = 3\times 10^5$ N/C, r = 2 cm = 0.02 m

We know that $E = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}$

$3\times 10^5 = 9\times 10^9 \times \frac{2\lambda}{0.02}$ or $\lambda = 3.3\times 10^{-7}$ C/m

3.



The electric field due to short dipole of electric dipole moment p_1 at the other dipole is-

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

The potential energy of dipole with dipole moments p_2 in the electric field is-

$$U = -p_2 E \cos\theta = -p_2 \times \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

$$= -\frac{1}{2\pi\varepsilon_0}\frac{p_1 p_2}{r^3}$$

Terminal Questions:

3. Given, R= 1 cm = 0.01 m, $E = 2.7\times 10^4$ N/C, r = 2 cm = 0.02 m

Using $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$, we get-

$2.7\times 10^4 = 9\times 10^9 \times \frac{Q}{(0.02)^2}$ or Q = 12 C

16. The two charges form a dipole. Here q = 3 μC = 3×10^{-6} C, 2l = 0.2 cm = 0.002 m

(i) Electric dipole moment, p = q × 2l = $3\times10^{-6} \times 0.002 = 6 \times 10^{-9}$ C-m

(ii) r = 60 cm =0.60 m

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} = 9 \times 10^9 \times \frac{6 \times 10^{-9}}{(0.60)^3} = 250 \text{ N/C}$$

(iii) Electric potential in broad-side-on position, V = 0

(iv) Electric field intensity at a distance of 60 cm from the dipole in end-side-on position is-

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{2p}{r^3}\right] = 9 \times 10^9 \times \frac{2\times 6 \times 10^{-9}}{(0.60)^3} = 500 \text{ N/C}$$

(v) Electric potential at a distance of 60 cm from the dipole in end-side-on position is-

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2} = 9 \times 10^9 \times \frac{6 \times 10^{-9}}{(0.60)^2} = 150 \text{ volt}$$

3.13 REFERENCES

1. Electricity and Magnetism with Electronics, K.K. Tewari, S. Chand & Company Pvt. Ltd., New Delhi

2. Nootan Physics, Kumar, Mittal, Nageen Prakashan Pvt. Ltd., Meerut

3.14 SUGGESTED READINGS

1. Engineering Physics, S.K. Gupta, Krishna Prakashan Media (P) Ltd, Meerut

2. Theory and Problems of College Physics, F.J. Bueche, McGraw-Hill Book Company

UNIT 4 DIELECTRIC POLARIZATION AND POLARIZATION CHARGES

Structure

4.1INTRODUCTION

Electrical insulator materials which will prevent the flow of current in an electrical circuit are being used since from the beginning of the science and technology of electrical phenomena. Dielectrics are insulating materials that exhibit the property of electrical polarization, thereby they modify the dielectric function of the vacuum. The first capacitor was constructed by Cunaeus and Mussachenbroek in 1745 which was known as Leyden jar. But there were no studies about the properties of insulating materials until 1837. Faraday published the first numerical measurements on these materials, which he called dielectrics. Hehas found that the capacity of a condenser was dependent on the nature of the material separating the conducting surface. This discovery encouraged further empirical studies of insulating materials aiming at maximizing the amount of charge that can be stored by a capacitor. Throughout most of the 19th century, scientists searching for insulating materials for specific applications have become increasingly concerned with the detailed physical mechanism governing the behavior of these materials. In contrast tothe insulation aspect, the dielectric phenomena have become more general and fundamental, as it has the origin with the dielectric polarization.

In this Unit we have consider the problems of electrostatics in the absence of matter. Now we consider the phenomena in the medium other than empty space (vacuum) such as solid or liquid insulator, alternatively called dielectric the theory of dielectric was begun by Michael Farady, in 1837, and subsequently developed by Maxwell.

The properties of dielectric may vary from point to point i.e., it may not be homogeneous and in the neighbourhood of a point, the properties of a dielectric may not be same everywhere i.e. it may not be isotropic.

4.2 OBJECTIVES

The Main objectives of the present unit are:

- (i) To know about the Dielectrics.
- (ii) To study about polarization vector P.
- (iii) To know about electric field of polarized piece of a dielectric.
- (iv) To know about potential of polarized piece of a dielectric.
- (v) Gauss's law of a dielectric.

4.3 DIELECTRIC

A dielectric is a substance in which all the electrons are tightly bound to the nuclei of the atom i. e., no free electron are available to carry current . Thus substances which do not permit the passage of electric charge are called dielectric or insulators. The electric conductivity of a dielectric is very low (the conductivity of a dielectric is zero). Example: Certain substances such as glass, plastic quartz, mica, resins, waxes and oil etc.

4.3.1 Dielectric Constant

The theory of dielectric was begun by Faraday and subsequently by Maxwell. Using a simple electroscope and two parallel plate capacitors Faraday, discovered that dielectric materials can conduct small conductivity. He constructed two identical capacitors, in one of which he placed a dielectric. When both capacitors were charged to the same potential difference, it was found that the charge on the capacitors with dielectric is greater than that without. Since q is large for same V it follows from $C = \frac{q}{V}$ that the capacitance of a capacitor increases if dielectric is between the plates.

The ratio of capacitors after and before introducing the dielectric is known as dielectric constant (K) of the material. Thus if c is the capacitance with dielectric materials and C_o that in vacuum

i.e., $K = \frac{C}{C_0}$ .... (1)

The constant K is also called relative permittivity, specific inductivity capacitance or dielectric coefficient.It is independent of the shape and size of the capacitor but its value varies widely for different materials. For vacuum $K = 1$ (by definition), for air 1.006, for glass around 6 and so on.

4.3.2 Classification of dielectrics

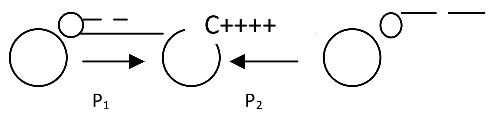
The molecules of dielectric may be classified as polar and non –polar.

Non Polar Molecules

In an atom the negatively charged electrons are distributed around the positively charged nucleus in such a way that the centre of electron cloud coincide with centre of nucleus. So in an atom there is no separation of positive and negative charge. The atom there for has no dipole moment. When two identical atoms combine to form a molecule, again there is no separation of negative and positive charge. The molecule formed by two identical atoms does not possess dipole moment. Such molecule are called non polar molecule and substance is made up of such molecule are called non polar substances.

H_2, N_2, O_2, CO_2, CCl_4, C_6H_6, C_6H_{12}, CS_2, etc. are **s**ome common example of non polar molecules. In a molecule of CO_2, the oxygen ions are symmetrically placedwith respect to the carbon ion, hence net dipole moment is zero. Thus CO_2 is non polar molecule.

$$\mathbf{p} = \mathbf{p}_1 - \mathbf{p}_2 = 0 \qquad .... (2)$$



Polar molecules

When two electrons of different electro negativities combine to form a covalent bond the shared electron pair is shifted towards the more electronegative atom as result of which a separation of positive and negative charge takes place and the bond acquires a dipole moment. Such a bond is called polar bond. If the polar bond in a molecule is symmetrically distributed, the resultant dipole moment of various polar bonds comes out be to zero. Then the molecule is called non polar. On the other hand if the polar bonds in a molecule are not symmetrically distributed, the resultant dipole has finite value, then the molecule is polar.

HCL, CO, NH_3, H_2O, $CHCL_3$, C_6H_5Cl, $C_6H_5NO_2$, C_2H_5OH etc. are some common example of polar molecules. In HCl molecule, the electron cloud is slightly shifted towards the more electronegative atom Cl. The molecule is therefore, a dipole having dipole moment P directed from Cl atom to the H atom similarly the co molecule is a dipole having a moment O to C atom.

In water molecule the two OH bonds are inclined at 104^0. Because of higher electro negativity of oxygen, the O-H bond acquires polarity with negative ends at the oxygen atom and the positive at the hydrogen atom.

$$p = [p_1^2 + p_2^2 + 2p_1p_2\cos\theta]^{1/2} \quad .... (3)$$

Where $p_1 = p_2$ is the dipole moment of one O-H bond and $\theta = 104^0$

4.3.3 Polarization of dielectric

The phenomenon of polarization may be illustratedhere from elementary atomic view. When a dielectric material is placed in electric field, the positive and negative charge of non polar molecules or atoms experience electrostatic forces in opposite directions. Therefore the centres of gravity of the two charges are separated from each other. The molecules thus acquire an induced electric dipole moment in the direction of the field.

When an electric field is applied on polar molecules (permanent dipole), the forces on a dipole give rise to a couple, whose effect is to orient the dipole along the direction of electric field. The stronger field, the greater is the aligning effect. This alignment is however, incomplete due to the thermal agitation of the molecule (the alignment become more and more perfect as the electric field is increase or the temperature is decreased).Thus non polar molecules become induced dipole whereas polar molecules are oriented by the field and therefore have their dipole moments increased. The orientation of induced dipoles or of permanent dipoles in an external electric field is such as to set the axis of dipoles along the field. This phenomenon is known as electric polarization.

There is a main difference between these two mechanisms. The polarization of non-polar molecule is independent of temperature. As polar molecule are undergoing thermal motion hence are randomly oriented. Thus the polar molecule can aligned perfectly with the smallest external electric field at about absolute zero.

4.3.4 Effect of Polarization on Electric Field within the dielectric

Suppose a slab of dielectric material is placed in the uniform electric field E_0 set up between the parallel plates of a charge capacitor. The slab becomes electrically polarized i.e. its dipole are oriented in the direction of the field. The net effect is appearance of negative charge on one face of the slab and an equal positive charge on opposite face. The polarization charges induced on the two faces of the slab produces their own electric field E, which opposes the external field E_0. Hence the resultant field **E** within the dielectric is smaller than $\mathbf{E_0}$but point as in same direction as $\mathbf{E_0}$ ($\mathbf{E = E_0 - E'}$). The field in the rest of the free space is still $\mathbf{E_0}$**.** Hence we conclude thatwhen a dielectric is placed in an electric fieldthe field within the dielectric is weakened (but not reduced to zero).

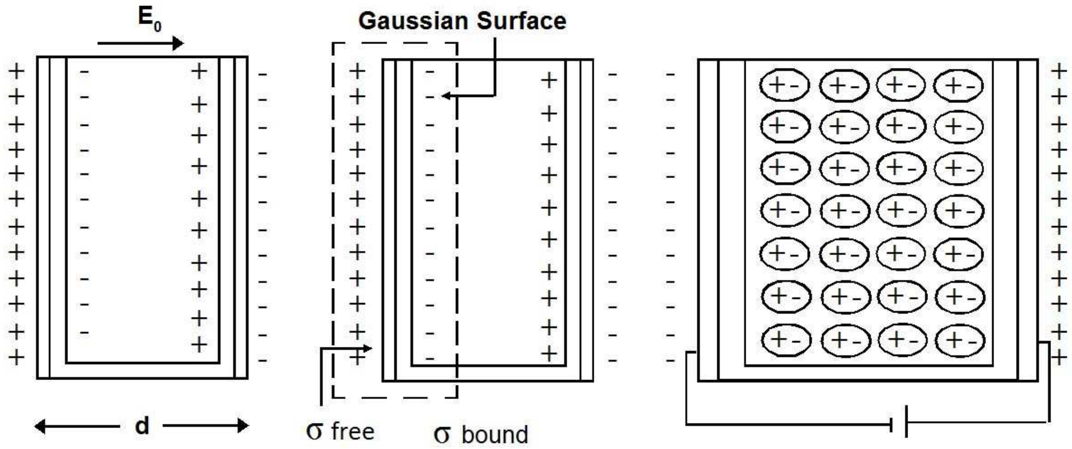


Figure 1

The charges within the polarised dielectric or those appearing at its surfaces are known as fictious charges or bound charges or polarisation charges and the charges on the plates of condenser are called free charges or real charges. If we assume that all the molecules are polarised to the same extent then the bound charges within the main body of dielectric will neutral one another because the negative side of one polarised molecule is adjacent to the positive side of its neighbour. However at the surface of the dielectric, in contact with the plates, the bound charges are not neutralised. This causes the field in the dielectric to become smaller than in the free space.

4.4 ELECTRIC POLARIZATION VECTOR P

When a dielectric is placed in an electric field, its molecules become electric dipoles and the dielectric is said to be electrically polarised. The state of polarisation is described by polarisation vector**P**, which is defined as the dipole moment per unit volume of dielectric material. The polarisation vector is related to bound charges.

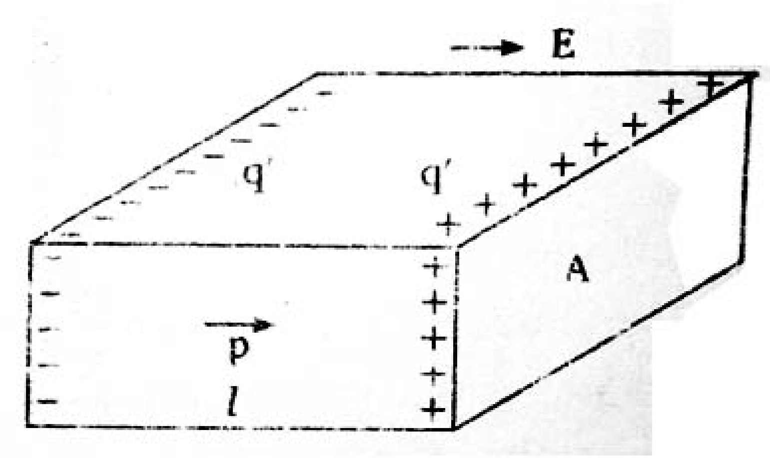


Figure 2

Let us consider a slab of homogeneous isotropic dielectric material of thickness *l* and face area A. Let it be placed perpendicular to a uniform electric field between the parallel plates of a capacitor having free charges +q and –q. The slab is polarised. Let *–q'* and *+q'* be the bound charges induced on its end faces (figure 2).

The induced electric dipole moment of the slab as a whole is *q'l* and its volume is *Al*. The magnitude of the electric polarisation is, therefore,

$$P = \frac{q'l}{Al} = \frac{q'}{A} \qquad .... (4)$$

Now *P* may also be defined as the induced surface charge per unit area i.e., the surface density of bound charges (σ_p) in dielectric

$$P = \sigma'_p \qquad ....(5)$$

Thus for a homogeneous isotropic dielectric, the electric polarisation *P* is numerically equal to the surface density of the induced charge appearing at the ends of dielectric block.

The unit of *P* is same as of charge density i.e., coulomb/m^2. It is zero for vacuum. The direction of **P** is from the negative induced charge –q' to the positive induced charge +q' as for any dipole.

Equation (5) can be generalised by considering the case when the dielectric surface is not to **P** (figure 2). Let the normal to the surface plane *XY* makes an angle θ with the direction of **P.**

Let σ_p be the surface charge of found charges. The dipole moment of the slab is $\sigma_p Al$ and its volume is (A cosθ)*l*. The magnitude of polarisation vector is

$$P = \frac{\sigma_P Al}{(A\cos\theta)l}$$

$$\sigma_p = P\cos\theta = \boldsymbol{P.n} \qquad .... (6)$$

where ***n*** is a unit vector normal to the surface.

4.5 FIELD OF A POLARIZED PIECE OF DEELECTRIC

If a dielectric is uniformly polarised, polarisation charges appear only at the surface. In case of non-uniform polarisation charges also appear within the body of the dielectric. We have seen that in case of uniform polarisation the surface density of polarisation charge is equal to the normal component of polarisation vector. In what follows we shall see that in case of non-uniform polarisation the volume density of polarisation charge is equal to the negative divergence of polarisation vector.

Consider a volume element dτ at point **r'** (x', y', z') inside the dielectric. The point of observation P lies at point **r"** (x'', y'', z''). The position vector of field point P relative to volume element (source point) dτ is **r** (=r" – r'). The dipole moment associated the volume element is Pdτ, where P is polarisation vector. The potential at point P due to charge in volume element is

$$d\varphi = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{P.r}}{r^3}d\tau$$

Hence the potential of entire piece of polarized material is

$$\varphi = \frac{1}{4\pi\varepsilon_0}\int\frac{\mathbf{P.r}}{r^3}d\tau$$

where the integration is to be performed over the volume occupied by the dielectric piece. The above expression for potential can be written as

$$\phi = -\frac{1}{4\pi\varepsilon_0}\int \mathbf{P}.\left(\nabla.\frac{1}{r}\right)d\tau \qquad \text{....... (7)}$$

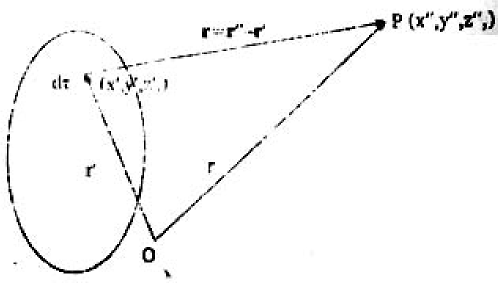


Figure 3

4.6 POTENTIAL OF A PIECE OF POLARIZED DIELECTRIC

Here the operator ∇ involves differentiation with respect to observer's coordinates (x'', y'', z'')

$$\vec{\nabla} = \hat{i}\frac{d}{dx''} + \hat{j}\frac{d}{dy''} + \hat{k}\frac{d}{dz''}$$

We define

$$\vec{\nabla}' = \hat{i}\frac{d}{dx'} + \hat{j}\frac{d}{dy'} + \hat{k}\frac{d}{dz'}$$

Which involves differentiation with respect to source coordinates (x', y', z'). It can be shown that

$$\nabla = -\nabla'$$

Making use of this result we can write equation (7) as

$$\phi = \frac{1}{4\pi\varepsilon_0}\int \mathbf{P}.\left(\nabla'.\frac{1}{r}\right)d\tau \qquad .... (8)$$

To transform equation (7) into more convenient from we make use of the following identity

$$\nabla'(\varphi\mathbf{A}) = \nabla'\varphi.\mathbf{A} + \boldsymbol{\varphi}\nabla'.\mathbf{A}$$

To use this result we make the following replacement. A→P, $\phi \rightarrow \frac{1}{r}$. Doing so we obtain

$$\mathbf{P}.\nabla'\frac{1}{r} = \nabla'\left(\frac{\mathbf{P}}{r}\right) - \frac{1}{r}\nabla'.\mathbf{P} \quad .... (9)$$

In view of equation (9) we can write equation (8) as

$$\phi = \frac{1}{4\pi\varepsilon_0}\int\left[\nabla'\left(\frac{\mathbf{P}}{r}\right) - \frac{1}{r}\nabla'.\mathbf{P}\right]d\tau$$

$$= \frac{1}{4\pi\varepsilon_0}\int \nabla'\frac{\mathbf{P}}{r}d\tau - \frac{1}{4\pi\varepsilon_0}\int\frac{\nabla'.\mathbf{P}}{r}d\tau \quad .... (10)$$

Transforming the first integral on the right hand side into surface integral by divergence theorem we have

$$\phi = \frac{1}{4\pi\varepsilon_0}\int\frac{\mathbf{P.n}}{r}dS + \frac{1}{4\pi\varepsilon_0}\int\frac{\nabla'.\mathbf{P}}{r}d\tau \qquad .... (11)$$

The first term on the right hand side of equation (11) looks like the potential due to a surface charge distribution with surface charge density

$$\sigma_b = \mathbf{P.n} \qquad .... (12)$$

And the second term looks like the potential due to a volume charge distribution with volume charge density

$$\rho_b = -\nabla.\mathbf{P} \qquad .... (13)$$

In terms of newly defined charge densities σ_b and ρ_b the potential of the polarized dielectric is

$$\phi = \frac{1}{4\pi\varepsilon_0}\int\frac{\sigma_b dS}{r} + \frac{1}{4\pi\varepsilon_0}\int\frac{\rho d\tau}{r} \quad .... (14)$$

Proof of result $\nabla = -\nabla'$

$\mathbf{r} = \mathbf{r''} - \mathbf{r'} = (x'' - x')\mathbf{i} + (y'' - y')\mathbf{j} + (z''-z')\mathbf{k}$

$r^2 = (x'' - x')^2 + (y'' - y')^2 + (z''-z')^2$

$2r\frac{\partial r}{\partial x'} = -2(x'' - x') \rightarrow \frac{\partial r}{\partial x'} = \frac{x''-x'}{r}$

Similarly, $\frac{\partial r}{\partial y'} = \frac{y''-y'}{r}$ and $\frac{\partial r}{\partial z'} = \frac{z''-z'}{r}$

Now $\nabla'\left(\frac{1}{r}\right) = i\frac{\partial}{\partial x'}\left(\frac{1}{r}\right) + j\frac{\partial}{\partial y'}\left(\frac{1}{r}\right) + k\frac{\partial}{\partial z'}\left(\frac{1}{r}\right)$

$= -\frac{1}{r^2}\frac{\partial r}{\partial x'}i - \frac{1}{r^2}\frac{\partial r}{\partial y'}j - \frac{1}{r^2}\frac{\partial r}{\partial z'}k$

$= \frac{1}{r^2}\frac{x''-x'}{r}\,\mathrm{i} + \frac{1}{r^2}\frac{y''-y'}{r}\mathrm{j} + \frac{1}{r^2}\frac{z''-z'}{r}\mathrm{k}$

$= \frac{\mathbf{r}}{r^3}$

$= -\nabla\left(\frac{1}{\mathrm{r}}\right)$

Hence $\nabla = -\nabla'$

4.7 GAUSS'S LAW IN DIELECTRIC

The well known gauss's law in electrostatics states that that electric flux through any closed surface is equal to 1/ ε_0 times the net charge enclosed by the surface.

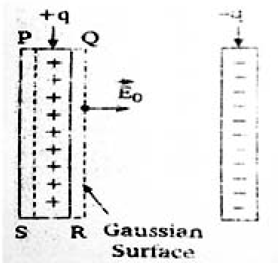


Figure 4

Let us consider a parallel plate capacitor with plate area A having vacuum between its plates (figure 4) +q and –q be the charges on the plates of the capacitor and E_0 be the uniform electric field between the plates. Let PQRS be a Gaussian surface. The electric flux through this surface is

$$\oint \mathrm{E}_0 . dS,$$

where dS is a small vector area on the surface. The net charge enclosed by the surface is $+q$. Therefore by Gauss's law

$$\oint \mathrm{E}_0 . dS = \frac{q}{\varepsilon_0}$$

But $\mathrm{E}_0 . d\mathrm{S} = E_0 A$

$\therefore E_0 A = \frac{q}{\varepsilon_0}$ .... (15)

Or $E_0 = \frac{q}{\varepsilon_0 A}$

Now, let us apply this law to a parallel plate capacitor filled with a dielectric material of dielectric constant *K*.

A negative charge –q' is induced on one surface and an equal positive charge +q' on the other. These induced charges produce their own field which oppose the external magnetic field E_0. Let E be the resultant field within the dielectric. The net charge enclosed by the Gaussian surface *PQRS* is now *q-q'*. In this case, Gauss's law gives

$\oint \mathrm{E}_0 . dS = \frac{q}{\varepsilon_0}$ .... (16)

Or $EA = \frac{q-q'}{\varepsilon_0}$

$E = \frac{q-q'}{\varepsilon_0 A}$ .... (17)

We know that $\frac{E}{E_0} = \frac{1}{K}$

Or $E_0 = EK$

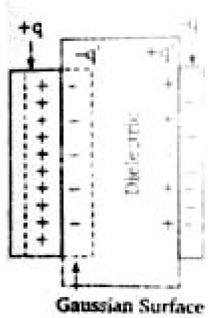


Figure 5

Putting this value of E_0 in equation (15), we have

$E_0 = \frac{q}{K\varepsilon_0 A}$

Inserting it in equation (17), we have

$\frac{q}{K\varepsilon_0 A} = \frac{q-q'}{K\varepsilon_0 A}$

or

q' = $q\left(1 - \frac{1}{K}\right)$ .... (18)

This equation shows that the induced surface charge q' is always less than free charge *q* and is zero when *K* = 1 or the dielectric is not present.

From equation (18) we find that q – q' = $\frac{q}{K}$.

Substituting this value of equation (17), The Gauss's law in presence of dielectric takes the following form

$$\oint \vec{E}.d\vec{S} = \frac{q}{\varepsilon_0}$$

or $\oint K\vec{E}.d\vec{S} = \frac{q}{\varepsilon_0}$ .... (19)

We note that while using this form of Gauss's law, the charge *q* contained within the Gaussian surface is taken to be 'free' charge only. The induced charge *q'* has been taken into account by the introduction of *K* on left hand side.

4.8 TERMINAL QUESTIONS

4.8.1 Long type Questions:

1. Differentiate between polar and non-polar molecules. Explain polarisation in them.
2. Differentiate between electronic, ionic and orientational polarisability.

3. What do you understand by dielectric polarisation? Explain partial and complete polarisation.
4. Deduce Gauss's Law in dielectrics.
5. Explain the effect of Polarization on electric field within the dielectric.

4.8.2 Short type Questions:

1. What is a dielectric? Give some examples.
2. What is dielectric constant?
3. Explain polar molecules.
4. Explain Non-polar molecules.
5. What is electric Polarisation of Vector P?
6. Explain Gauss's Law in Dielectrics.
7. Explain Field of a Polarized piece of Dielectric.

4.8.3 Objective type Questions:

1. Which one of the following substances is dielectric:
   (a) Copper (b) Mica (c) Germanium (d) Tungsten
2. CO_2 molecules is:
   (a) Polar (b) Non-Polar (c) Natural (d) Basic
3. HC*l* molecule is :
   (a) Polar (b) Non-Polar (c) Neutral (d) Conductor
4. Centres of positive and negative charges are not coincident in :
   (a) O_2 (b) N_2 (c) CO_2 (d) NH_3
5. Polar molecule is:
   (a) O_2 (b) N_2 (c) H_2 (d) H_2O
6. Non-Polar molecule is:
   (a) H_2O (b) CCl_4 (c) $CHCl_3$ (d) H_2O
7. Following is not a dielectric:
   (a) Wax (b) Mercury (c) Glass (d) Mica
8. Unit of Polarisation vector P is:
   (b) Coulomb (b) Coulomb metre (c) Coulomb/metre^{-2} (d) Newton/coulomb

4.9 ANSWERS

Objective type Questions:

(b) 2. (b) 3. (c) 4. (c) 5. (d) 6. (b) 7. (b) 8. (c)

4.10 REFERENCES

1. P.Van Musschen Broek, Introduction Philosophion Naturalem, Luchtman, Leiden (1762).
2. M.Faraday, *Phil.Trans* 128:1 79 265 (1837).

4.11 SUGGESTED BOOKS

1. D.C. Tayal, Electricity & Magnetism published by Himalaya publishing House
2. S.I. Ahmad and K.C. Lal, Electricity, Magnetism and Electronics published by Unitech House, Lucknow

UNIT 5 BOUNDARY CONDITIONS OF FIELD VECTORS, CAPACITORS FILLED WITH DIELECTRICS

Structure

5.1 INTRODUCTION

In this Unit we have discussed three electric vectors (electric fields strength, electric polarization and electric displacement vectors), restatement of Gauss's law, dielectric strengths and concept of capacitance in details.

5.2 OBJECTIVE

The Main objectives of the present unit are:

(i) To define three electric vectors
(ii) To define Restatement of Gauss's Law
(iii) To define capacitors
(iv) To define Dielectric constant and their strength
(v) To define combination of capacitors and their types
(vi) To define Energy stored in a capacitor

5.3 ELECTRIC FIELD STRENGTH $\vec{E}$

The electric field strength at any point in an electric field is defined as the force experienced per unit infinitesimal positive charge (q_0). If F is the force on small charge $q_{0,}$ then

i.e. $$E= \lim_{q0\to 0} \frac{F}{q0}$$

The direction of E is along the direction of force. The unit of E is Newton/coulomb or volt/metre.

5.4 ELECTRIC POLARIZATION $\vec{P}$

When a dielectric is placed in an external electric field, its molecules gain electric dipole moment and dielectric is said to be polarised. The electric dipole moment induced per unit volume of the dielectric material is called the electric polarisation of the dielectric. It is denoted by a vector P.

If σ_p is the surface charge densities of fictitious charges appearing at the end faces of a dielectric block, then P= σ_p

The unit of polarisation is coulomb/metre^2.

5.5 ELECTRIC DISPLACEMENT VECTOR $\vec{D}$

Let σ be the surface density of free charges on the capacitor plates and σ' of the bound charges on the dielectric. The magnitude of the electric fields due to σ and σ' are

$$E_0 = \frac{\sigma}{\varepsilon 0} \quad \text{and } E' = \frac{\sigma'}{\varepsilon 0}$$

The magnitude of the resultant field within the dielectric is therefore,

$E=E_0 - E'$ (the fields are oppositely directed)

Or

$E = \sigma/\varepsilon_0 - \sigma'/\varepsilon_0$

Or

$\varepsilon_0 E = \sigma - \sigma'$

Or

$\sigma = \varepsilon_0 E + \sigma'$ .... (1)

The last term of above equation (σ') is the induced charge density which is equal to the magnitude of electric polarisation P. So the above equation may be written as

$\sigma = \varepsilon_0 E + P$ .... (2)

The quantity on the right hand side of above equation is known as *electric displacement* D. Thus

$D = \varepsilon_0 E + P$ .... (3)

From above two equations we find

$D = \sigma$ .... (4)

Since E and P are vectors, D is also a vector. This displacement vector is an important addition which is of great use is Maxwell's electromagnetic equation to explain displacement current. In vector form equation (3) becomes

$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$ .... (5)

5.6 THREE ELECTRIC VECTORS

E, P and **D** are three electric vectors related to each other as shown in equation 5. These vectors may vary in magnitude and direction from point to point in complicated problems of electrostatics. But in simple case of a parallel plate capacitor filled with dielectric, each of three has a constant value or every point in the dielectric.

From the definition of **D, P** and **E,** we note the following-

(1) **D** is connected with the free charge only. The displacement field can be represented by lines of displacement just as electric field is represented by lines of force. The lines of **D** begin and end on the free charges (figure 1).

(2) **P** is connected with the induced surface charge or polarisation charge only. It can also be represented by lines known as lines of **P.** These lines begin and end on the polarisation charges *i.e.*, induced charges due to polarisation. The flux of **P** equals the negative of the bound (induced) charge. Clearly **P** is zero except inside the dielectric.

(3) The electric field intensity **E** is connected with the charge actually presents (free and bound charge). The lines of E depend upon the presence of both kinds of charges.

(4) Unlike the electric field **E** and the polarisation **P,** the electric displacement **D** has no clear physical meaning. The only reason for introducing it is that it enables one to calculate the electric field in the presence of dielectric without knowing the distribution of polarisation charges. The introduction of **D** is a convenience and not a necessity.

(5) The unit of **E** is Newton/Coulomb while that of **P** and **D** is Coulomb/meter^2.

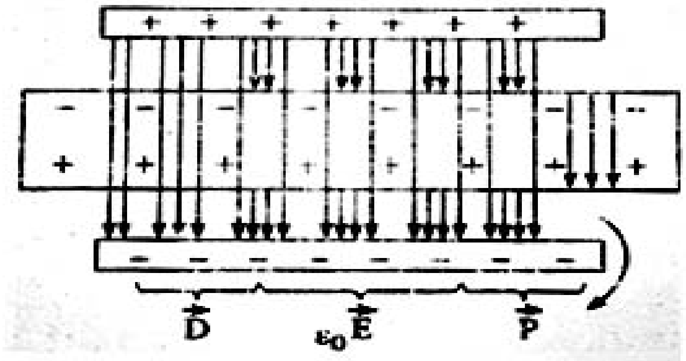


Figure1

5.6.1 D and P in terms of E

The vectors **D** and **P** can both be expressed in terms of **E** alone.

We know that $E_0 = \sigma/\varepsilon_0$ and $D = \sigma$ .....(see equation 4)

$E_0 = D/\varepsilon_0$ or $\boldsymbol{D} = \varepsilon_0\boldsymbol{E_0}$ .... (6)

Also $\boldsymbol{E_0} = \boldsymbol{KE}$ or $\boldsymbol{D} = K\varepsilon_0\boldsymbol{E}$....... (7)

Equation (6) and (7) also show that the displacement D has the same value in the dielectric and in vacuum (where $K = 1$, $E = E_0$). Hence the use of **D** is more convenient rather than **E.**

Similarly we can also write a relation between $\boldsymbol{P}$ and $\boldsymbol{E}$.

Equation (2) gives $P = \sigma - \varepsilon_0 E$

But we have seen above that

$D = \sigma = K\varepsilon_0 E$

$\boldsymbol{P} = K\varepsilon_0\boldsymbol{E} - \varepsilon_0\boldsymbol{E}$

or $\boldsymbol{P} = (K - 1)\,\varepsilon_0\boldsymbol{E}$

This clearly shows that in vacuum ($K = 1$), the polarisation **P** is zero.

5.7 RESTATEMENT OF GAUSS"S LAW

The Gauss's law in presence of dielectric has the following form

$$\oint_S K\vec{E}.d\vec{S} = \frac{1}{\varepsilon_0}\int_V \rho_f dV$$

where V is the volume enclosed by the surface S.

But the relation $\boldsymbol{D} = K\varepsilon_0\boldsymbol{E}$allows us to write the Gauss's law in another form

$\oint \mathbf{D}.dS = q = \int \rho_f dV$ .... (8)

Where q represents the free charge only, this tells us that the surface integral of the normal component of **D** over a closed surface equals the free charge enclosed by the surface.

Transforming the surface integral into volume integral using divergence theorem we have

$$\int_V div\, \boldsymbol{D}\, dV = \int_V \rho_f dV$$

Or $\int_V (div\, \boldsymbol{D} - \rho_f) dV = 0$

Since V is arbitrary, we have

$$\text{Div } D = \rho_f \quad \text{or} \quad \nabla . \boldsymbol{D} = \rho_f \qquad .... (9)$$

This is Gauss's law in differential form in a dielectric.

5.8 CONCEPT OF CAPACITANCE

When water is poured in a vessel, the level of the water in the vessel rises. When heat is given to a conductor, the temperature (i.e.,thermal level) of conductor increases. In the same way when electrical charge is given to a conductor, its electrical potential (i.e.electrical level) increases. It is observed that the increase in potential (V) of conductor is directly proportional to charge (Q) given to it,i.e.,

$V \alpha Q$

Or $Q \alpha V$

Or $Q = C V$ .... (10)

Where C is a constant for a given conductor and depends on the shape and size of the conductor, the surrounding medium and the presence of the other neighbouring conductors. This constant is called the capacitance of the conductor.

Form the equ (10),

$C = Q/V$ .... (11)

i.e, the capacitance of the conductor is the ratio of the charge given to and rise in potential of the conductor.

If V=1 volt,C=Q,i.e., the capacitance of the conductor is numerically equal to the charge required to be given to conductor which raises its potential level by 1 volt.

In SI system the unit of capacitance is coulomb/volt, called the farad (F).

i.e. $1 \text{ farad} = \frac{1\ coulomb}{1\ volt}$

Thus, the capacitance of conductor is 1 Farad if 1 coulomb of charge raises its potential by 1 volt.

In practice farad is a very big unit, therefore usual units used are micro farad(μF) and picofarad (pF)

$1\mu F = 10^{-6}$ F

and $1 \text{ pF} = 1\mu\mu F = 10^{-12}$ F

When water is poured in a vessel continuously, we observe that initially level of water in the vessel rises, then vessel is completely filled and finally water begins to flow out. In the same way when charge is given to the conductor continuously, its potential rises, becomes maximum and finally when insulation capacity of the surrounding medium vanishes, the charge begins to leakin the medium. Thus a given conductor in a given medium cannot attain the amount of charge more than a definite maximum amount of charge. This definite maximum charge is determined by capacitance of conductor. Thus the capacitance conductor is its capacity of collecting the charge.

Dimensions of capacitance:

Capacitance C= Charge (Q)/Potential (V)

As charge =current x time

So dimensions of charge, Q= [AT]

Potential, V=Work (W)/Charge (Q)

$$\text{Dimensions of potential } V = \frac{[ML^2T^{-2}]}{[AT]} = [ML^2T^{-3}A^{-1}]$$

$$(C) = \frac{\text{Dimensions of charge (Q)}}{\text{Dimensions of potential (V)}} = \frac{[A\ T]}{[ML^2T^{-3}A^{-1}]}$$

So Dimensions of capacitance $=[M^{-1}L^{-2}T^4A^2]$

5.8.1 Capacitance of an isolated spherical conductor

Suppose aninsulated spherical conductor of radius R is placed in air. The word isolated implies that there is no other conductor near by the sphere. Suppose a charge +Q Coulomb is given to spherical conductor. As charge given to a conductor spreads on its outer surface such that the potential on each point of conductor becomes 'same'. Thus, the surface of sphere becomes equipotential surface. As the electric lines of force are always perpendicular to equipotential surface; therefore, the electric lines of force emerge normally from the surface of sphere; and they appear to come from centre O radially outward. Consequently to determine the effect of charged sphere; at the surface points and external points, we can assume that the whole charge (Q) given to sphere may be supposed to be concentrated at its centre. Hence assuming charge (+ Q) situated at centre O of sphere, the potential at the surface of sphere,

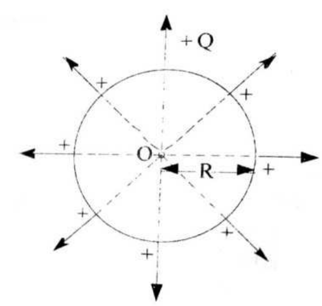


Figure 2

$$V=\frac{1}{4\pi\varepsilon_0}.Q/R$$

Where $\frac{1}{4\pi\varepsilon_0}$ =9.0 x 10^9 newton-meter^2/coulomb^2

Capacitance of isolated sphere

$$C=\frac{Q}{V}=\frac{Q}{(\frac{1}{4\pi\varepsilon 0}\frac{Q}{R})}$$

$$C=4\pi\varepsilon_0 R \qquad .... (12)$$

If R is in meter, C is in Farad,then

$$C \alpha R$$

Clearly,capacitance of a spherical conductor is directly proportional to its radius.

Remark: From (12), $\varepsilon_0=\frac{C_0}{4\pi R}$

From this expression,the unit of permittivity of free space is farad/meter.

From the Coulomb's law of electrostatic force

$F=\frac{1}{4\pi\varepsilon_0}q_1q_{2/r}{}^2$; the unit of ε_0 coulomb^2/newton-meter^2.

Thus, farad/meter and coulomb^2/newton-meter^2 units of same physical quantityε_0.

Example 1: If 10 microcoulomb charge given to a conductor increases its potential by 2.5 volt. What is the capacitance of the conductor?

Solution: Here Q=10 μC = 10x10^{-6} coulomb, V=2.5 volt

Capacitance $C=\frac{Q}{V}$

= 10x10^{-06}/ 2.5

= 4.0x10^{-6} farad

= 4.0 μF

Example 2: Assuming the earth be a spherical conductor of radius 6400 km, calculate its capacitance.

Solution: The capacitance of a spherical conductor in air

$C=4\pi\varepsilon_0 R$

$4\pi\varepsilon_0 = 1/9\text{x}10^9\ C^2/N\text{-}m^2$

R= 6400x10^3m

C= 6.4x10^6/9x10^9 =7.11x10^{-4}F

5.8.2 Parallel Plate Capacitor with Dielectric

It consist of two plate metallic plates A and B,placed parallel to each other .The plate may be of any shape,e.g., circular ,square and rectangular. The plates must be similar and at small separation. The plates carry equal and opposite charge +Q and –Q respectively.For this the plate is given a charge +Q and outer surface of plate B is earthed. When charge Q is given to plate A,the charge (-Q)is induced at inner surface of plate B and charge (+Q) at outer surface as outer surface of plate B is earthed,its charge (+Q) is transferred to earth. Thus the net charge on plate A is –Q and on the plate B it is (-Q).

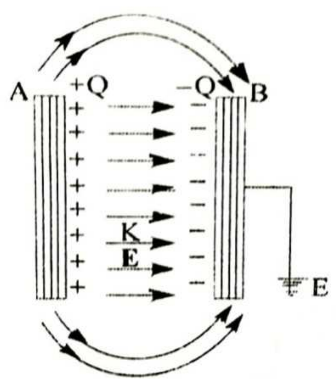


Figure 3

In general the electric field between the plates due the charges +Q and -Q remains uniform, but at edges,the electric lines of the force deviate outward. If separation between the plates is much smaller than the size of the plates,the electric field strength between the plates may be assumed uniform.

Suppose A is the area of each plate, d the separation between the plates,K the dielectric constant of the medium filled between the plates. If σ is the magnitude of charge density of plates,then

$$\sigma = Q / A$$

The electric field strength between the plates

$$E = \sigma / K \varepsilon_0$$

Where ε_0 =permittivity of free space.

The potential difference between the plates

$$V_{AB} = Ed = \sigma d/K \varepsilon_0 \quad \text{....... (13)}$$

Putting the value of σ, we get

$$V_{AB} = \frac{\left(\frac{Q}{A}\right)d}{K\varepsilon 0} = \frac{Qd}{K\varepsilon 0A}$$

Capacitance of capacitor

$$C = Q / V_{AB} = Q/(Qd/K \varepsilon_0 A)$$

$$C = \frac{K\varepsilon 0A}{d} \quad \text{------(14)}$$

This expression for the capacitance for the parallel plate capacitor, clearly the capacitance of a parallel plate capacitor is

(i) Directly proportional to the area of each plate.
(ii) Directly proportional to the dielectric constant (or permittivity)of the medium.
(iii) Inversely proportional to the distance between the plates.
(iv) Independent of metal of plates.

Thus for high capacitance of a parallel plate capacitor.

(i) Area(A) of the plates should be large
(ii) The separation(d) between the plates should be small
(iii) The medium between the plates should be of high electric constant (K).

5.9 DIELECTRIC CONSTANT

If medium between the plates be air (or vacuum); then K=1,therefore capacitance of air capacitor

$$C_0 = \frac{\varepsilon 0 A}{d} \qquad \text{...... (15)}$$

Dividing equation(14) by (15)

$$\frac{C}{C0} = K$$

$$\text{Or } C = KC_0 \qquad \text{....... (16)}$$

This shows that medium of dielectric constant K is introduced between plates of parallel plate capacitor,the capacitance of the capacitor increases K-times.

From the equation(16), the dielectric constant of the medium may be define as the ratio of capacitance of the capacitor filled with medium to the capacitance of the same capacitor filled with the vacuum (or air).

5.10 INCREASE OF CAPACITANCE WITH IN THE DIELECTRIC MEDIUN

Suppose a dielectric medium is filled between the plates of the parallel plate capacitor.Every matter is constituted of molecules or atoms. In an atom positive charge is concentrated at the nucleus and the negatively charged electrons revolve around the nucleus in orbits. In dielectric medium the electron are strongly bound to the nucleus and in general the centre of positive and negative charges in each atom/ molecules coincide. When capacitor is charged, an electric field is established between the plates of the capacitor. Due to this electric field, the centres of positive charges are displaced along the direction of electric field or towards plate B; while the centres of negative charges are displaced opposite to the direction of electric field or towards the plate A. Thus the centres positive and negatively charges of each molecules/atom are displaced and molecule is said to be polarized. This causes an electric field E_i between the dielectric medium,which is opposite to direction of electric field produced due to charges on the plates. Thus,due to presence of dielectric medium,the resultant electric field between the plates is reduced and hence the potential difference (V=Ed) across the plates is reduced. Consequently the capacitance of capacitor ($C=Q/V_{AB}$) is increased.

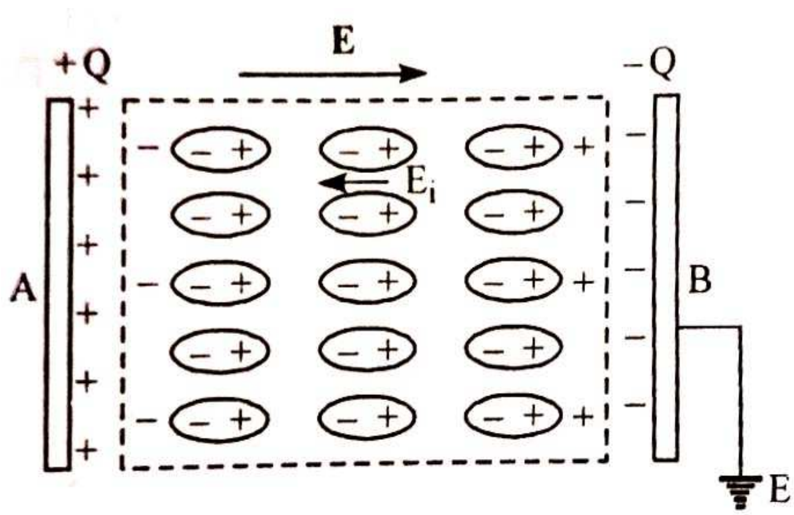


Figure 3

5.11 DIELECTRIC STRENGTH

When potential difference between the plates of capacitor is increased continuously the electric field between capacitor plates will go on increasing and consequently the separation between positive and negative charges will go on increasing and a stage will come when the opposite charges of molecule will break off from molecule and become free. In this situation the dielectric willnot remain insulating conductor. As a result capacitor will be discharged.

The minimum electric field strength applied to dielectric at which its electric breakdown takes place is called the dielectric strength. In other words, "The dielectric strength of a dielectricis the maximum electric field that it can withstand without breakdown of its insulation property". It is constant for a given dielectric. The minimum value of potential difference across capacitor plates at which dielectric breaks down is called the breaking potential difference. It is to be noted that dielectric strength for a material remains fixed,but the breaking potential depend on the thickness of dielectric, i.e.

Breaking potential difference =dielectric strength x thickness.

The dielectric strength of the vacuum is infinity,for air it is 3×10^6V/m,for plastic it is 10^7V/m and for mica it is 1.6×10^8V/m.

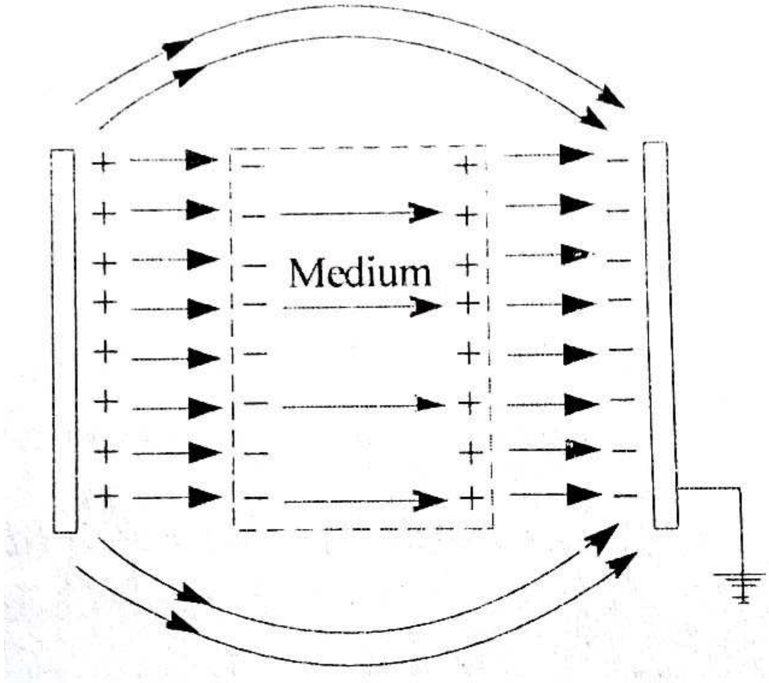


Figure 4

5.12 PARALLEL PLATE CAPACITOR WITH A DIELECTRIC

Consider a parallel plate capacitor, area of each plate being A, the separation between the plates being d. Let a dielectric slab of dielectric constant K and thickness t< d be placed between the plates. Thickness of air between the plates = (d-t). If charge on plates be +Q and –Q, then surface charge density

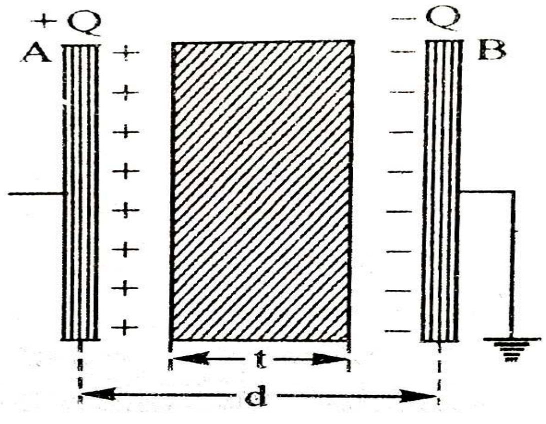


Figure 5

$\sigma = \frac{Q}{A}$

The electric field between the plates in air

$E_1 = \frac{\sigma}{\varepsilon 0} = \frac{Q}{\varepsilon 0 A}$

The electric field between the plates in slab

$E_2 = \frac{\sigma}{K\varepsilon 0} = \frac{Q}{K\varepsilon 0 A}$

The potential difference between the plates

V_{AB}= Work done in carrying unit positive charge from one plate to another

= ΣEx (as field between the plates is not constant)

$= E_1(d-t)+E_2 t = \frac{Q}{\varepsilon 0A}(d-t) + \frac{Q}{K\varepsilon 0A}t$

$V_{AB} = \frac{Q}{\varepsilon 0A}[\, d-t+ \frac{t}{k}]$

Capacitance capacitor, $C = \frac{Q}{VAB} = \frac{Q}{\frac{Q}{\varepsilon 0A}[\, d-t+\frac{t}{k}]}$ ……(17)

$C = \frac{\varepsilon 0A}{[\, d-t+\frac{t}{k}]} = \frac{\varepsilon 0A}{d-t(1-\frac{1}{k})}$

This is the required expression. As K> 1, it is obvious that due to introduction of slab of thickness t and dielectric constant K between the plates of a parallel plate capacitor, the effective distance in air is reduced by $(1-\frac{1}{K})t$; and so the capacitance of capacitor increases.

5.13 FORCE BETWEEN PLATES OF A CHARGED PARALLEL PLATE CAPACITOR

The plates of the parallel plate capacitor are oppositely charged, hence each plate must experience force of attraction.

Consider parallel plate capacitor of plate area A and separation between the plates d. Each charged plate produces an electric field and the other plates are placed in the vicinity of this electric field. Let Q be the charge and σ the surface charge density on each plate. Clearly

$\sigma = \frac{Q}{A}$

The electric field produced due to either charged plates,

$E_1 = \frac{\sigma}{2\varepsilon 0}$ .... (18)

Because charge on plate is accumulated on one side, Due to this electric field,the force of attraction on other plate =QE_1

$= Q\frac{\sigma}{2\varepsilon 0}$ (using (18) .... (19)

If E is the electric field between the plates, then

$E = \frac{\sigma}{\varepsilon 0}$

or

$\sigma = \varepsilon_0 E$ .... (20)

So Force of attraction between the plates

$$F = (1/2)QE \quad .... (21)$$

This is required expression .The factor ½ appears because the electric field in the vicinity of charge Q is produced by one plate only,so

$E_1 = 1/2\ E.$

If we put $E = \frac{\sigma}{\varepsilon 0} = \frac{Q}{A\ \varepsilon 0}$ in equation (21),we get

$$F = \frac{Q2}{2\ \varepsilon 0\ A} \quad ....(22)$$

5.14 COMBINATION OF THE CAPACITORS

If the capacitor of required capacitance is not available,then the two and more capacitors may be combined to provide the required capacitance.There are two main method of combination.

1. **Series combination**

The reduced the capacitance,the capacitors are connected in series. In this combination the first plate of first capacitor is connect to first plate of second capacitor,the second plate of second capacitor is connected to first plate to third plate capacitor and so on ;the second plate of last capacitor is connected to the earth.In fig.6 three capacitors of capacitance C_1,C_2,C_3 are connected in series between point A and D.

Suppose by means of electric source a charge +Q is given to first plate of first capacitor C1. By induction –Q charge is induced on the inner surface of second plate of first capacitor and a +Q on inner surface of first plate of second capacitor $C_2$and so on (fig 6).Thus first plate of each capacitor has charge +Q and second plate of each capacitor has charge –Q.

Let the potential difference across the capacitor C_1,C_2,C_3 be V_1,V_2,V_3 respectively. As the second plate of first capacitor C_1 and first plate of second capacitor C_2 are connected together, therefore their potentials are equal. Let this common potential be V_B. Similarly the common potential of second plate of C_2 and first plate of C_3 is V_c. The second plate of capacitor C_3 is connected to the earth,therefore its potential V_D =0. As charge flows from higher potential to lower potential,therefore $V_A>V_B>V_C>V_D$.

For the first capacitor $V_1 = V_A - V_B = \frac{Q}{C1}$ .... (23)

For the second capacitor $V_2 = V_B - V_c = \frac{Q}{C2}$ .... (24)

For the third capacitor $V_3 = V_c - V_D = \frac{Q}{C3}$ .... (25)

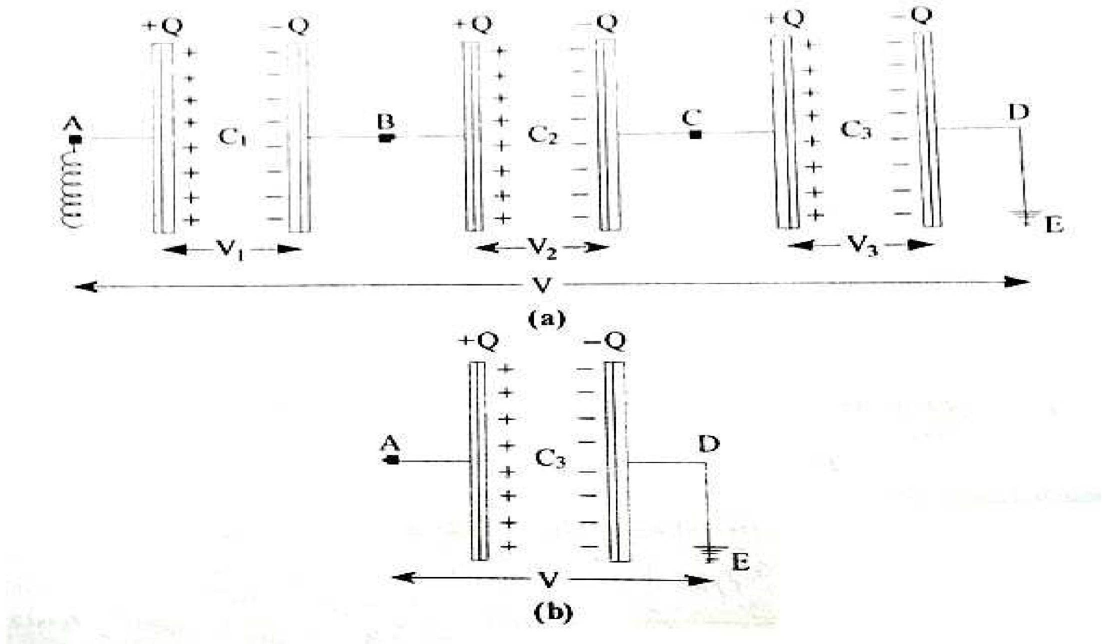


Figure6

Adding equations (23),(24) and (25),we get

$V_1+V_2+V_3 = V_A - V_D = [1/C_1+1/C_2+1/C_3]$ .... (26)

If V be the potential difference between A and D, then

$V_A - V_D = V$

From (26) we get

$V= (V_1+V_2+V_3) = Q\ [1/C_1+1/C_2+1/C_3]$ ......(27)

Three capacitor, only one capacitor placed between A and D such that on given in charge Q,the potential difference between its plates become V , then it will be called equivalent capacitor.If its capacitance be C then

$V = \frac{Q}{C}$ .... (28)

Comparing equation (27) and (28), we get

$\frac{Q}{C}$ = Q [$1/C_1+1/C_2+1/C_3$] or $1/C = 1/C_1+1/C_2+1/C_3$ ....(29)

(i) Thus in series arrangement, “the reciprocal of equivalent capacitance is equal to the sum of reciprocal of the individual capacitor”. Infact the equivalent capacitance is even less than the lowest capacitance in series.

(ii) The charge of each capacitor is same.

(iii)The total potential difference applied across the combination is equal to the sum of potential difference across the individual capacitors,i.e.,$V=V_1+V_2+V_3$. Therefore the series arrangement is used to divide a high voltage (which cannot be tolerated by single capacitor)among several capacitors.

Remarks: If n capacitors of capacitance C_1, C_2, C_3, ------------C_n are connected in series,the net capacitance C will be given by

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}=...........+\frac{1}{C_n}$$

2. **Parallel Arrangement:** To increase the capacitance,the capacitors are connected in parallel in this combination the first plate of each capacitor is connect to a common point A and second plate to another common point B. The point A is connected to electric source and point B is connected to earth. In figure 7 three capacitors of capacitance C_1,C_2,C_3 are connected in parallel.

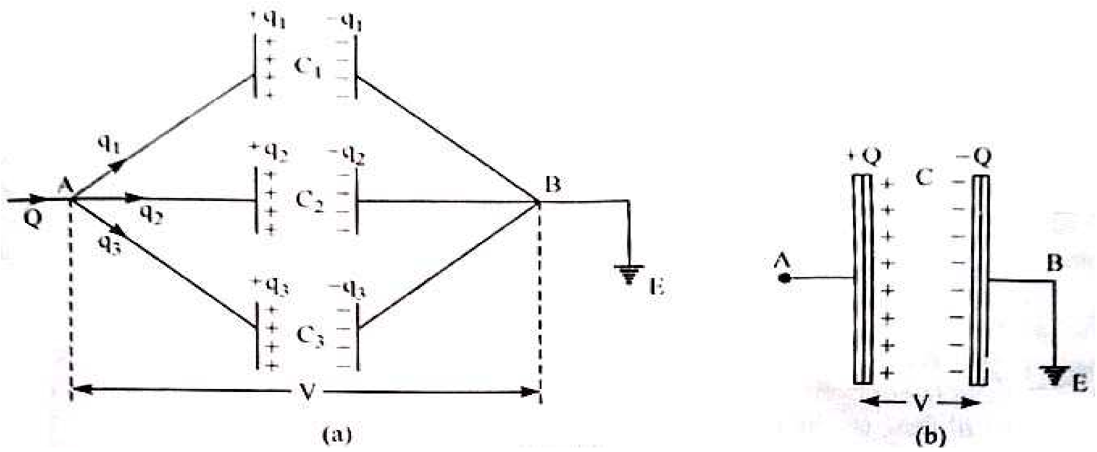


Figure7

Let a charge Q be given to point A by means of an electric source. The first plate of each capacitor will be at potential A and second plate will be at zero potential, because it is connected to each other. Clearly the potential difference between the plates of each capacitors.

$$V_A - V_B = V_A = V(say)$$

The charge Q will be divided on capacitors C_1, C_2, C_3 .
The charge q_1, q_2, $q_3$be the charge on capacitors C_1,C_2,C_3 respectively.

$$Q=q_1+q_2+q_3 \quad .... (30)$$

$$q_1=C_1V,\ q2=C_2V,\ q_3=C_3V$$

Substituting these values in (30), we get

$$Q=C_1V+C_2V+C_3V \quad .... (31)$$

If, in place all three capacitors,only one capacitor of capacitance C be connected between A and B;such that on giving it charge Q,the potential difference between its plates be V,then it will be called equivalent capacitor.
If C be the capacitance of equivalent capacitor, then

$$Q = CV \quad .... (32)$$

Comparing equations (31) and (32), we get

$$CV= (C_1+C_2+C_3)\ V$$

$$C= C_1+C_2+C_3 \quad .... (33)$$

Thus in parallel arrangement

(i) The equivalent capacitance is equal to the sum of capacitances of individual capacitors ($C= C_1+C_2+C_3$).
(ii) The total charge is equal to the sum of charges onindividualcapacitors ($Q=q_1+q_2+q_3$).
(iii) The potential difference across each capacitor is same .

Remarks: If n-capacitors of capacitance C_1,C_2,C_3**-------**C_n be connected in parallel, the net capacitance

$$C=C_1+C_2+C_3+\text{-----------}+Cn$$

5.15 SPHERICAL CAPACITOR

Aspherical capacitor consists of two concentric metallic spheres.A and B of radii a and b respectively (b>a) insulated from each other by dielectric of permittivity ε. Let us find the capacitance of spherical capacitor in the following cases.
Case (i).When the sphere is earthed. If the inner sphere A be given a charge +Q,then a charge –Q will be induced on the inner surface of the sphere B and a charge +Q on the outer surface of outer sphere. As the sphere is earthed,the charge +Q induced on the outer surface of outer sphere B will flow to the earth.

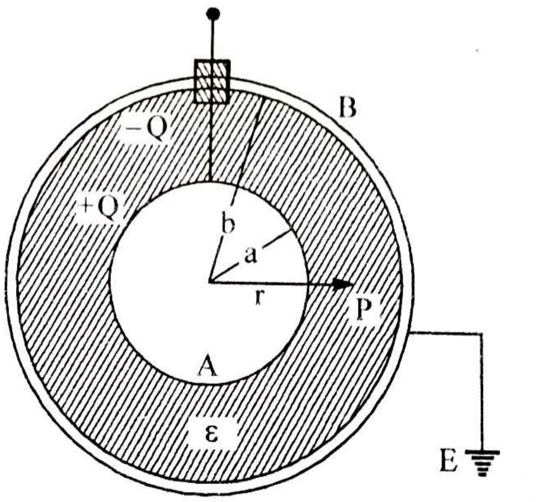


Figure8

Now the electric field strength at a point P distant r from the centre O and within the concentric spheres is entirely due to the charge +Q on the inner sphere and is given by

$$E = \frac{1}{4\pi\varepsilon 0}\frac{Q}{r2}\,r^{\wedge}$$

Where $r^{\wedge}$ is the unit vector along OP

The potential difference between two sphere is then,given by

$$V=-\int_b^a E.dr=-\int_b^a \frac{1}{4\pi\varepsilon}.\frac{Q}{r2}r^{\wedge}.dr$$

$$= - -\frac{Q}{4\pi\varepsilon}\int_b^a \frac{1}{r^2}dr = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r}\right]$$

$$=\frac{Q}{4\pi\varepsilon}\left[\frac{1}{a}-\frac{1}{b}\right]$$

If K is dielectric constant of medium,then $\mathcal{E} = K\,\mathcal{E}_0$

From which the capacitance of spherical capacitor is given by

$$C=\frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi K\varepsilon 0}\left[\frac{1}{a}-\frac{1}{b}\right]}=\frac{4\pi K\varepsilon 0}{\left[\frac{1}{a}-\frac{1}{b}\right]} = \frac{4\pi K\varepsilon 0}{\left[\frac{1}{a}-\frac{1}{b}\right]} = 4\pi K\varepsilon 0\,\frac{ab}{b-a}$$

$$C=4\pi K\varepsilon 0\,\frac{ab}{b-a}$$

Case (ii). When inner surface is earthed: If charge +Q be given to the outer spherical shell B of innerand outerradii b and c respectively,the charge +Q is distributed into two parts (i) charge $+Q_i$ spreads on the inner surface of radius b and (ii) charge $+Q_0$ spreads on the outer surface of radius c such that

$Q=Q_i+Q_0$

Due to induction the Charge –Qi is induced on the inner sphere is earthed,the inner sphere is at zero potential.

If the surrounding objects are at infinite distance from the outer sphere and at zero potential, also if the medium between the outer sphere and infinity of permittivity Ɛ, the electric field strength at a point for which

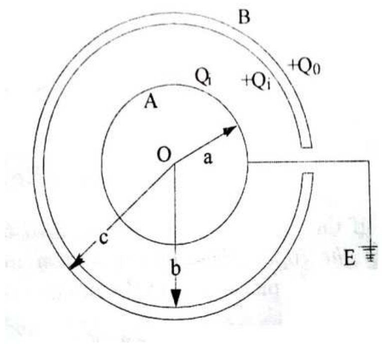


Figure 9

r > c is

$$\vec{E}_0 = \frac{1}{4\pi\varepsilon_0}\frac{Q_0}{r^2}\hat{r} \qquad ....(34)$$

and electric field strength at a point for which a<r<b is

$$\vec{E}_i = \frac{1}{4\pi\varepsilon_0}\frac{Q_i}{r^2}\hat{r} \qquad ......(35)$$

As the potential at infinity and also that of inner sphere is zero,the potential of outer shell may be written as

$$V=-\int_{\infty}^{0} E_0 \,.\, dr = -\frac{1}{4\pi\varepsilon 0}\int_{\infty}^{0}\frac{Q_0}{r^2}\hat{r}.dr$$

$$= -\frac{Q_0}{4\pi\varepsilon 0}\int_{\infty}^{0}\frac{1}{r^2}dr = \frac{1}{4\pi\varepsilon 0}.\frac{Q_0}{c} \qquad .... (36)$$

We have $V = \frac{Q_i}{4\pi\varepsilon}\left[\frac{1}{a}-\frac{1}{b}\right]$ .... (37)

Comparing equation (36) and (37), we have

$$\frac{1}{4\pi\varepsilon 0}.\frac{Q_0}{c} = \frac{Q_i}{4\pi\varepsilon}\left[\frac{1}{a}-\frac{1}{b}\right]$$

$$\frac{Q_i}{Q_0} = \frac{K}{c}\frac{ab}{b-a}....(38)$$

As the total charge given to the shell B is $Q=Q_0+Q_i$, the capacitance of the arrangement is given by

$$C = \frac{Q_0+Q_i}{V} = \frac{Q_0+Q_i}{\frac{1}{4\pi\varepsilon 0}.\frac{Q_0}{c}} = 4\pi\varepsilon_0 c\left[1+\frac{Q_i}{Q_0}\right]$$

$$= \quad 4\pi\varepsilon_0 c\left[1+\frac{K}{c}.\frac{ab}{b-a}\right] \text{ from equation (38)}$$

$$= \quad 4\pi\varepsilon_0 c+4\pi\varepsilon_0 K\frac{ab}{b-a} \qquad .... (39)$$

If the outer sphere is surround by concentric earthed sphere of radius d, then capacitance of the system may be calculated by similar procedure,keeping d in place of ∞ in the integral of eq. (36) this result is

$$C = 4\pi\varepsilon_0 K\frac{ab}{b-a} + 4\pi\varepsilon_0\frac{cd}{d-c} \qquad .... (40)$$

5.16 CYLINDRICAL CAPACITOR

The cylindrical capacitor consists of a long metal cylinder A of radius a surrounded by an earthed metallic concentric cylindrical shell B of inner radius b. The space between the two cylinders is small in comparison with their lengths and is filled with a dielectric of permittivity ε. If the charge +Q is given to the inner cylinder,then an equal charge –Q is induced on inner surface of outer cylinder and a charge +Q on the out surface of the outer cylinder. As the outer cylinder is earthed,the charge +Q induced on the outer surface of outer cylinder flows to earth. If the length *l* of the two cylinders is large compared with separation (b-a),the charge Q can be considered to be distributed uniformly over the two cylinders. The charge per unit length is thus $\lambda=\frac{Q}{l}$.

The electric field strength at a point P in the space between the two cylinders at a distance r from the axis is entirely due to charge +Qon the inner cylinder and is directed radially away. It is given by

$$E=\frac{\lambda}{4\pi\varepsilon r}\hat{r} \qquad .... (41)$$

The potential difference between the outer and inner cylinders i.e., the potential of the inner cylinder is now given by

$$V=-\int_b^a E.dr = -\int_b^a \frac{\lambda}{4\pi\varepsilon r}\hat{r}.dr$$

$$=\frac{\lambda}{4\pi\varepsilon}\int_b^a \frac{1}{r}dr$$

$$=\frac{\lambda}{4\pi\varepsilon}\log_e\frac{b}{a}$$

The capacitance per unit length of cylindrical capacitor is given by so

$$C = \frac{chargeperunitlength}{potentialdiffrencebetweentwocylinder}$$

$$\frac{\lambda}{\frac{\lambda}{4\pi\varepsilon}\log_e\frac{b}{a}} = \frac{2\pi\mathcal{E}}{\log_e\frac{b}{a}} \qquad \text{......... (42)}$$

If K is dielectric constant of medium between the places, then$\mathcal{E}$ =ε_0K

$$C = \frac{2\pi\varepsilon_0 K}{\log_e\frac{b}{a}}$$

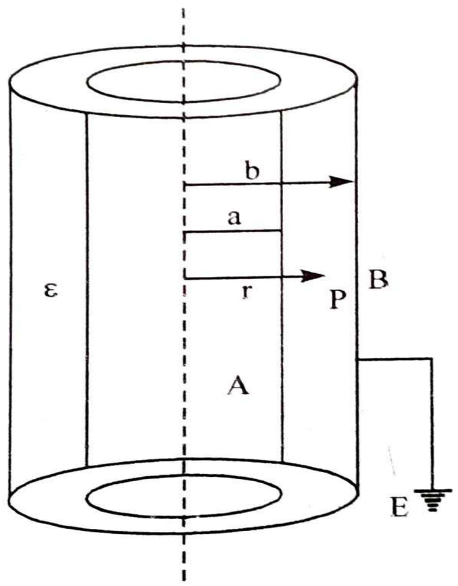


Figure 10

The capacity of the cylindrical conductor of the length l is , therefore, given by

$$C_l = \frac{2\pi\varepsilon_0 Kl}{\log_e\frac{b}{a}} \qquad \text{........ (43)}$$

This type of cylinder is of great practical importance. For example,coaxial cables consist of cylindrical metal shield a coaxial central conductor and an interposed dielectric. These are widely used in the transmission of high frequency signals. In their usethecapacitance introduced by them is taken into account.

A submarine cable is also an example of cylindrical conductor. The copper cable forms the inner cylinder and sea water works as outer earthed cylinder. The insulating material plays the role of the dielectric between two cylinders. The capacitance per unit length of the submarine cable and co-axial cables is given by equation (42).

5.17 ENERGY STORED IN A CAPACITTOR

Let us consider a capacitor of capacitance C which is given total charge Q coulombs in small instalments. Suppose during the process of charging, the charge at any instant on the capacitor is q. At this instant the potential difference between the plates of the capacitor is v=$\frac{q}{c}$. If a further charge dq is given to the capacitor,the work will have to be done against this potential difference. This work done is

$$dW = v.dq = \frac{q}{c}dq \left[since v = \frac{q}{c}\right]$$

Therefore the total amount of work done in charging the capacitor from charge 0 to Q coulombs is

$$W=\int_0^Q \frac{q}{c}dq = \frac{1}{c}\left[\frac{q^2}{2}\right]_0^Q = \frac{Q^2}{2c} \text{ joules} \qquad .... (44)$$

If v is potential difference between the plates of the capacitor when it has charge Q, then

$$Q=CV$$

$$W=\frac{(CV)^2}{2c} = \frac{1}{2}CV^2 \text{joules} \qquad .... (45)$$

This is energy stored in the capacitor .This energy resides in the dielectric.

For a parallel plate capacitor having area of each plates A,separation between the plates d and the medium between the plates of permittivity Ɛ

Capacitance C= $\frac{\varepsilon A}{d}$; and electric field strength,E=$\frac{V}{d}$

So that

$$W=\frac{1}{2}.\frac{\varepsilon A}{d}.(Ed)^2 = \frac{\varepsilon E^2}{2}.Ad \qquad .... (46)$$

Therefore the energy stored per unit volume in the electric field of strength E

$$W = \frac{\varepsilon E^2}{2} \text{ joules/meter}^3 = \frac{\varepsilon_0 K E^2}{2} Jm^{-3} \qquad ......(47)$$

5.18 Terminal Questions

Long type Questions:

1. What do you understand by dielectric polarisation? Explain the electric field vector E, Electric polarisation vector P and Electric displacement vector D in a dielectric material and deduce a relation between them.
2. Define displacement vector D and deduce relation between D and E.
3. Deduce relation D = ε0E for dielectric material filled in parallel plate condenser.
4. Derive an expression for the capacity of a parallel plate capacitor with space between the plates partly filled with of dielectric substance.
5. Derive an expression for the energy stored by a charged capacitor.

Short type Questions:

1. What is dielectric? Give some examples.
2. Define electric polarisation vector P and displacement vector D.
3. What is the relation between vector P and vector E? What is unit of vector P?
4. Differentiate between vector D, E and P.
5. Write an expression for the capacitance of a parallel plate capacitor. On what factors does it depend?

Objective type Questions:

1. Unit of Polarisation vector Pis:
(a) Coulomb (b) Coulomb metre (c) Coulomb/metre-2 (d) Newton/Coulomb
2. Unit of Displacement vector Dis:
(a) Coulomb (b) Coulomb- metre (c) Coulomb/metre^{-2} (d) Coulomb/metre^{2}
3. Displacement vector D depends upon:
(a) Charge (b) Medium (c) Dielectric (d) None of above
4. Relation between Vector P and Vector E:
(a) $P = \chi E \varepsilon 0 E$ (b) $P = \varepsilon 0KE$ (c) $P = \chi E E$ (d) $P = (\chi E-1)E$
5. The relation between the three electric vectors E, D and P is:
(a) $D=P+E$ (b) $D= P/E$ (c) $D= \varepsilon E+P$ (d) $D = \varepsilon (E+P)$
6. When a dielectric is introduced between the plates of a parallel plate air capacitor, its capacitance:
(a) Decreases (b) increases (c) remains unchanged
(d) may decrease or increase depending on the nature of dielectric

5.19 Answers

1. (c) 2. (d) 3.(c) 4. (a) 5. (c) 6. (b)

5.20 SUGGESTED READINGS

1. Electricity & Magnetism, D.C. Tayal, Himalaya publishing House
2. Electricity, Magnetism and Electronics, S.I. Ahmad and K.C. Lal,
Unitech House, Lucknow
3. Fundamental of Electricity and Magnetism,R.G. Mendiratta and B.K.Sawhney, East-West Press Pvt Ltd

UNIT 6 MAGNETIC MATERIALS, MAGNETIC SUSCEPTIBILITY, HYSTERESIS LOOP

Structure

6.1 INTRODUCTION

Mossotti and Clausius have done a systematic investigation about the dielectric properties of materials. They attempted to correlate the specific inductive capacity, a macroscopic characteristic of the insulator introduced by Faraday which is now popularly termed as dielectric constant with the microscopic structure of the material. Following Faraday in considering the dielectrics to be composed of conducting spheres in a non-conducting medium, Clausius and Mossotti succeeded in deriving a relation between the real part of the dielectric constantand the volume fraction occupied by the conducting particles in the dielectric.

In the beginning of 20th century, Debye realized that some molecules had permanent electric dipole moments associated with them, and this molecular dipole moment is responsible for the macroscopic dielectric properties of such materials. Debye succeeded in extending the Clausius - Mossotti theory to take into account the permanent moments of the molecules, which allowed him and others to calculate the molecular dipole moment from the measurement of dielectric constant. His theory was later extended by Onsager and Kirkwood and is in excellent agreement with experimental results for most of the polar liquids. Debye's other major contribution to the theory of dielectrics is his application of the concept of molecular permanent dipole moment to explain the anomalous dispersion of the dielectric constant observed by Drude. For an alternating field, Debye deduced that the time lag between the average orientation of moments and the field becomes noticeable when the frequency of the field is within the same order of magnitude as the reciprocal relaxation time. This way the molecular relaxation process leads to the macroscopic phenomena of dielectric relaxation, i.e., the anomalous dispersion of the dielectric constant and the accompanying absorption of electromagnetic energy over certain range of frequencies.

6.2 OBJECTIVE

The Main objectives of the present unit are-

- To define the electric susceptibility
- To define the permittivity
- To define the Polarizability and their types
- To define about Claussius –Mossotti Equation
- To Explain Debye Equation or Langevin-Debye Theory of Polarisation in Polar Dielectrics
- Behaviour of dielectric material in an alternating electric field

6.3 ELECTRIC SUSCEPTIVILITY

When a dielectric material is placed in an electric field, it becomes electrically polarised. In most cases *i.e.* for isotopic dielectrics (whose electrical properties are identical in all directions), the degree of polarisation **P** is found to be proportional to the intensity of electric field **E** at a given point of dielectric provided the field is not very strong.

$\boldsymbol{P} \propto \boldsymbol{E}$

$\boldsymbol{P} = \chi_e \boldsymbol{E}$ .... (1)

$\boldsymbol{P} = \varepsilon_0 \chi_e \boldsymbol{E}$ .... (2)

The constant χ_e is called the electric susceptibility of the dielectric material and vector E is the electric field within the dielectric.
The total proportionality factor $\varepsilon_0 \chi = \chi_e$ is known as absolute susceptibility or dielectric susceptibility.
The electric susceptibility of a dielectric may be defined as the ratio of the polarisation to the electric intensity in the dielectric. Since polarisation P equals the surface density of induced charge, the susceptibility may also be defined as the ratio of induced charge density to the electric intensity.

Hence the units of susceptibility are those of surface density divided by electric intensity

$$\chi_e = \varepsilon_0 \chi = \frac{P}{E} = \frac{Coulomb/\mathrm{m2}}{Newton/Coulomb} \quad \text{....... (3)}$$

χ_e = Coulomb^2 / Newton-meter^2

AlsoFarad = Coulomb^2 / Newton-meter

$[\because \text{volt} = \text{Newton}\frac{\text{m}}{\text{coul}} \text{ and Farad} = \text{Coul./Volt}]$

$$\therefore \quad \chi_e = \frac{Farad}{meter} \quad \text{....... (4)}$$

It value for vacuum is zero. The polarization of dielectrics whose molecules are permanent dipoles depend on temperature. Hence such dielectrics show a dependence of susceptibility on temperature while non-polar dielectrics do not.

6.4 RELATION BETWEEN DIELECTRIC CONSTANT AND DIELECTRIC SUSCEPTIVILITY

We have following two equations

$\boldsymbol{P} = (K - 1)\, \varepsilon_0 \boldsymbol{E}$

and $\boldsymbol{P} = \chi\, \varepsilon_0 \boldsymbol{E}$

Hence by comparison of above two equations, we can write

$\chi = (K - 1)$ or $K = \chi + 1$ .... (5)

Equation (3) can also be written in another form by using relation $\chi = \chi_e / \varepsilon_0$

$$K = 1 + \frac{\chi_e}{\varepsilon_0} \quad \text{....... (6)}$$

The value of K for all dielectrics is greater than one. Since for empty space χ_e is zero, the value of K is 1.

6.5 PERMITTIVITY

We have $D = K\,\varepsilon_0 E$

The product $K\,\varepsilon_0$ is called permittivity of the dielectric and is represented by ε that is

$$\varepsilon = K\,\varepsilon_0 \quad ....\ (7)$$

in empty space $K = 1$, so that $\varepsilon = \varepsilon_0$. The quantity ε_0 is therefore correctly described as 'permittivity of empty space'.

Also $$K = \frac{\varepsilon}{\varepsilon_0} ....(8)$$

K is also known as 'relative permittivity' of the dielectric. When a dielectric is placed in electric field, the distribution of field changes to a degree depending upon relative permittivity.

Now, we can write

$D = K\,\varepsilon_0 E$

Or $D = \varepsilon E$

Or $$\varepsilon = \frac{D}{E} \quad ....\ (9)$$

Hence the permittivity of a dielectric medium is the ratio of electric displacement to the electric intensity in the dielectric.

Problem 1: The electrical susceptibility of a material is $35.4\text{x}10^{-12} C^2 N^{-1} m^{-2}$. What are the value of the dielectric coefficient and the permittivity of the material?

Solution**:** The dielectric coefficient k of a material is related to its electric susceptibility χ_e by

$$K = 1 + \chi_e / \varepsilon_0$$
$$= 1 + 35.4\text{x}10^{-12} / 8.85\text{x}10^{-12}$$
$$= 1 + 4 = 5$$

The permittivity is

$$\varepsilon = K\,\varepsilon_0$$
$$= 5\,(8.85\ \text{x}10^{-12})$$
$$= 44.3\text{x}\ 10^{-12}\ \text{coul}^2 / \text{newton-m}^2$$

6.6 MICROSCOPIC VIEW OF POLARIZATION

When a dielectric substance is subjected to an external electric field $\mathbf{E_0}$, the electric field acting on an atom or molecule within the substance is not the same as the external field. It is somewhat different. The calculation of electric field acting on an atom or molecule is a major problem of dielectric theory. We call this field local or internal electric field $\mathbf{E}_{local}$ or $\mathbf{E}_i$. It is this field which acting on an atom or molecule induces dipole moment $\mathbf{P_i}$. Obviously, the induced dipole moment is proportional to the local electric field.

$$p_i \propto E_{local}$$
$$p_i = \alpha E_{local}$$

Here α is proportionality constant and is called polarisability. It is a microscopic parameter of dielectric and cannot be measured directly in laboratory. If the substance contains n atoms/molecules per unit volume then polarisation vector **P** is given by

$$p = n\, p_i = n\alpha E_{local}$$

Unit of α: $\alpha = \frac{p}{E} = \frac{coul.meter}{volt/meter} = \frac{coul.}{volt}\text{meter}^2 = \text{farad.meter}^2$

6.7 KIND OF POLARIZABILITY

The magnitude of polarisability is measure of ease with which an atom/molecule undergoes distortion under the action of an electric field. There are three kinds of polarisability:

- **(i)** Electronic
- **(ii)** Ionic or atomic
- **(iii)** Dipolar or orientational.

6.7.1 Electronic Polarisability (α_e)

In absence of any electric field on an atom the centre of negatively charged electron cloud coincides with the centre of positively charged nucleus. So the dipole moment of atom is zero. When an electric field $\mathbf{E}_0$is applied on a dielectric, its constituent atoms experience electric field $\mathbf{E}_{local}$. Under the action of this field, electron cloud shifts slightly in a direction opposite to the electric field and nucleus in opposite direction. The nucleus being much heavier than the electron cloud, its shift is negligibly small. The centres of negative and positive charge no longer coincide. This charge separation is attended with an induced dipole moment. The charge separation also results in a force of attraction between them which opposes the action of the electric field as a result of which equilibrium is soon established. Let x be the separation of centres of positive and negative charge. The electric field of electron cloud at the location of nucleus is

$$E = \frac{\rho x}{3\varepsilon_0} = \frac{Zex}{4\pi\varepsilon_0 a^3} = \frac{p_i}{4\pi\varepsilon_0 a^3}$$

Here $\rho = \frac{Ze}{(4/3)\pi a^3}$ is the volume density of charge, $p_i = Zex$ is the induced dipole moment Z = atomic number of atom, α = radius of atom. In equilibrium, $E = E_{local}$

$$E_{local} = E = \frac{p_i}{4\pi\varepsilon_0 a^3}$$

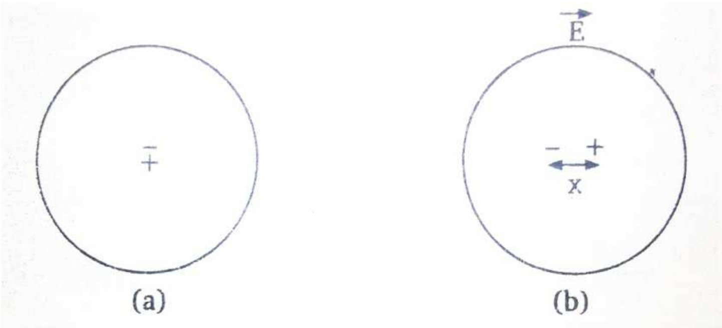


Figure 1 (a) Centre of positive and negative charge coincide (b) The electric field creates a separation between of centres of positive and negative charge.

The electronic polarisabilty α_e of the atom is given by

$$\alpha_e = \frac{p_i}{E_{local}} = 4\pi\, \varepsilon_0 \alpha^3 \qquad \text{....... (10)}$$

This shows that electronic polarisability is proportional to the volume of the atom.

6.7.2 Ionic (atomic) Polarisability(α_i)

This kind of polarisability occurs in ionic solids which are made up of ions (NaCl, HCl). Let a_0 be the separation of ions in absence of electric field. Under the action of electric field the positive ions are pulled on one side and the negative ions on the other. Thus the separation of cautions and anions is increased. This creates an induced dipole moment. The induced dipole moment p_i is proportional to the local field acting on the ions.

$$p_i = \alpha_i E_{local} \qquad \text{....... (11)}$$

where α_i is ionic polarisabilty.

6.7.3 Orientational Polarizability (α_0)

The orientational contribution to polarizabilty arises when the substance is built up of molecules possessing permanent dipole moment. In the absence of external electric field, the dipole moments are randomly oriented in all directions. When an electric field is applied, this is a tendency for the permanent dipoles to orient (align) themselves in the direction of the applied field thus producing a net dipolemoment. This mechanism is called orientational dipolar or polarisability. The induced dipole moment is expressed as

$$p_0 = \alpha_0 E_{local}$$

Where p_0 is the average value of induced dipole moment per molecule and α_0 is a constant called dipolar or orientational polarisability. Generally interfacial polarisability is neglected. Polarisability of such type is due to large number of defects in the structure of crystal (lattice vacancies, impurity centres, dislocation etc.)

Thus, dielectric polarisation is a sum of three contributions

$$P = P_e + P_i + P_o$$

Correspondingly

$\alpha = \alpha_e + \alpha_i + \alpha_0$

Or $\alpha = \alpha_d + \alpha_0$

Where $\alpha_d = \alpha_e + \alpha_i$ and called deformation polarisability. It results from the deformation of molecules caused by electric field.

Non – polar molecules can have only deformation (electronic and ionic) polarisabilty while polar molecules can have both deformation as well as orientational polarisability.

6.8 MOLECULAR FIELD OR LORENTZ LOCAL FIELD IN A DIELECTRIC

The polarisabilty α as a scalar quantity and has the dimension of volume *i.e.*, meter^3. Each of the three types of polarisability is a function of frequency of the applied voltage. The electric field which is responsible for polarising a molecule of the dielectric is called themolecular field or polarising field. If the dielectric is a gas (whose molecules are at large distances from one another), the polarising field is simply the externally applied field. In case of solid or liquid dielectrics, however, the actual field acting on a molecule of a dielectric is different from the external field. It includes not only the external field but also the field produced exception of the molecule under consideration (because it will not be polarised by its own field). This is known as local or internal or microscopic electric fieldacting on a molecule and is responsible for the polarisation of this particular molecule. Lorentz (1909) was the first to evaluate this field and hence it is named after him.

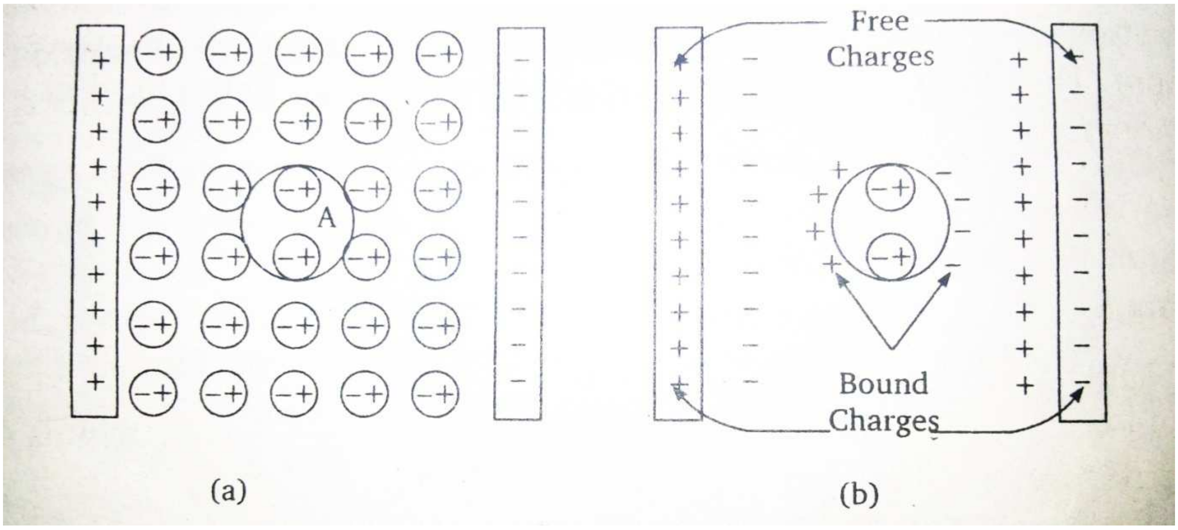


Figure 2

The following method suggested by Lorentz can be used to calculate the local field at a molecular position. Let the dielectric sample be polarised placing it in the uniform electric field between two parallel plates of a capacitor (figure 2). Suppose we want to calculate the field at position *A* of the molecule assuming that this molecule is not present at all. We draw a sphere around *A*, the size of which is big enough to contain a large number of molecules but small compared to the distance between the plates of the capacitor. The dielectric outside the sphere may be treated as a continuum of dipoles (macroscopic point of view). The molecules inside the sphere are, however, to be treated as individual dipoles. Now the local field at *A* is due to three sources:

(i) The external field E_0 which is determined by the free charges on the plates between which dielectric is placed.

$E_0 = \frac{\sigma}{\varepsilon_0}$,

Where σ = free charge density on plates of capacitor.

(ii) E_2 is the field at centre (*A*) of the sphere due to the bound charges on its surface. In order to calculate it *dS* be a surface element of the sphere with polar coordinates (*r*, θ). The component of the electric polarisation **P** normal to *dS* is *P* cos θ. The induced charge density the field due to polarised molecules (dipoles) of the dielectric outside the sphere.

To evaluate this contribution, the dielectric sample outside the sphere may be replaced by bound (induced) charges on the outer faces of the dielectric and also on the surface of the sphere as shown in fig.

Let E_1 be the depolarising field due to the bound charges on the outer faces of the dielectric and E_2 be the field due to the bound charges on the surface of the sphere. Then $E_1 = -\frac{\sigma'}{\varepsilon_0}$, where σ' is the bound charge density. The negative sign signifies that E_1 opposes E_0. E_2 shall be evaluated shortly.

(iii) The field (E_3) due to the polarised molecules within the sphere. This field is zero for many practical cases of gases, liquids and cubic crystals. We shall therefore ignore it here.

Hence the total electric field at *A* may be written as

$E_{local} = E_0 + E_1 + E_2$

$E_{local} = \frac{\sigma}{\varepsilon_0} - \frac{\sigma'}{\varepsilon_0} + E_2$

We know that the net macroscopic electric field within the dielectric is given by

$$E = \frac{\sigma}{\varepsilon_0} - \frac{\sigma'}{\varepsilon_0} \quad .... (12)$$

Therefore, $E_{local} = E + E_2$ .... (13)

Over *dS* is therefore *P* cos θ. The charge on *dS* is *P* cos θ *dS* where θ is the angle between direction of *P* (or *E*) is and the radius of the sphere.

The field at *A* due to the charge on *dS* is $1/4\pi\varepsilon_0(P \cos\theta\, dS/ r^2)$, where r is the radius of sphere. The field is directed from *A* to *dS*. The component of this field in the direction of **E** is

$$\left(\frac{1}{4\pi\varepsilon_0}\frac{P\cos\theta dS}{r^2}\right)\cos\theta = \frac{P\cos^2\theta dS}{4\pi\varepsilon_0 r^2}$$

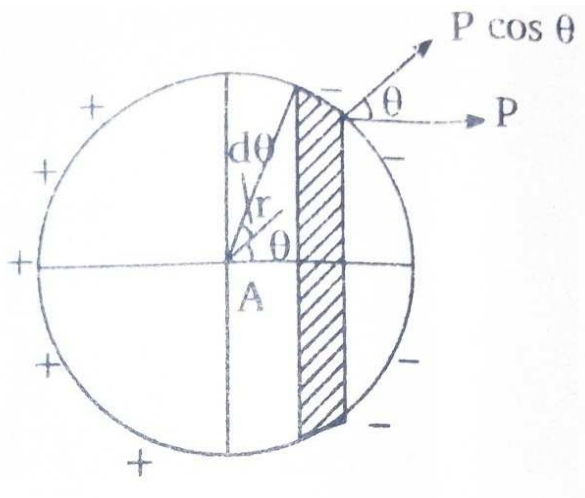


Figure 3

Now, suppose that dS is a ring shaped element (shown shaded) of radius $r\sin\theta$ and width $rd\theta$ on the surface of the sphere. The area of the element is
$dS = 2\pi(r \sin\theta)\, rd\theta = 2\pi r^2 \sin\theta d\theta$.
The component of the field at A perpendicular to **E** due to this ring is zero, since such components are symmetrically distributed around the axis. The component of the field along the direction of **E** is

$$\frac{P\cos^2\theta dS}{4\pi\varepsilon_0\, r^2} = \frac{P\cos^2\theta r^2(2\pi \sin\theta d\theta)}{4\pi\varepsilon_0\, r^2}$$

$$= \frac{P}{2\varepsilon_0} \cos^2\theta\ \sin\theta d\theta$$

The field E_2 at A due to the entire induced charge on the surface of the sphere is
$E_2 = \int_0^{\pi} \frac{P}{2\varepsilon_0} \cos^2\theta\ \sin\theta d\theta$

$$= \frac{P}{2\varepsilon_0} \int_0^{\pi} \cos^2\theta\ \sin\theta d\theta$$

$$= \frac{P}{2\varepsilon_0} \left[-\frac{\cos^3\theta}{3}\right]_0^{\pi} = \frac{P}{3\varepsilon_0}$$

In vector notation $\mathbf{E}_2 = \frac{\mathbf{P}}{2\varepsilon_0}$ .... (14)

Substituting $\mathbf{E}_2$ in equation (13)

$$\boldsymbol{E}_{\mathbf{local}} = \mathbf{E} + \frac{\mathbf{P}}{3\varepsilon_0} \qquad .... (15)$$

This is the actual field at the position of a molecule within the dielectric and is called 'Lorentz field equation.'

6.9 CLAUSIUS-MOSSOTTI EQUATION

Clausius and Mossotti tried to correlate the macroscopic properties of a dielectric with its microscopic character. They established a relation between the dielectric constant (a macroscopic parameter) and the molecular polarisability (microscopic parameter) of a non-polar dielectric. This relation is known as 'Clausius-Mossotti Equation.'

The polarisabilty (α) of a molecule is the dipole moment (p) induced in the molecule per unit polarising (local) field. That is

$$p = \alpha E_{\text{local}}$$

if there are n molecules per unit volume of the dielectric, then the polarisation P is given by

$P = \text{n}p = \text{n}\alpha E_{\text{local}}$.

We know that $E_{\text{local}} = \text{E} + \frac{\text{P}}{3\varepsilon_0}$, where E is the macroscopic field within the dielectric.

$\therefore P = \text{n}\alpha\,(E + \frac{P}{3\varepsilon_0})$

Now, the polarisation P is related to the dielectric constant K by the equation $P = (K - 1)\,\varepsilon_0 E$. Then we have

$$(K - 1)\varepsilon_0 E = \text{n}\alpha\,[E + \frac{(K-1)\varepsilon_0 E}{3\varepsilon_0}]$$

or $$(K - 1)\varepsilon_0 = \text{n}\alpha\,[1+ \frac{(K-1)}{3}] = \text{n}\alpha\,(\frac{K+2}{3})$$

or $$\alpha = \frac{3\varepsilon_0\,(K-1)}{\text{n}(K+2)} \qquad \text{....... (16)}$$

This is known as the 'Clausius – Mossotti Equation. If n is known, α can be calculated by measuring Kexperimentally.

Equation (16) reduces in another simple form by using the relation $K = \frac{\varepsilon}{\varepsilon_0}$.

That is

$$\frac{\varepsilon - \varepsilon_0}{\varepsilon + 2\varepsilon_0} = \frac{\text{n}\alpha}{3\varepsilon_0}$$

The magnitude of $\frac{\varepsilon - \varepsilon_0}{\varepsilon + 2\varepsilon_0}$ or $\frac{(K-1)}{(K+2)}$ is known as specific polarisation of a dielectric.

The Clausius-Mossotti Equation can also be written interms of the dielectric suscepetibilty χ_e by putting $K = 1 + \frac{\chi_e}{\varepsilon_0}$. That is

$$\alpha = \frac{3\varepsilon_0}{\text{n}} \frac{\chi_e}{\chi_e + 3\varepsilon_0} \qquad \text{.....(17)}$$

The Clausius – Mossotti Equation has been verified experimentally for a number of gases (hydrogen etc.). Since α is constant for a particular gas, $\frac{1}{\text{n}}\frac{(K-1)}{(K+2)}$ must be a constant. The value of n was varied by changing the pressure of hydrogen gas and the dielectric constant K was measured for various pressures. It was found that $\frac{(K-1)}{\text{n}(K+2)}$ was independent of pressure, thus verifying the equation.

Limitation of the Clausius-Mossotti Equation

In deriving above equation the field (E_3) due to polarised molecule within the sphere is supposed to be zero because of the following assumption:

(1) Since polarisation is considered as proportional to the field, it means the polarisation of the molecules is by elastic displacement only.

(2) Absence of short range interaction.

(3) Isotropy of the polarisability of the molecules.

All these condition are satisfied with neutral molecules having no constant dipoles *i.e.*, non-polar molecules. Thus the equation is valid for non-polar liquids and gases only. It does not hold for crystalline solids and polar molecules.

Atomic Radius: It can be shown that α is proportional to the cube of the radius of the molecule. Hence if K is found and n is known for a gas at a given temperature and pressure, the radius of the atom may be found for monoatomic gases.

6.10 DEBYE EQUATION OR LANGEVIN-DEBYE THEORY OF POLARIZATION IN POLAR DIELECTRIC

When a polar dielectric is placed in the electric field, two things happen. First, it displaces the centre of gravity of protons and electrons so that an extra dipole moment is induced giving the electronic polarisability. For the moment, we shall ignore this induced dipole contribution, but its effect will be added later. Second, the individual molecules experience torque which tends to align them with the field. But the alignment is not complete because of the thermal motion of the molecules which favour random orientations. The average alignment produced gives rise to a net dipole moment per unit volume. If the temperature of the specimen is raised, the polarisation becomes even smaller due to the increase in thermal agitation of the molecules. Thus, for polar dielectrics, the orientation polarisability and hence the dielectric constant and the electric suscepetibility do depend on temperature.

Let us now calculate the net dipole moment per unit volume created by alignment of the molecules at a temperature T. Let n be the number of molecules per unit volume of the specimen and θ be the angle which the permanent dipole moment $\mathbf{p_0}$of a molecule makes with the polarising field $\mathbf{E}$ (Here $\mathbf{E}$ means $\mathbf{E}_{local}$, the effective electric field for simplicity we avoid the subscript 'local').

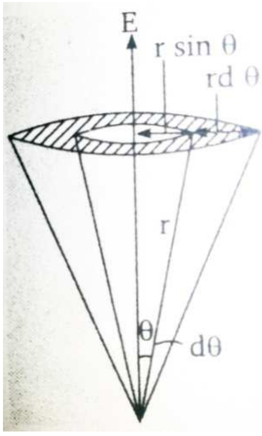


Figure 4

The potential energy of this molecule in the field is

$U = -p_{0}.\mathbf{E} = p_0 E cos\theta$ .....(18)

The average effective dipole moment per unit volume may be calculated from statistical probability law (Maxwell Boltzmann law) which states that at absolute temperature $T^{o}K$, the probability of finding a particular molecular energy *U* is proportional to $e^{-U/kT}$, where k is Boltzmann's constant.

Then the number of molecules (dipoles) per unit volume, *dn*, having energy and oriented at angles between θ and θ + *d*θ with respect to the direction of **E** is given by

$$dn = Ce^{-U/kT} d\omega \quad .....(19a)$$

Where *C* is a constant and *d*ω is the solid angle contained between θ and θ + *d*θ (figure 4). This angle is given by

$$d\omega = \frac{ring area between \theta \text{ and } \theta + d\theta}{r^2}$$

$$= \frac{2\pi\,(r\sin\theta)(rd\theta)}{r^2}$$

$$d\omega = 2\pi \sin\theta d\theta$$

Substituting this value of *d*ω in equation we get

$$dn = C^{(p0E\ cos\Theta)/kT}\ 2\pi\ \sin\theta d\theta$$

Let us write $\frac{p_0 E}{kT}$ =a and $2\pi c' = C$ (a new constant). Then

$$dn = Ce^{a\cos\theta}.\sin\theta.d\theta$$

The total number of molecules per unit volume is

$$n = \int dn = C' \int_0^{\pi} e^{a\cos\theta}.\sin\theta.d\theta$$

Now each of the *dn* molecules has a component of dipole moment $p_o \cos\theta$ along the direction of the field. The dipole moment of *dn* molecules along the field direction is thus $p_0 \cos\theta\ dn$. (By symmetry, the sum of the components at right angles of the field is zero).

Average dipole moment in the direction of applied electric field is given by

$$\overline{p} = \frac{\text{total dipole moment in the direction of field}}{\text{total number of dipoles}}$$

$$= \frac{\int_0^{\pi} p_0 \cos\theta dn}{\int_0^{\pi} dn} = \frac{p_0 \int_0^{\pi} e^{a\cos\theta} \cos\theta \sin\theta d\theta}{\int_0^{\pi} e^{a\cos\theta} \sin\theta d\theta}$$

Putting $\cos\theta = x$, $-\sin\theta d\theta = dx$ we have

$$\overline{p} = \frac{p_0 \int_{-1}^{1} x e^{ax} dx}{\int_{-1}^{1} e^{ax} dx} = p_0 \frac{d}{da} [\ln \int_{-1}^{1} e^{ax} dx]$$

$$= p_0 \frac{d}{da} [\ln (e^a - e^{-a}) - \ln a]$$

$$= p_0 \left[\frac{e^a + e^{-a}}{e^a - e^{-a}} - \frac{1}{a}\right]$$

$$= p_0 \left(\coth a - \frac{1}{a}\right) \qquad .....(19b)$$

$$= p_0 \boldsymbol{L}(a) \qquad .....(20)$$

Where $\boldsymbol{L}(a) = \coth a - \frac{1}{a}$ is called Langevin function. The polarisation P of the dielectric is

$$P = n\overline{p} = np_o\boldsymbol{L}(a) = np_o\left[\coth a - \frac{1}{a}\right] \qquad .....(21)$$

A plot of Langevin function $L(a)$ against a is shown in the figure (5). For large value of $a = \frac{p_0 E}{kT}$ *i.e.*, at large field strengths or low temperature

$$L(a) = \left(\coth a - \frac{1}{a}\right) \rightarrow 1 .....(22)$$

So $P \rightarrow np_0$ = saturation value of polarisation.

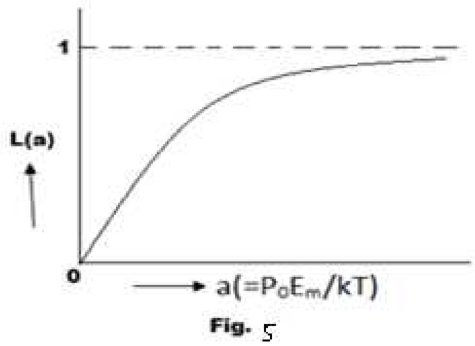


Fig. 5

It means the maximum value of the dipole moment per unit volume can be produced in the dielectric when all the molecular dipoles are perfectly aligned in the field direction. Thus large fields and low temperature causes P to approach its saturation value. This is clear from the latter part of the curve. In practice, however, the dielectric would break down (*i.e.* would become conducting) at such large fields.

At ordinary temperature, for fields even upto the dielectric strength a is small and the curve is linear. The dipole moment p_o of most polar materials is such that $a \ll 1$ ($\approx 10^{-3}$) for a full range of field strengths.

Since it is the linear region, which is important, it is appropriate to expand $L(a)$ in a power series of a and keep only the important terms.

$L(a) = \left(\coth a - \frac{1}{a}\right) = \frac{a}{3} - \frac{a^3}{45}$

Practically a is very small, hence,

$L(a) = \frac{a}{3}$

Now equation (19) reduces to

$P = np_0\frac{a}{3} = \frac{np_0^2 E}{3kT}$.....(23)

It follows from this equation that the polarisation P is linear function of polarising field E, as shown by the initial (Straight line) part of the curve. Thus a dielectric material containing polar molecules is, in general, linear.

From equation (21), the electric susceptibility χ_e of the dielectric material is given by

$\chi_e = \frac{\boldsymbol{P}}{\boldsymbol{E}} = \frac{np_0^2}{3kT}$

Thus the electric susceptibility and hence also the dielectric constant of a polar dielectric is inversely proportional to the absolute temperature. It is this feature which distinguishes a polar dielectric from a non-polar dielectric for which both the susceptibility and the dielectric constant are independent of temperature.

Now, the polarisability is defined as the dipole moment of a molecule per unit polarising field. Therefore, the polarisability α_0 due to the alignment of the molecular dipoles of the polar dielectric is given by, from equation (21).

$\alpha_0 = \frac{P}{nE} = \frac{np_0^2}{3kT}$

This result has been derived by neglecting induced dipole moments and represents the orientational polarisability. In fact, the induced dipole moments (which are responsible for the polarisation of non polar dielectrics) are also present in polar dielectrics. They give rise to 'deformation polarisability, α_d'. Thus the total polarisability of a molecule of polar dielectric is

$\alpha = \alpha_d + \alpha_0$

Or $\alpha = \alpha_d + \frac{p_0^2}{3kT}$ .....(24)

This equation is known as Langevin Debye equation and it has been of great importance in interpreting molecular structures.

The magnitude of α_d for both polar and non-polar dielectrics is of the same order. At ordinary temperatures, α_0 is much larger that α_d. This is because the permanent dipole moments, where they exits, are enormously larger than any induced moment. This is why the dielectric constant for a polar dielectric is higher than that for a non-polar dielectric. For example, the dielectric constant of water is about 80, while a typical non-polar liquid might have a dielectric constant around 2.

The above theory is valid for liquids and gases. In a solid dielectric the molecules are very density packed and so their mutual interactions cannot be ignored.

6.11 BEHAVIOUR OF DIELECTRIC MATERIAL IN AN ALTERNATING ELECTRIC FIELD: COMPLEX DIELECTRIC CONSTANT

When a dielectric material is kept in an alternating field, the macroscopic electrical field **E** as well as the polarisation vector **P** and displacement vector **D** become time dependent. In general, the polarisation vector **P** lags in phase over the electrical field **E** and thereby **D** too. This phenomenon can be represented geometrically by representing them in Argand plane as below.

Let the complex electric field be represented as

$\tilde{E} = E_0 e^{j\omega t}$ .....(25)

With peak value E_0 and frequency ω, then the polarisation vector **P** is represented as

$\tilde{P} = P_0 e^{j(\omega t - \theta)}$....(26)

Where θ is the phase angle.

The displacement vector (complex) is defined as

$$\tilde{D} = \varepsilon_0 \tilde{E} + \tilde{P}$$

$$= D_0 e^{j(\omega t - \varphi)} \quad .....(27)$$

Now, the complex permittivity of the dielectrics defined as

$$\tilde{\varepsilon} = \frac{\tilde{D}}{\tilde{E}} = \frac{D_0}{E_0} e^{-\omega\phi}$$

Or $\varepsilon_0 \tilde{K} = \frac{D_0}{E_0} e^{-j\phi}$ .....(28)

Where $\tilde{K}$ is the complex dielectric constant. $\tilde{K}$ may be represented as $K_r - jK_i$, where K_r and K_i are the real and imaginary parts of $\tilde{K}$.

Thus, with the help of equation (26), we have

$K_r - jK_i = \frac{D_0}{\varepsilon_0 E_0} e^{-j\phi}$....(29)

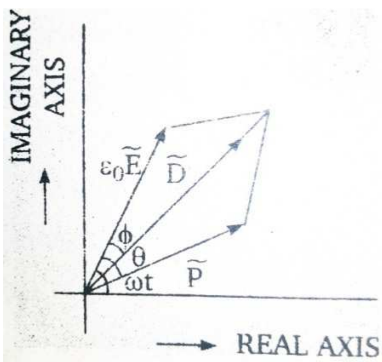


Figure 6

Now, equating real and imaginary parts, we have

$K_r = \frac{D_0}{\varepsilon_0 E_0} \cos\phi$

and $K_i = \frac{D_0}{\varepsilon_0 E_0}$ sinϕ.....(30)

It can be shown that imaginary part of dielectric constant is related to the power loss in the dielectric, which is delivered by the source. The heating effect of water in an alternating electric field is an example of this.

6.12 TERMINAL QUESTIONS

Long Type Question

1. Define polar and non-polar molecules. Deduce Clausius –Mossotti relation for non-polar dielectrics.
2. What is dielectric polarisation ? Give the Langevin's theory of polarization in polar dielectrics.
3. Explain Langevin-Debye theory of polarisation in polar dielectrics. Show that for polar dielectrics the susceptibility is inversely proportional to the absolute temperature.
4. Differentiate between electronic, ionic and orientational polarisability.
5.

Short Type Questions

1. Deduce relation between dielectric constant and electrical susceptibility of dielectric.
2. State Clausius-Mossotti relation.
3. What is the Electronic polarisability ?
4. What is the atomic polarisability ?
5. What is the orientational polarisability ?
6. Write Langevin-Debye equation of polar dielectrics.
7. Write the Limtation of Clausius-Mossotti equation.

Objective type questions

1. Langenin's Functions L(x) is :
   (a) Coth x + 1/x
   (b) Coth x-1/x
   (c) X coth x
   (d) Coth x-x
2. Electric susceptibility of a polar dielectric at absolute temperature T is :
   (a) Directly proportional to T
   (b) Inversely proportional to T
   (c) Directly proportional to T^3
   (d) Inversely proportional to T^3
3. Claussius- Mossotti equation does not hold for :
   (a) gases
   (b) liquid
   (c) crystalline solids

(d) none of these

4. The relation between dielectric constant K and dimensionless electric susceptibility χ_e is :
   (a) $K=1+ \varepsilon_0 \chi_e$
   (b) $K=1- \chi_e$
   (c) $K=1+ \chi_e$
   (d) $K= \varepsilon_0 \chi_e$

6.13 ANSWERS

Objective type questions:

1. (b) 2.(b) 3. (c) 4.(c)

6.14 REFERENCES

1. O.F. Mossoti, *Bibl.univ.modena* **6**, 193 (1847).
2. O.F. Mossoti, *Mem. di Mathem e.di.fisica in modena,* **24(2)** 49 (1850).
3. R. Clausius **Volume 2** Vieweg. Braunschweigh (1879).
4. M. Faraday, *Phil.Trans* **128:1** 79 265 (1837).
5. P. Debye, *Phys.Z*, **13**,97 (1912).
6. L. Onsager, *J. Amer.Chem.Soc*, **58**,1486 (1936).
7. J.G. Kirkwood J. Chem. Phys, **7**, 911 (1939).
8. J.G. Kirkwood and R.M.Fuoss, *J. Chem.Phys*, **9**,329 (1941).
9. P. Drude, Z. Phys.Chem, **23**,267 (1897).

6.15 SUGGESTED READINGS

1. Electricity & Magnetism, D.C. Tayal, Himalaya publishing House

2. Electricity, Magnetism and Electronics, S.I. Ahmad and K.C. Lal, Unitech House, Lucknow

3. Fundamental of Electricity and Magnetism, R.G. Mendiratta and B.K.Sawhney, East-West Press Pvt Ltd

UNIT 7 LORENTZ FORCE, BIOT-SAVART LAW, MAGNETIC FORCE, AMPERE'S CIRCUITAL LAW

Structure

7.1 INTRODUCTION

The magnetic effects can be produced by a magnet or by a current carrying conductor. The region around a magnet or current carrying conductor, in which a magnetic needle experiences a torque and rests in a definite direction, is called 'magnetic field'. A charge moving in a magnetic field experiences a deflecting force. Of course, if a charge moving through a point experiences a deflecting force, then a magnetic field is said to exist at that point. This field is represented by a vector quantity $\vec{B}$, called magnetic field or magnetic induction. The magnetic induction can be defined in terms of lines of induction as the number of lines of induction passing through a unit area placed normal to the lines measures the magnitude of magnetic induction or magnetic flux density $\vec{B}$. Obviously, in a region smaller is the relative spacing of the lines of induction, the greater is the magnetic induction. The tangent to the line of induction at any point gives the direction of magnetic induction $\vec{B}$ at that point. The lines of induction simply represent graphically how $\vec{B}$ varies throughout a certain region of space. In the present unit, you will study the force on a moving charge in simultaneous electric and magnetic fields, Biot-Savart law, magnetic force between current elements, Ampere's circuital law and its applications.

7.2 OBJECTIVES

After studying this unit, you should be able to-

- understand Lorentz force
- apply Biot-Savart law
- apply Ampere's circuital law
- solve problems using Biot-Savart law and Ampere's circuital law

7.3 LORENTZ FORCE

Let us consider a charged particle of charge q which is moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$, then the magnetic force acting on that charged particle is given by -

$$\vec{F} = q(\vec{v} \times \vec{B}) \qquad \text{.....(1)}$$

The direction of $\vec{F}$ will be perpendicular to both the direction of velocity $\vec{v}$ and the direction of magnetic field $\vec{B}$. Its exact direction is given by the law of vector product of two vectors.

The magnitude of magnetic force is given as-

$$F = qvB \sin\theta \qquad \text{.....(2)}$$

where θ is the angle between velocity $\vec{v}$ and magnetic field $\vec{B}$.

$\vec{v}$

θ

+q $\vec{B}$

Figure 1

If the angle between velocity $\vec{v}$ and magnetic field $\vec{B}$ is 90^0 then-

$F_{max} = qvB \sin 90^0 = qvB$

i.e. if velocity $\vec{v}$ and magnetic field $\vec{B}$ are at right angle then the magnetic force acting on the charged particle is maximum that is equal to qvB.

If $\theta = 0^0$ or 180^0 i.e. velocity $\vec{v}$ and magnetic field $\vec{B}$ are parallel to each other then-

$F = qvB \sin 0^0 = 0$

i.e. if the charged particle is moving parallel to the magnetic field, it does not experience any force.

If v = 0, then F = 0. This means that if the charged particle is at rest in the magnetic field, then it does not experience any force.

If a charged particle is moving in space where both an electric field $\vec{E}$ and a magnetic field $\vec{B}$ are present, then the total force acting on the charged particle is called the Lorentz force.

The electric force acting on charged particle, $\vec{F}_e = q\vec{E}$ .....(3)

The magnetic force acting on the charged particle, $\vec{F}_m = q(\vec{v} \times \vec{B})$

The total force acting on the charged particle, $\vec{F} = \vec{F}_e + \vec{F}_m$

$$= q\vec{E} + q(\vec{v} \times \vec{B})$$

or $\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$ .....(4)

The force given by equation (4) is called the Lorentz force and the equation is known as Lorentz force equation.

If a charged particle enters perpendicular to both the electric and magnetic fields, then it may cancel each other and therefore, the charged particle will pass undeflected. In this situation,

$\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})] = 0$

or $\vec{E} = -(\vec{v} \times \vec{B})$ .....(5)

In magnitude, $E = v \times B$ or $v = \frac{E}{B}$ .....(6)

Thus a charged particle entering in simultaneous electric and magnetic field may pass undeflected. Such an arrangement of simultaneous electric and magnetic fields is called velocity-selector. Because the charged particle of only specified velocity given by v = E/B can pass undeflected. The particle of velocity v < E/B will be deflected towards electric force and those with velocity v > E/B will be deflected towards magnetic force.

7.4 BIOT-SAVART LAW

Oersted's experiment showed that a current-carrying conductor produces a magnetic field around it. French scientists Biot and Savart, in the same year 1820, performed a series of experiments to study the magnetic fields produced by various current-carrying conductors and formulated a law to determine the magnitude and direction of the magnetic fields so produced. This law is known as 'Biot-Savart law'.

Let us consider a conductor of an arbitrary shape carrying electric current i and P a point in vacuum at which the magnetic field is to be determined. Let us divide the conductor into infinitesimal current-elements. Let us consider a small current element XY of length dl.

P' •Y × P

r θ r

dl

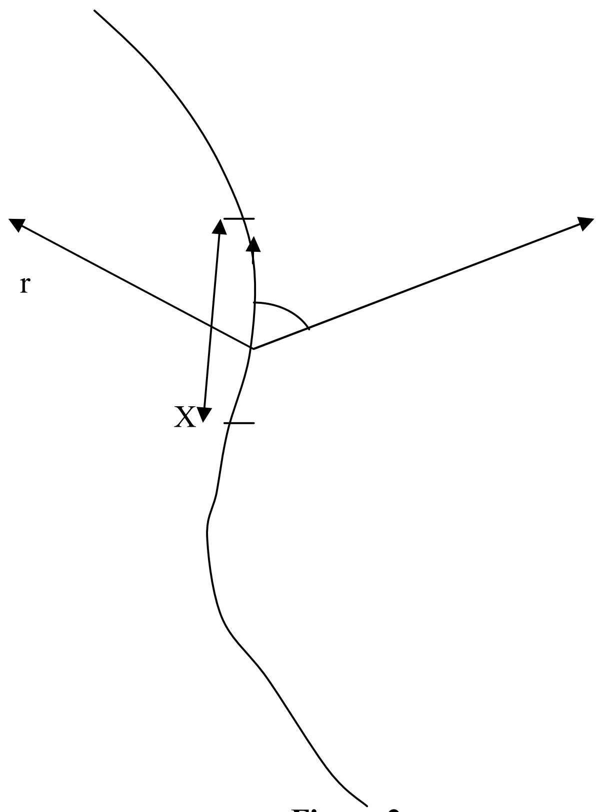


Figure 2

According to Biot-Savart law, the magnetic field dB produced due to this current element at point P at a distance r from the element is-

(i) directly proportional to the current flowing in the element i.e. $dB \propto i$
(ii) directly proportional to the length of element i.e. $dB \propto dl$
(iii) directly proportional to sin of angle between current element and the line joining current element to point P i.e. $dB \propto \sin\theta$
(iv) inversely proportional to the square of the distance of the element from point P i.e. $dB \propto \frac{1}{r^2}$

Combining these, we get-

$dB \propto \frac{idl\sin\theta}{r^2}$

or $dB = \frac{\mu}{4\pi}\frac{idl\sin\theta}{r^2}$.....(7)

where, $\frac{\mu}{4\pi}$ is a dimensional constant of proportionality whose value depends upon the units used for the various quantities. It depends on the medium between the current element and point of observation (P). Here, μ is called the permeability of medium. Equation (7) is called Biot-Savart law. The product of current i and the length of element dl i.e. idl is called the current element. Current element is a vector quantity; its direction is along the direction of current.

If you place the conductor in vacuum or air, then μ is replaced by $\mu_0$and thus Biot-Savart law can be written as-

$$dB = \frac{\mu_0}{4\pi}\frac{idl\sin\theta}{r^2}\text{.....(8)}$$

μ_0 is called the permeability of free space or air.Its value in the SI system is assigned as-

$\mu_0 = 4\pi \times 10^{-7}$ weber/ampere-meter ($WbA^{-1}m^{-1}$)

Thus,$\frac{\mu_0}{4\pi} = 10^{-7}\ WbA^{-1}m^{-1}$

μ_0 or $\frac{\mu_0}{4\pi}$ may also be expressed in Newton/Ampere^2 (N/A^2).

The direction of magnetic field is perpendicular to the plane containing current element and the line joining point of observation to current element. Therefore, in vector form, Biot-Savart law can be expressed as-

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{l}\times\vec{r}}{r^3}$ .....(9)

The resultant magnetic field at P due to the whole conductor can be found by integrating equation (9) over the entire length of the conductor. Thus

$\vec{B} = \int d\vec{B}$

Direction of magnetic field dB: The direction of magnetic field $d\vec{B}$ is perpendicular to both the current element $id\vec{l}$ and the position vector $\vec{r}$ of point P relative to current element and may be found by the law of vector cross product or by Maxwell's right hand screw rule. Thus in figure 2 the direction of magnetic field at point P is shown by × (cross) i.e. vertically inward (downward perpendicular to the plane of the paper) and at point P', the direction of magnetic field is shown by •(dot) i.e. vertically outward(upward perpendicular to the plane of the paper).

7.4.1 Maxwell's Right Hand Screw Rule:

If we hold the screw driver in our right hand and rotate a screw in such a way that the point of screw moves along the direction of electric current in the conductor, then the direction of rotation of the thumb will be the direction of magnetic lines of force.

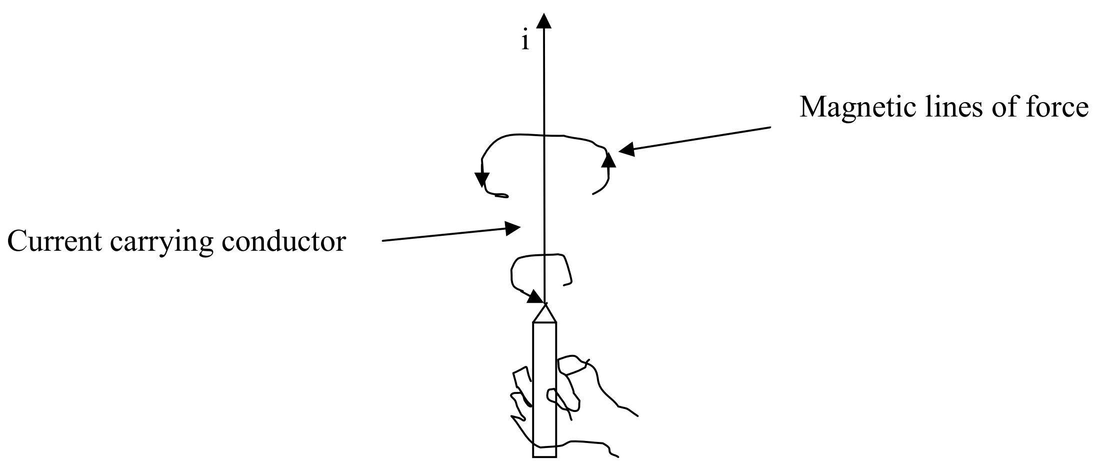


Figure 3

7.4.2 Comparison of Coulomb's Law and Biot-Savart Law

A current generates a magnetic field in the surrounding space while a stationary charge generates an electric field. Coulomb's law gives the electric field due to a distribution of charges while

Biot-Savart law gives the magnetic field due to a current element. According to Coulomb's law, the magnitude of electric field at a point distant r due to a charge element dq is given as-

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

According to Biot-Savart law, the magnitude of magnetic field at a point distant r due to a current element i dl is given as-

$$dB = \frac{\mu_0}{4\pi}\frac{idl\sin\theta}{r^2}$$

where θ is the angle between the length of the element and the line joining the element to the point.

We, thus, see that Biot-Savart law is the magnetic equivalent of Coulomb's law and both are inverse square laws. However, these two laws differ in certain respect. The charge element dq is a scalar while the current element i d*l* is a vector ($id\vec{l}$) whose direction is in the direction of the current. According to Coulomb's law, the magnitude of electric field depends only upon the distance of the charge element from the point. According to Biot-Savart law, the magnitude of magnetic field at the point also depends upon the angle between the current element and the line joining the current element to the point. Secondly, according to Coulomb's law, the direction of electric field is along the line joining the charge element and the point. According to Biot-Savart law, the direction of magnetic field is perpendicular to the current element as well as to the line joining the current element to the point.

Example 1: An electron moving with velocity 5×10^7 m/sec enters a magnetic field of 1 Wb/m^2 at an angle of 90^0 to the magnetic field. Estimate the magnetic force acting on the electron.

Solution: Here $v = 5\times10^7$ m/sec, B = 1 Wb/m^2, $\theta = 90^0$, $q = e = 1.6\times10^{-19}$ C

Using $F = qvB \sin\theta$

$F = 1.6\times10^{-19}\times5\times10^7\times1\times\sin90^0$

$= 8\times10^{-12}$ Newton

Example 2:A proton is moving northwards with a velocity of 3×10^7 m/sec in a uniform magnetic field of 10 Tesla directed eastward. Find the magnitude and direction of the magnetic force on the proton. Charge on proton= 1.6×10^{-19} C

Solution: Given $v = 3\times10^7$ m/sec, B= 10Tesla, $q = 1.6\times10^{-19}$ C

The magnetic force on proton $F = qvB \sin\theta$

$= 1.6\times10^{-19}\times3\times10^7\times10\times\sin 90^0 = 1.6\times10^{-19}\times3\times10^7\times10\times1 = 4.8\times10^{-11}$ Newton

The magnetic field is directed eastward and the direction of motion of proton is northward i.e. the direction of flow of current is northward. By Fleming's left-hand rule, the force on the proton will be directed vertically downwards.

Self Assessment Question (SAQ) 1: An electron is moving vertically upward with a speed of 2×10^8 m/sec. Find out the magnitude and direction of the force on the electron exerted by a horizontal magnetic field of 0.50 Wb/m^2 directed towards west? Also calculate the acceleration of the electron.

Self Assessment Question (SAQ) 2: An electron moving with velocity $\vec{v}$ along +x-axis enters a uniform magnetic field $\vec{B}$ directed along + y-axis. What is the magnitude and direction of the force on the electron?

Self Assessment Question (SAQ) 3:A 2 MeV proton is moving perpendicular to a uniform magnetic field of 2.5 Tesla. Find the force on the proton. The mass of proton = 1.65×10^{-27} Kg.

Self Assessment Question (SAQ) 4: Choose the correct option-

The force on a charged particle moving in a magnetic field is maximum when the angle between direction of motion and field is-

(i) 45^0(ii) 180^0(iii) zero (iv) 90^0

Self Assessment Question (SAQ) 5: Choose the correct option-

A moving electric charge produces-

(i) electric field only (ii) magnetic field only (iii) both electric and magnetic fields (iv) neither of these two fields

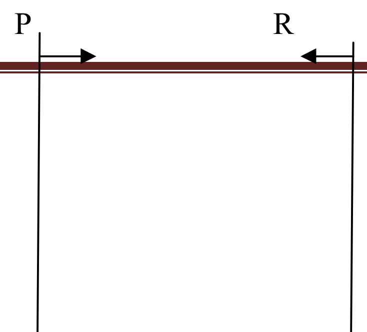
7.5 MAGNETIC FORCE BETWEEN TWO PARALLEL CURRENT CARRYING CONDUCTORS

Let us consider two long, straight and parallel current carrying conductors PQ and RS separated by a distance r. Let i_1 and i_2 be the currents flowing in two conductors in the same direction respectively. Now, let us find expression for the force acting between the conductors.

The magnitude of the magnetic field at a point P on conductor RS is –

$$B = \frac{\mu_0}{4\pi}\frac{2i_1}{r}$$

By Maxwell's right hand screw rule, the direction of this field is perpendicular to the plane of the page directed downward.



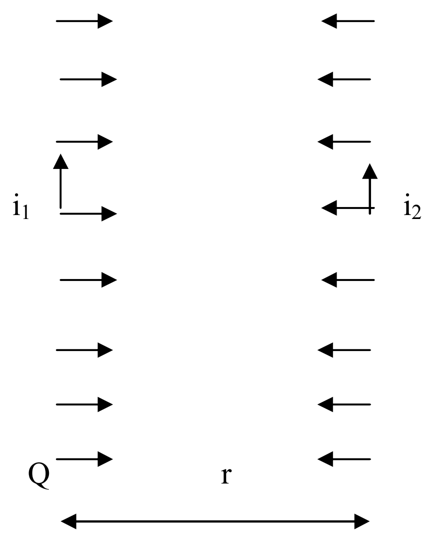


Figure 4

Obviously, the conductor RS is situated in magnetic field B perpendicular to its length. It, therefore, experiences a magnetic force. Using formula, F = iBl sinθ, the magnitude of magnetic force acting on a length l of conductor RS is given as-

$F = i_2 B\, l \sin\theta = i_2 \frac{\mu_0}{4\pi}\frac{2i_1}{r} l \sin 90^0$

Or $$F = \frac{\mu_0}{4\pi}\frac{2i_1 i_2 l}{r} \quad .....(10)$$

The force per unit length of conductor RS is given by-

$$F/l = \frac{\mu_0}{4\pi}\frac{2i_1 i_2}{r} \quad .....(11)$$

By Fleming's left hand rule, the direction of this force is towards conductor PQ if i_2 is flowing in the same direction as i_1(Figure 4). Similarly, the force per unit length of conductor PQ due to current i_2 in conductor RS will be same i.e. $F/l = \frac{\mu_0}{4\pi}\frac{2i_1 i_2}{r}$ and is directed towards conductor RS . Thus, if the currents are in the same direction, then the nature of the force is attractive. The two conductors will have a tendency to move towards each other. If the two ends of the conductors are fixed, then the shape of two conductors will be concave.

If the direction of currents in two conductors is opposite, the force on two conductors will be outwards i.e. repulsive in nature (Figure 5) and now the conductors will repel to each other. If the ends of two conductors are fixed, then the shape of these conductors will be convex.

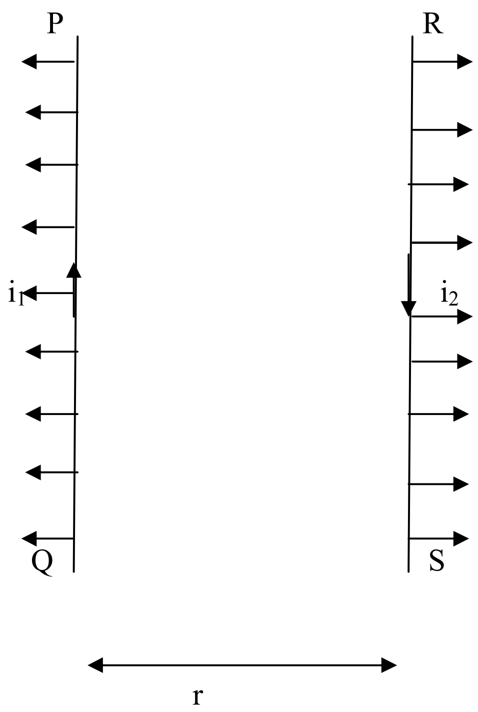


Figure 5

7.5.1 Definition of Ampere:

The force of attraction or repulsion between two long, parallel and straight conductors in vacuum has been used to define ampere.

$$F/l = \frac{\mu_0}{4\pi}\frac{2i_1i_2}{r}$$

Let $i_1 = i_2 = 1$ Amp. and $r = 1$ meter, then

$$F/l = \frac{\mu_0}{4\pi}\frac{2i^2}{r} = 1 \times 10^{-7} \times \frac{2\times(1)^2}{1}$$

$= 2 \times 10^{-7}$ N/meter

Thus, 1 ampere is the current which when flowing in each of two infinitely long parallel conductors 1 meter apart in vacuum produces between them a force of exactly 2×10^{-7} N/meter of length.

Example 3: Estimate the force per unit length on a long straight wire carrying a current of 4 Amp due to a parallel wire carrying a current of 6 amp. If the direction of currents in two wires is same, then find the nature of force acting between them. The distance between the wires is 3 cm.

Solution: Given i_1 = 4 amp, i_2 = 6 amp, r = 3 cm = 3 × 10^{-2} m

Using formula $F/l = \frac{\mu_0}{4\pi}\frac{2i_1i_2}{r}$, we get-

Force per unit length $F/l = 1\times 10^{-7}\times\frac{2\times4\times6}{3\times10^{-2}}$

$= 1.6\times10^{-4}$ N/m^{-1}

Since the direction of currents in two wires is same, therefore the force acting between them is attractive in nature.

Example 4: Two parallel wires, 1 m apart, carry currents of 1 amp and 3 amp in opposite directions. Calculate the magnitude and nature of force acting between them on a length of 2 m.

Solution: Given r = 1 m, i_1 = 1 amp, i_2 = 3 amp, l = 2 m

Using $F = \frac{\mu_0}{4\pi}\frac{2i_1i_2l}{r}$, we get-

$F = 1\times 10^{-7}\times\frac{2\times1\times3}{1}\times2$

$= 12\times10^{-7}$ N/m (repulsive i.e. away from each other)

Self Assessment Question (SAQ) 6: The parallel wires each of length 200 cm and carrying a current of 0.4 amp in the same direction, are kept 40 cm apart in air. Find the force per unit length on each wire.

Self Assessment Question (SAQ) 7:"Two parallel wires carrying current in the same direction repel each other". Is this statement true or false? Give reason.

7.6 AMPERE'S CIRCUITAL LAW

According to Ampere's circuital law, "The line integral of magnetic induction around a closed path is equal to μ_0 times the net current enclosed by the path" i.e.

$$\oint \vec{B}.\vec{dl} = \mu_0\, i \qquad .....(12)$$

where i is the current enclosed by the path.

Let us suppose that the magnetic field induction B arises due to a long wire carrying a current of i ampere. Now let us consider a circular path of radius r centred on this current carrying wire.

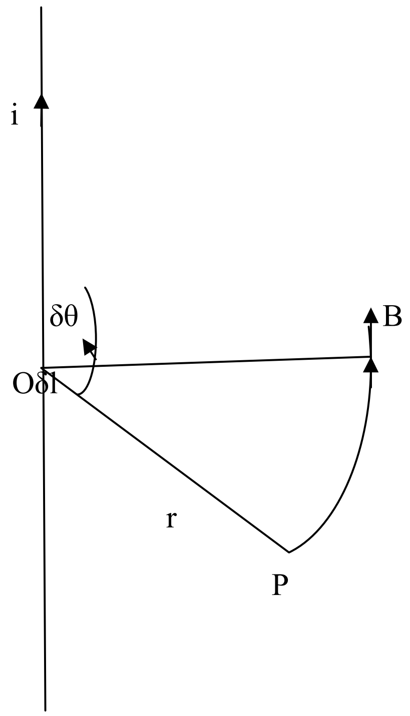


Figure 6

The magnitude of magnetic induction at any point P on the circular path is given by-

$$B = \frac{\mu_0}{4\pi}\frac{2i}{r} \qquad .....(13)$$

For all points on the circular path, the magnetic induction $\vec{B}$ has the same magnitude given by equation (13) and it is parallel to the tangent to the circular path. Therefore, the line integral of the magnetic induction B around the circular pathcentred on the current carrying wire is given by-

$$\oint \vec{B}\, dl = \oint \vec{B} dl = \oint \frac{\mu_0}{4\pi}\frac{2i}{r} r d\theta$$

$$= \frac{\mu_0}{4\pi} 2i \oint \delta\theta$$

$$= \frac{\mu_0}{4\pi} 2i\, (2\pi) = \mu_0\, i$$

Thus we have- $\oint \vec{B}.\vec{dl} = \mu_0\, i$

The sign of integral depends upon the direction in which the current is enriched. The sign is positive if the path followed for line integral is parallel to B and negative if the path followed is anti-parallel.

If the path enclosing the current is not circular but is irregular of any shape, then we divide the path into large number of small elements. Ampere's law holds for closed path of any shape.

7.6.1 Differential form of Ampere's Law

Ampere's circuital law can be expressed in terms of magnetic field intensity ($\vec{H}$). We know that-

$\vec{B} = \mu_0\vec{H}$

Therefore from equation (12) we have-

$\oint \mu_0\vec{H}.\vec{dl} = \mu_0\, i$

Or $\oint \vec{H}.\vec{dl} = i$ .....(14)

But current $i = \iint \vec{J}.\vec{dS}$ .....(15)

Where $\vec{J}$ is the current density and $\vec{dS}$ is small element of area at the point of current density $\vec{J}$ inside the closed path.

Therefore, equation takes the form as-

$\oint \vec{H}.\vec{dl} = \iint \vec{J}.\vec{dS}$ .....(16)

Using Stoke's theorem, we have-

$\oint \vec{H}.\vec{dl} = \iint \text{curl}\,\vec{H}.\vec{dS}$

Therefore, equation (16) becomes-

$\iint \text{curl}\,\vec{H}.\vec{dS} = \iint \vec{J}.\vec{dS}$

i.e. $\iint (\text{curl}\vec{H} - \vec{J}).\vec{dS} = 0$ .....(17)

As the surface is arbitrary, therefore integrand must vanish i.e.

$\text{curl}\,\vec{H} - \vec{J} = 0$

or $\text{curl}\,\vec{H} = \vec{J}$ .....(18)

Multiplying both sides by μ_0 in equation (18), we get-

$\mu_0 \text{curl}\,\vec{H} = \mu_0\vec{J}$

or $\text{curl}\,\mu_0\vec{H} = \mu_0\vec{J}$

or $\quad \text{curl } \vec{B} = \mu_0 \vec{J}$ .....(19)

Equation (18) or (19) is the differential form of Ampere's circuital law. The above relation (19) indicates that the magnetic induction at a point is derived from the given value of $\vec{J}$ at that point by integration. However this equation is not enough to derive $\vec{B}$ at a point because for the same value of $\vec{J}$ at the point another term may be added to $\vec{B}$. We, therefore, need another condition.

7.6.2 Applications of Ampere's Law

Magnetic Field due to Long Straight Current Carrying Wire

Let us consider a long straight wire carrying a current i. From the symmetry of wire, it is clear that the magnetic lines of force are concentric circles centred on the wire

$\vec{dl}$

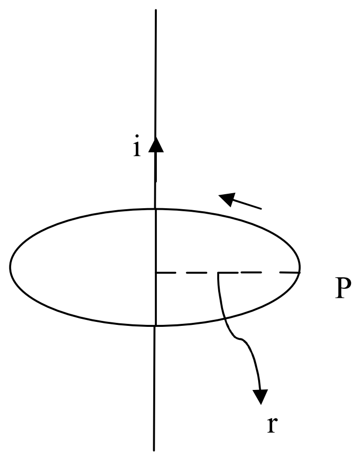


Figure 7

Let P be a point at distance r from the wire at which magnetic field is to be required. Let us consider a circular path of radius r passing through P. By symmetry, the value of magnetic field is same at each point on the circular path. $\vec{B}$ and $\vec{dl}$ are always directed along the same direction. Therefore, the line integral of $\vec{B}$ along the boundary of circular path is-

$$\oint \vec{B}.\vec{dl} = \int B dl \cos 0^0 = B \int dl = B\,(2\pi r)$$

Using Ampere's circuital law-

$$\oint \vec{B}.\vec{dl} = \mu_0\, i$$

Putting for $\oint \vec{B}.\vec{dl}$, we get-

$$B\,(2\pi r) = \mu_0\, i$$

Or $$B = \frac{\mu_0}{2\pi}\frac{i}{r}$$

Or $$B = \frac{\mu_0}{4\pi}\frac{2i}{r}$$

This is the required magnetic field.

7.7 MAXWELL CORRECTION IN AMPERE'S LAW

Let us examine the validity of this equation for time varying fields. Since divergence of curl of any vector quantity is always zero, therefore div curl $\vec{H}$ = 0. Then equation (18) curl $\vec{H}=\vec{J}$ implies-

$$\text{div}\,\vec{J} = 0 \qquad .....(20)$$

We knowthe equation of continuity-

$$\text{div}\,\vec{J} + \frac{\partial \rho}{\partial t} = 0 \qquad .....(21)$$

or $$\text{div}\,\vec{J} = -\frac{\partial \rho}{\partial t} \qquad .....(22)$$

Here ρ is the charge density.

From equations (20) and (22), we get-

$$\frac{\partial \rho}{\partial t} = 0$$

or $$\rho = \text{constant}$$

i.e. charge density is static. Thus we conclude that Ampere's circuital law $\oint \vec{H}.\vec{dl} =\; i$ is valid only for steady state conditions and is insufficient for the cases of time varying fields. Hence to include time varying fields, Ampere's law must be modified. Maxwell investigated mathematically how one could alter Ampere's equation $\oint \vec{H}.\vec{dl} =\; i$ so as to make it consistent with the equation of continuity.

Maxwell assumed that the definition of current density $\vec{J}$ is incompleteand hence something say $\vec{J_d}$must be added to it.Thus, the total current density, which must be solenoidal, becomesequal to $\vec{J} + \vec{J_d}$. Using this assumption, equation (18) curl $\vec{H}=\vec{J}$becomes-

$$\text{curl}\,\vec{H}=\vec{J} + \vec{J_d} \qquad .....(23)$$

Now let us identify $\vec{J_d}$. Let us take divergence of equation (23) as-

$$\text{div curl } \vec{H} = \text{div } (\vec{J} + \vec{J_d}) \qquad .....(24)$$

But we know that the divergence of curl of any vector quantity is always zeroi.e. div curl $\vec{H} = 0$, therefore, the above equation takes the form as-

$$\text{div } (\vec{J} + \vec{J_d}) = 0$$

or $$\text{div } \vec{J} + \text{div } \vec{J_d} = 0$$

or $$\text{div } \vec{J_d} = - \text{div } \vec{J} \qquad .....(25)$$

We know the equation of continuity-

$$\text{div } \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

or $$\text{div } \vec{J} = - \frac{\partial \rho}{\partial t}$$

Putting for div $\vec{J}$ in equation (25), we get-

$$\text{div } \vec{J_d} = \frac{\partial \rho}{\partial t} \qquad .....(26)$$

But by differential form of Gauss theorem we have-

$$\text{div } \vec{D} = \rho \qquad .....(27)$$

where $\vec{D}$ is electric displacement vector.

Using equation (27) in equation (26), we get-

$$\text{div } \vec{J_d} = \frac{\partial (\text{div } \vec{D})}{\partial t}$$

$$= \text{div } \left(\frac{\partial \vec{D}}{\partial t}\right)$$

or $$\vec{J_d} = \left(\frac{\partial \vec{D}}{\partial t}\right) \qquad .....(28)$$

Therefore, the modified form of Ampere's law becomes-

$$\text{curl } \vec{H} = \vec{J} + \vec{J_d} = \vec{J} + \left(\frac{\partial \vec{D}}{\partial t}\right) \qquad .....(29)$$

The additional term which Maxwell added in Ampere's circuital law to include time varying fields is called 'displacement current' because it arises when electric displacement vector $\vec{D}$

changes with time. By the addition of this term Maxwell assumed that this term i.e. displacement current is as effective as the conduction current $\vec{J}$ for producing magnetic field.

Characteristics of displacement current

(a) Displacement current is a current only in the sense that it produces a magnetic field. It has none of the other properties of current since it is not related with the motion of a charge.
(b) Displacement current has a finite value even in a perfect vacuum where there is no charge at all.
(c) The magnitude of displacement current is equal to the rate of change of electric displacement vector i.e. $\vec{J_d} = \left(\frac{\partial \vec{D}}{\partial t}\right)$
(d) Displacement current in a good conductor is negligible as compared to the conduction current at any frequency less than optical frequencies.

Example 5:A 50 V voltage generator at 20 MHz is connected to the plates of air dielectric parallel plate capacitor with plate area 2.8 cm^2 and distance of separation is 0.02 cm. Find the maximum value of displacement current density and displacement current.

Solution: V_o = 50 Volt, f = 20 MHz = 20×10^6 Hz, S = 2.8 cm^2 = 2.8×10^{-4} m^2 , d = 0.02 cm = 2×10^{-4} m

$V = V_o \sin \omega t = V_o \sin 2\pi ft = 50 \sin (2\pi\times20\times10^6 t)$

Displacement current density $\vec{J_d} = \left(\frac{\partial \vec{D}}{\partial t}\right)$

$$= \frac{\partial(\varepsilon_0 \vec{E})}{\partial t} = \frac{\partial}{\partial t}\left(\varepsilon_0 \frac{V}{d}\right)$$

$= \frac{\varepsilon_0}{d}\frac{\partial V}{\partial t}$

$= \frac{\varepsilon_0}{d}\frac{\partial\{50 \sin (2\pi\times20\times10^6 t)\}}{\partial t}$

$= \frac{\varepsilon_0}{d}\{50\cos (2\pi \times 20 \times 10^6 t)\}\times 2\pi \times 20 \times 10^6$

$= \frac{8.85\times10^{-12}}{2\times10^{-4}}\{50 \cos(2\pi \times 20 \times 10^6 t)\}\times 2\pi \times 20 \times 10^6$

$= 277.8 \cos (4\pi\times10^7 t)$ Amp/m^2

Displacement current $i_d = J_d\times S = 277.8 \cos (4\pi\times10^7 t)\times2.8\times10^{-4}$

$=0.0778\times2.8 \cos(4\pi\times10^7 t)$ Amp

Self Assessment Question (SAQ) 8: Choose the correct option-

The concept of displacement current was proposed by-

(i) Faraday (ii) Gauss (iii) Ampere (iv) Maxwell

Self Assessment Question (SAQ) 9: Choose the correct option-

Maxwell's modified Ampere's law is valid-

(i) only when electric field does not change (ii) only when electric field varies with time

(iii) in both of the above situations (iv) none of these

Self Assessment Question (SAQ) 10: Choose the correct option-

The displacement current arises due to-

(i) negative charges only (ii) positive charges only (iii) both negative and positive charges (iv) time varying electric field

Self Assessment Question (SAQ) 11: Choose the correct option-

Displacement current goes through the gap between the plates of a capacitor when the charge of a capacitor is-

(i) zero (ii) decreasing (iii) increasing (iv) remaining constant

Self Assessment Question (SAQ) 12: Choose the correct option-

Displacement current is a current only in the sense that-

(i) it produces a magnetic field (ii) it produces electric field (iii) it produces both fields (iv) none of these

7.8 SUMMARY

In this unit, you have studied about Lorentz force and Biot-Savart law. You have studied that a current carrying conductor produces magnetic field around it. You have also studied about the magnetic force between two current carrying conductors and established its expression and deduced the definition of ampere. You have seen that the conductors attract each other if currents in them are in the same direction and repel each other if currents are in opposite directions. In this unit, you have studied and analyzed Ampere's circuital law and Maxwell's correction in it. According to Ampere's circuital law, the line integral of magnetic induction around a closed path is equal to μ_0 times the net current enclosed by the path. You have also seen that Ampere's law holds for closed path of any shape. You have known about displacement current and its peculiar characteristics. To present the clear understanding and to make the concepts of the unit clear, many solved examples are given in the unit. To check your progress, self assessment questions (SAQs) are given place to place.

7.9 GLOSSARY

Magnetic field- the region surrounding a magnetic

Magnetic induction- a vector which specifies the magnitude and direction of magnetic field at a point

Simultaneous – concurrent, coincident

Electric force- the force experienced by a charge placed at a point in an electric field

Magnetic force- the force experienced by a charge in a magnetic field

Infinitesimal- minute, tiny

Vacuum- emptiness, vacuity

Characteristics- features, qualities

7.10 TERMINAL QUESTIONS

1. Explain the magnitude and direction of the force acting on a charge moving in a magnetic field. When is the force maximum and when minimum?

2. Explain Biot Savart law.

3. Establish the expression for magnetic force acting between two long, parallel and straight current carrying conductors.

4. Both the electric and magnetic field can deflect an electron. What is the difference between these deflections?

5. Explain Ampere's circuital law. Give its significance. Derive its differential form.

6. Explain Maxwell's correction in Ampere's circuital law.

7. Explain the concept of Maxwell's displacement current and show how it led to the modification of the Ampere's law.

8. Obtain the generalized form of Ampere's circuital law. Comment on the concept of the displacement.

9. Throw the light on characteristics of displacement current.

10. Using Ampere's circuital law, establish the expression of magnetic field due to a long current carrying wire.

11. Give a comparison between Coulomb's law and Biot-Savart law.

7.11 ANSWERS

Self Assessment Questions (SAQs):

1. Given $v = 2\times10^{8}$ m/sec, $B = 0.50$ Wb/m^2 , $q = e = 1.6\times10^{-19}$ C, $m= 9\times10^{-31}$ Kg

Using $F = qvB \sin\theta$, we get-

$F = 1.6\times10^{-19}\times2\times10^{8}\times0.50\times\sin 90^{0} = 1.6\times10^{-11}$ N (towards north, Using Fleming's left hand rule)

Using $F = ma$

Or $a = F/m = 1.6\times10^{-11}/ 9\times10^{-31} = 1.8 \times10^{19}$ m/sec^2

2. Using $F = qvB \sin\theta = evB \sin 90^{0} = evB$

Using Fleming's left hand rule, the direction of the force is along –z- axis.

3. Given $K = 2\text{MeV} = 2\times10^{6}\times1.6\times10^{-19}= 3.2\times10^{-13}$ J, $B = 2.5$ T, $m = 1.65\times10^{-27}$ Kg

$K = \frac{1}{2} mv^2$ or $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\times3.2\times10^{-13}}{1.65\times10^{-27}}} = 6.23\times10^{4}$ m/sec^2

Using $F = qvB \sin\theta = 1.6\times10^{-19}\times6.23\times10^{4}\times2.5\times\sin 90^{0} = 7.88\times10^{-12}$ N

4. (iv) 90^{0}

5. (iii) both electric and magnetic fields

6. Given $l = 200$cm $= 2$ m, $i_1 = i_2 = 0.4$ amp, $r = 40$ cm $= 0.4$ m

$F/l = \frac{\mu_0}{4\pi}\frac{2i_1i_2}{r} = 1\times 10^{-7}\times\frac{2\times0.4\times0.4}{0.4} = 8\times10^{-8}$ N/m (attractive)

7. The statement is false because one current carrying wire will experience force of attraction due to the magnetic field produced by the other current carrying wire.

8. (iv) Maxwell

9. (iii) in both of the above situations

10. (iv) time varying electric field

11. (ii) decreasing (iii) increasing

12. (i) it produces a magnetic field

Terminal Questions:

4. The force exerted by a magnetic field on a moving charge is perpendicular to the motion of the charge; hence the work done by this force on the charge is zero and therefore the kinetic energy of the charge does not change. In an electric field the deflection is in the direction of the field, hence the kinetic energy changes.

7.12 REFERENCES

1. Electric and Magnetism, Satya Prakash, Pragati Prakashan, Meerut

2. Engineering Physics, Satya Prakash, Meerut

3. Concepts of Physics, Part II, HC Verma, Bharati Bhawan, Patna

4. Introduction to Engineering Physics, A.S. Vasudeva, S. Chand and Company Ltd., New Delhi

5. Nootan Physics, Part-II, Kumar Mittal, Nageen Prakashan Pvt. Ltd., Meerut

7.13 SUGGESTED READINGS

1. Modern Physics, Beiser, Tata McGraw Hill

2. Fundamentals of Physics, David Halliday, Robert Resnick, Jearl Walker, John Wiley & Sons

3. Engineering Physics, S.K. Gupta, Krishna Prakashan Media (P) Ltd., Meerut

4. Electromagnetic Fields and Waves, R. Gowri, S.K. Kataria and Sons, Delhi

5. Electricity and magnetism with electronics, K.K. Tewari, S. Chand Publishing, New Delhi

UNIT 8 CURL AND DIVERGENCE OF $\vec{B}$,VECTOR POTENTIAL, MAGNETIC FLUX, TORQUE ON A CURRENT LOOP IN A UNIFORM MAGNETIC FIELD

Structure

8.1 INTRODUCTION

In the previous unit, you have studied and learnt about Lorentz force, Bio-Savart law and magnetic force between current carrying conductors. In that unit, you have also studied about Ampere's circuital law and Maxwell's correction in that. In the previous unit, you have learnt about displacement current and its peculiar characteristics. In the present unit, you will learn about curl and divergence of magnetic induction $\vec{B}$, vector potential and its importance, magnetic flux etc. You will also study about magnetic fields due to circular and solenoidal currents and establish the expressions for the field. When a current loop is placed in a uniform magnetic field, then it experiences a torque. In this unit, you will learn about this torque and establish an expression for the torque acting on that current carrying loop in a uniform magnetic field.

8.2 OBJECTIVES

After studying this unit, you should be able to-

- understand curl and divergence of $\vec{B}$
- understand vector potential and magnetic flux
- calculate the magnetic fields for circular and solenoidal currents
- understand torque on a current carrying loop and solve problems

8.3 CURL OF $\vec{B}$

The curl of a vector field at any point is defined as a vector quantity whose magnitude is equal to the maximum line integral per unit area along the boundary of an infinitesimal test area at that point and whose direction is perpendicular to the plane of the test area. The curl of vector field is sometimes called circulation or rotation.

According to Ampere's circuital law, "The line integral of magnetic induction around a closed path is equal to μ_0 times the net current enclosed by the path" i.e.

$$\oint \vec{B}.\vec{dl} = \mu_0\, i \qquad .....(1)$$

where i is the current enclosed by the path.

Let us consider a region in which there is a steady flow of charge. The current density in this region remains constant i.e. it does not change with time however its value may vary from place to place. Now let us consider a closed path in this region as shown in figure (1). The total current enclosed by this path is the flux of current density through the surface bounded by closed path i.e. the total current enclosed by the path given as-

$$i = \iint \vec{J}.\vec{dS} \qquad .....(2)$$

where $\vec{J}$ is the current density and $\overrightarrow{dS}$ is small element of area at the point of current density $\vec{J}$ inside the closed path.

Putting the value of i from equation (2) in equation (1), you get-

$\oint \vec{B}.\overrightarrow{dl} = \mu_0 \, [\iint \vec{J}.\overrightarrow{dS}\,]$

Using Stoke's theorem, you can convert line integral into surface integral as-

$\iint \text{curl}\, \vec{B}.\overrightarrow{dS} = \mu_0 \, [\iint \vec{J}.\overrightarrow{dS}\,]$

$\iint [\text{curl}\, \vec{B} - \mu_0\vec{J}].\, \overrightarrow{dS} = 0$

As the surface is arbitrary, therefore you have-

curl $\vec{B}$ - $\mu_0\vec{J} = 0$

$$\text{curl}\, \vec{B} = \mu_0\vec{J} \qquad \text{…..(3)}$$

Thus the curl of $\vec{B}$ is equal toμ_0 times current density. The above equation (3) is the differential form of Ampere's circuital law.The above relation indicates that the magnetic induction at a point is derived from the given value of $\vec{J}$ at that point by integration. However this equation is not enough to derive $\vec{B}$ at a point because for the same value of $\vec{J}$ at the point another term may be added to $\vec{B}$. We, therefore, need another condition.

8.4 DIVERGENCE OF $\vec{B}$

The divergence of a vector function at certain point is defined as the outward flux of the vector field per unit volume enclosed through an infinitesimal closed surface surrounding the point. The divergence of a vector function is scalar quantity. It should be noted that the divergence itself is simply an operator and has no physical meaning in itself. After operating on suitable physical vector functions, it represents various significant physical scalar quantities. If the divergence of any vector function in a region is zero, it means that the flux of the vector function entering any element of this region is equal to that leaving it.

According to Biot-Savart law the magnetic field at a point due to a current element $i\overrightarrow{dl}$ at a point having position vector $\vec{r}$ relative to current element is given by-

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{i\,\overrightarrow{dl}\times\vec{r}}{r^3} \qquad \text{…..(4)}$$

The magnetic field due to complete circuit current is given as-

$\vec{B} = \frac{\mu_0}{4\pi}\oint \frac{i\,\vec{dl}\times\vec{r}}{r^3}$ …..(5)

Taking divergence on both sides, you get-

$\text{div}\,\vec{B} = \nabla.\vec{B} = \nabla.\left\{\frac{\mu_0}{4\pi}\oint \frac{i\,\vec{dl}\times\vec{r}}{r^3}\right\}$ …..(6)

or $\text{div}\,\vec{B} = \frac{\mu_0}{4\pi}\oint \nabla.\left\{\frac{i\,\vec{dl}\times\vec{r}}{r^3}\right\}$

But $\nabla\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$

Hence the above relation can be written as-

$$\text{div}\,\vec{B} = -\frac{\mu_0}{4\pi}\oint \nabla.\left\{i\vec{dl}\times\nabla\left(\frac{1}{r}\right)\right\}$$

Using vector identity $\nabla.(\vec{A}\times\vec{B}) = \vec{B}.(\nabla\times\vec{A}) - \vec{A}.(\nabla\times\vec{B})$, the above expression becomes-

$$\text{div}\,\vec{B} = -\frac{\mu_0}{4\pi}\oint \nabla\left(\frac{1}{r}\right).(\nabla\times i\vec{dl}) - (i\vec{dl}).\left\{\nabla\times\nabla\left(\frac{1}{r}\right)\right\} \quad …..(7)$$

Now let us interpret the result. You that the magnetic field is specified at field point and the current element $i\vec{dl}$ is due to source point. The field point depends on variables (x,y,z) but on the other hand the field source $i\vec{dl}$ does not depend on variables (x,y,z), therefore it is obvious that

$\nabla\times(i\vec{dl}) = 0$ …..(8)

Also you know that the curl of gradient of a scalar function is always zero i.e.

$$\text{curl grad}\left(\frac{1}{r}\right) = 0 \quad \text{or } \nabla\times\nabla\left(\frac{1}{r}\right) = 0 \quad …..(9)$$

Now using relation (8) and (9) in equation (7), you get-

$$\text{div}\,\vec{B} = -\frac{\mu_0}{4\pi}\oint \nabla\left(\frac{1}{r}\right).0 - (i\vec{dl}).\{0\} = 0$$

i.e. $\text{div}\,\vec{B} = 0$ …..(10)

The above condition holds for all superposition of such fields or for the field of any distribution of currents. The equation (10) implies that the magnetic field is solenoidal.

8.5 VECTOR POTENTIAL

The vector identity div curl $\vec{A} \equiv 0$ shows that the solution of the equation div $\vec{B} = 0$ can be represented in the form as-

$\vec{B} = \text{curl}\,\vec{A}$ .....(11)

The vector field $\vec{A}$, the curl of which is equal to the magnetic field $\vec{B}$ is known as vector potential of a magnetic field $\vec{B}$.

$\vec{A}$ will be specified uniquely only if its divergence as well as its curl is given. We choose

$$\text{div}\,\vec{A} = 0 \qquad .....(12)$$

This choice is called Lorentz gauge- the gauging condition for the potential. The arbitrariness in the choice of the vector potential indicates that the vector potential plays only an auxiliary role and cannot be measured experimentally.

Let us derive equation for vector potential. We know that $\nabla \times \vec{B} = \mu_0\vec{J}$

Putting the value of $\vec{B}$ from equation (11), the above equation becomes-

$\nabla \times (\nabla \times \vec{A}) = \mu_0\vec{J}$

Using vector identity $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla.\vec{A}) - (\nabla.\nabla)\vec{A}$, the above equation becomes-

$\nabla(\nabla.\vec{A}) - (\nabla.\nabla)\vec{A} = \mu_0\vec{J}$

Or $$\text{grad div}\,\vec{A} - \nabla^2\vec{A} = \mu_0\vec{J}$$

Using Lorentz gauge given by equation (12), the above relation becomes-

$$0 - \nabla^2\vec{A} = \mu_0\vec{J}$$

Or $\nabla^2\vec{A} = -\mu_0\vec{J}$ .....(13)

In terms of Cartesian components of $\vec{A}$, we can write-

$$\left.\begin{aligned} \nabla^2 A_x &= -\mu_0 J_x \\ \nabla^2 A_y &= -\mu_0 J_y \\ \nabla^2 A_z &= -\mu_0 J_z \end{aligned}\right\} \qquad .....(14)$$

Each component of the vector potential thus satisfies Poisson's equation ($\nabla^2\varphi = -\frac{\rho}{\varepsilon_0}$) which has the solution as-

$$\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \iiint \frac{\rho(r')}{|\vec{r}-\vec{r'}|} dV' \qquad .....(15)$$

If all currents are concentrated in a finite region of space, then by analogy with equation (15), the solution of equations (14) can be written as-

$$A_i(\vec{r}) = \frac{\mu_0}{4\pi} \iiint \frac{J_i(r')}{|\vec{r}-\vec{r'}|} dV' \qquad .....(16)$$

where i stands for x, y and z. In vector form, we have-

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint \frac{\vec{J}(r')}{|\vec{r}-\vec{r'}|} dV' \qquad .....(17)$$

In a case of a filamentary current i through a differential length dl' along the wire, we have-

$$J\, dV' = (i/S)(Sdl') = i\, dl'$$

Now the above equation becomes-

$$d\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{i(\vec{r'})dl'}{|\vec{r}-\vec{r'}|} \qquad .....(18)$$

Summing up over all volume elements of the filament, we get-

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint \frac{i(r')}{|\vec{r}-\vec{r'}|} dV' \qquad .....(19)$$

The components of $\vec{A}$ vary as 1/r, like electric potential, which does not diverge with in a charge distribution. As divergence of a curl of a vector is always zero and div $\vec{B} = 0$ can be written as a curl of a vector and thus $\vec{A}$ is a vector. Due to these reasons $\vec{A}$ is called by the name of vector potential.

8.6 MAGNETIC FLUX

Let us consider a plane placedin a magnetic field. The magnetic flux linked with that plane is defined as the dot(scalar) product of magnetic field ($\vec{B}$) and the area of the plane ($\vec{A}$) i.e.

$$\text{The magnetic flux } \varphi = \vec{B}.\vec{A} \qquad .....(20)$$

If the perpendicular to the plane makes an angle θ with the direction of magnetic field, then-

The magnetic flux $\varphi = BA\cos\theta$ .....(21)

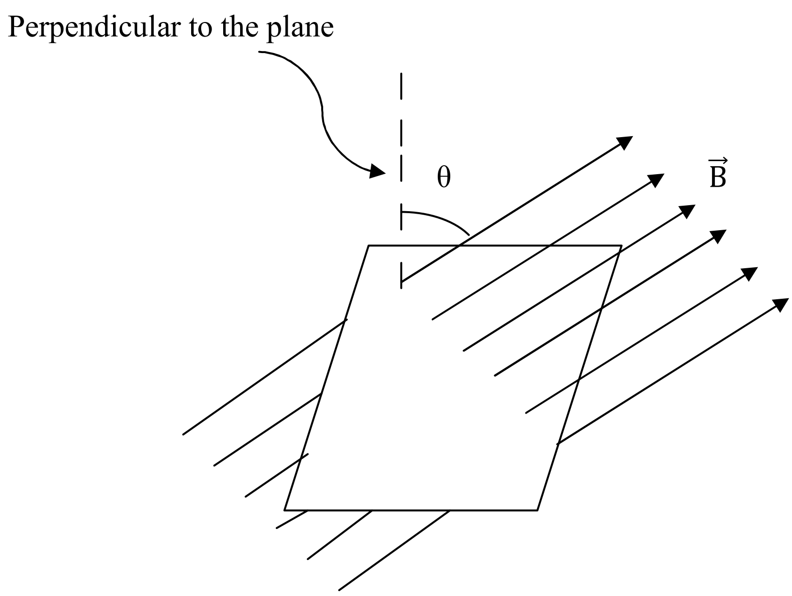


Figure 1

From equation (21), you can write-

$\varphi = (B\cos\theta)\,A$ .....(22)

= component of the magnetic field perpendicular to the plane× area of the plane

Thus, you can define the magnetic flux as the product of the component of the magnetic field perpendicular to the plane and the area of the plane.

If you consider the plane perpendicular to the uniform magnetic field, then the product of the magnitude of the field and the area of the plane is called the magnetic flux φ linked with the plane i.e.

$$\Phi = BA \quad (\text{since } \theta = 0^0) \qquad .....(23)$$

If infinitesimal small surface area ($\overrightarrow{dS}$) is considered, then magnetic flux linked with that surface area is given as-

$$d\varphi = \vec{B}.\overrightarrow{dS} \qquad .....(24)$$

The total magnetic flux linked with the entire surface-

$$\varphi = \iint \vec{B}.\vec{dS} \qquad .....(25)$$

φ is positive if the outward normal to the plane is in the same direction as $\vec{B}$ and is negative if the outward normal is opposite to $\vec{B}$.

The SI unit of the magnetic flux φ is weber (Wb).

Since from equation (23), you have-

$$B = \varphi/A$$

Thus the unit of magnetic flux is also expressed in weber/meter^2 (Wb/meter^2). That is why the magnetic field induction B is also called the magnetic flux density.

The CGS unit of magnetic flux is Maxwell.

$$1 \text{ weber} = 10^8 \text{ maxwell}$$

The magnetic flux is a scalar quantity while magnetic flux density is a vector quantity.

You may also express the magnetic flux in terms of the magnetic lines of force. We can represent a magnetic field by magnetic lines of force. If you draw limited lines of force so that in a magnetic field B = 1 Wb/meter^2 only one line of force passes per meter^2 through an area perpendicular to $\vec{B}$in a magnetic field B = 2 Wb/meter^2 only two lines of force pass per meter^2 perpendicular to B, and so on, then these lines are called the lines of flux. In a magnetic field the number of lines of flux passing per meter^{2}through an area perpendicular to the magnetic field is equal to the magnetic flux linked with that plane.

If $\theta = 90^0$ i.e. the plane is parallel to the magnetic field, then no flux-line will pass through it and the magnetic flux linked with that plane will be zero.

Example 1: A coil having 1000 turn and area 0.20 meter^2 is placed normally in a uniform magnetic field. The magnetic field changes from 0.20 Wb/meter^2 to 0.60 Wb/meter^2 uniformly over a period of 0.01 sec. Calculate the change in magnetic flux associated with the coil.

Solution: Given Area of coil A = 0.20 meter^2, B_1 = 0.20 Wb/meter^2, B_2 = 0.60 Wb/meter^2

The magnetic flux $\varphi = BA \cos\theta$

Since the coil is placed normally in a magnetic field, therefore $\theta = 0^0$

Therefore, the magnetic flux $\varphi = BA \cos 0^0 = BA$

The change in magnetic flux due to a change in magnetic field B-

$$\Delta\varphi = (\Delta B)\times A$$

$$= (B_2 - B_1)\times A = (0.60 - 0.20)\times 0.20$$

$$= 0.08 \text{ Wb}$$

Example 2: Find the magnetic flux linked with a rectangular coil of size 6 cm × 10 cm placed at right angle to a magnetic field of 0.5 Wb/meter^2.

Solution: Given, The area of coil A = 6 cm × 10 cm = 60 cm^2 = 60× 10^{-4} meter^2 = 6×10^{-3} meter^2

Magnetic field B = 0.5 Wb/meter^2

Since the coil is placed at right angle to a magnetic field, therefore the angle between the normal to the plane of coil and the direction of magnetic field $\theta = 0^0$

The magnetic flux linked with the coil $\varphi = BA\cos\theta = 0.5\times6\times10^{-3}\times\cos 0^0 = 3\times10^{-3}$ Wb

Example 3: 5.5× 10^{-4} magnetic flux lines are passing through a coil of electrical resistance 20 ohm. If the number of magnetic flux lines reduces to 5×10^{-5} in a short time, find the change in magnetic flux.

Solution: Given, Initial magnetic flux $\varphi_1 = 5.5\times 10^{-4}$ Wb, Final magnetic flux $\varphi_2 = 5\times10^{-5}$

Therefore, the change in magnetic flux $\Delta\varphi = \varphi_2 - \varphi_1$

$$= 5\times10^{-5} - 5.5\times 10^{-4} = -5.0\times10^{-4} \text{ Wb}$$

Self Assessment Question (SAQ) 1: Choose the correct option-

The divergence of a vector quantity is always-

(i) a vector (ii) a scalar (iii) sometimes a scalar and sometimes a vector (iv) none of these

Self Assessment Question (SAQ) 2: Choose the correct option-

The curl of a vector function is always-

(i) a vector (ii) a scalar (iii) sometimes a scalar and sometimes a vector

(iv) neither a scalar nor a vector

Self Assessment Question (SAQ) 3: Choose the correct option-

For a solenoidal vector-

(i) curl of that vector = 0 (ii) gradient of that vector = 1 (iii) divergence of that vector = 0 (iv) divergence of that vector =1

Self Assessment Question (SAQ) 4: Choose the correct option-

(i) 1 weber = 10^8 maxwell (ii) 1 maxwell = 10^8 weber (iii) 1 weber = 10 maxwell (iv) none of these

Self Assessment Question (SAQ) 5:A coil of wire enclosing an area of 200 cm^2 is placed at an angle of 30^0 with a magnetic field of 0.10 Wb/meter^2. What is the magnetic flux linked with the coil?

Self Assessment Question (SAQ) 6: If the divergence of any vector function in a region is zero, what does it mean?

Self Assessment Question (SAQ) 7: Why is the magnetic field induction B also called the magnetic flux density?

8.7 MAGNETIC FIELD FOR CIRCULAR CURRENTS

Let us consider a circular loop of radius a, carrying current i (Figure 2). Let P be a point at the axis of the loop at a distance x from the centre at which the magnetic field is required.

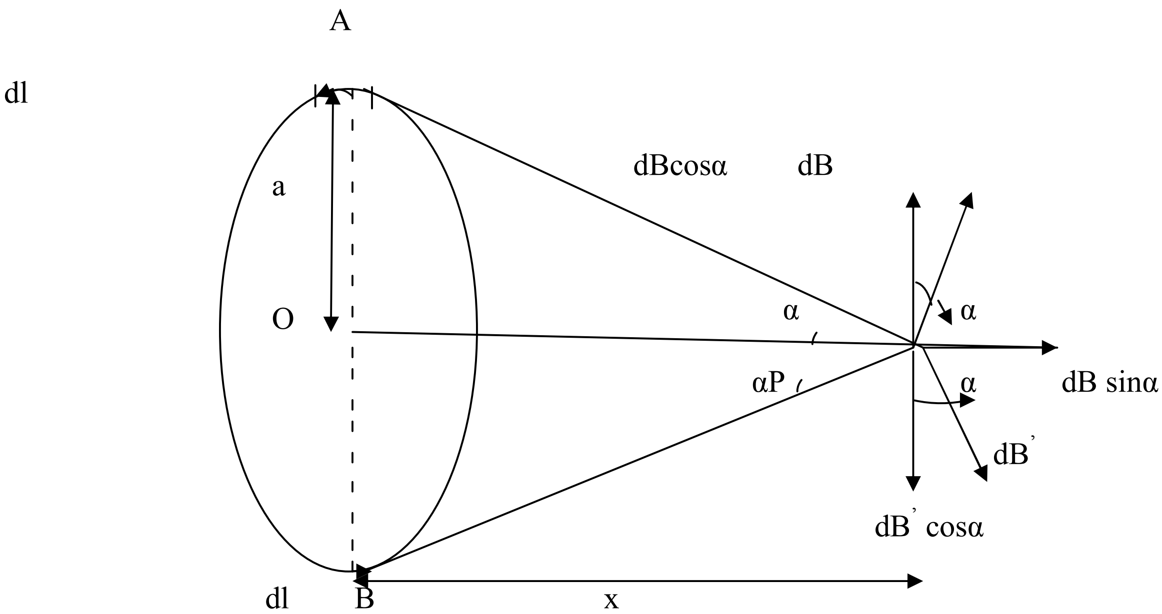


Figure 2

Let us consider a small current element of length dl at point A (at the top) of the loop, at right angles to the plane of the page and directed outward. Let r be the distance of this small element from the point P.

Using Biot-Savart law, the magnetic field due to this element at point P is-

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,\vec{dl}\times\vec{r}}{r^3} \qquad .....(26)$$

The direction of $d\vec{B}$ isperpendicular to the plane containing dl and r and is given by right hand screw rule. As the angle between $\vec{dl}$ and $\vec{r}$ is 90^0, therefore the magnitude of the magnetic field (magnetic induction) is given by-

$$dB = \frac{\mu_0}{4\pi}\frac{idl\sin 90^0}{r^2} = \frac{\mu_0}{4\pi}\frac{idl}{r^2} \qquad .....(27)$$

The direction of magnetic field dB is in the plane of the paper and at right angles to the line r, as shown. Let us resolve this magnetic field dB into its components-

(i) The component of dB along the axis of loop = dB sin α (horizontal component)
(ii) The component of dB at right angles to the axis of loop = dB cos α (vertical component)

Now let us consider another identical current element at point B (at the bottom of the loop) just opposite to the previous element of same length dl, which is at right angle to the plane of the page but directed inward. The magnetic field dB' due to this current element at point P will be equal in magnitude to dB but directed as shown. It is obvious that the components of dB and dB' at right angles to the axis (i.e. vertical components) are equal in magnitude but opposite in direction. Hence they cancel to each other. But the horizontal components i.e. the components along the axis are in same direction and hence they are added up. Thus the resultant magnetic field at point P is due to horizontal components only.

Let us imagine that the entire loop is divided into such current elements, the resultant magnetic field B at point P is directed along the axis and its magnitude is given by-

$$B = \oint dB\sin\alpha$$

Putting for dB in the above, you get-

$$B = \oint \frac{\mu_0}{4\pi}\frac{idl}{r^2}\sin\alpha$$

$$= \frac{\mu_0}{4\pi}\frac{i}{r^2}\oint dl\sin\alpha \qquad .....(28)$$

In right angled triangle AOP, $\sin\alpha = \frac{AO}{AP} = \frac{a}{r} = \frac{a}{\sqrt{a^2+x^2}}$

Therefore, from equation (28), you have-

$$B = \frac{\mu_0}{4\pi}\frac{i}{r^2}\oint dl \frac{a}{\sqrt{a^2+x^2}}$$

$$= \frac{\mu_0}{4\pi}\frac{i}{r^2}\frac{a}{\sqrt{a^2+x^2}}\oint dl$$

But $\oint dl = 2\pi r$ (the circumference of the loop)

Therefore, $$B = \frac{\mu_0}{4\pi}\frac{i}{r^2}\frac{a}{\sqrt{a^2+x^2}}\times 2\pi a$$

$$= \frac{\mu_0}{2}\frac{i}{r^2}\frac{a^2}{\sqrt{a^2+x^2}}$$

Or $$B = \frac{\mu_0 i a^2}{2(a^2+x^2)^{\frac{3}{2}}} \quad \text{.....(29)}$$

If there are n turns in the loop, then each turn will contribute equally to B. Therefore,

$$B = \frac{\mu_0 n i a^2}{2(a^2+x^2)^{\frac{3}{2}}} \quad \text{.....(30)}$$

The direction of the magnetic field B is along the axis of the loop.

At the centre of the loop, x =0, therefore, from equation (31), you have-

$$B = \frac{\mu_0 n i a^2}{2(a^2+0)^{\frac{3}{2}}} = \frac{\mu_0 n i a^2}{2a^3} = \frac{\mu_0 n i}{2a} \quad \text{.....(31)}$$

Again the direction of the magnetic field is perpendicular to the plane of the loop i.e. along the axis of the loop.

If the loop is small, then $x \gg a$, (i.e. a can be neglected in comparison of x)therefore, from equation (30), you have-

$$B = \frac{\mu_0 n i a^2}{2(a^2+x^2)^{\frac{3}{2}}} = \frac{\mu_0 n i a^2}{2x^3} \quad \text{.....(32)}$$

8.8 MAGNETIC FIELD FOR SOLENOIDAL CURRENTS

A solenoid is a long insulated copper wire wound over a tube of card-board or china clay in a close-packed cylindrical helix. When electric current is passed through the solenoid, a magnetic field is produced around and within the solenoid.

Figure 3 shows the lines of force of the magnetic field due to a solenoid. The magnetic lines of force inside the solenoid are nearly parallel which indicate that the magnetic field within the solenoid is uniform and parallel to the axis of the solenoid.

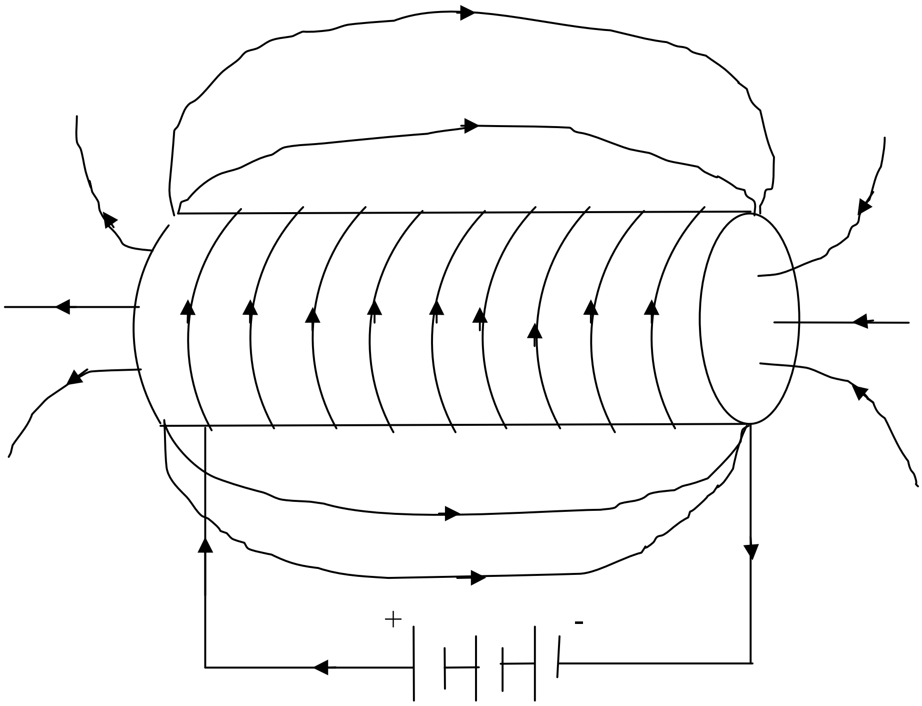


Figure 3

Let there be a long solenoid of radius *a* metre and carrying a current of *i* ampere. Let the number of turns per unit length of the solenoid be n. Let P be a point on the axis of the solenoid (Figure 4).

aαC r

O α P P'

x

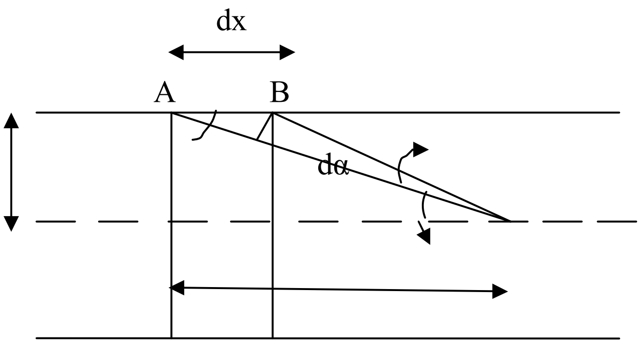


Figure 4

Let us imagine that the solenoid is divided up into a number of narrow coils and let us consider one such coil AB of width dx. The number of turns in this coil is n dx. Let x be the distance of the point P from the centre O of this coil. The magnetic field at P due to this elementary coil is given by-

$$dB = \frac{\mu_0 (n\,dx) i a^2}{2(a^2+x^2)^{\frac{3}{2}}} \qquad .....(33)$$

Let r be the distance of the coil AB from P and dα the angle subtended by the coil at P. Then in Δ ABC, we have-

$$\sin\alpha = \frac{BC}{AB} = \frac{r\,d\alpha}{dx}$$

or

$$dx = \frac{r\,d\alpha}{\sin\alpha}$$

But in right angled triangle APO, $a^2 + x^2 = r^2$

Putting for dx and $(a^2 + x^2)$ in equation (33), you get-

$$dB = \frac{\mu_0\left(n\frac{r\,d\alpha}{\sin\alpha}\right)ia^2}{2(r^2)^{\frac{3}{2}}} = \frac{\mu_0 n\, i\, a^2\, d\alpha}{2r^2 \sin\alpha}$$

$$=\left(\frac{a^2}{r^2}\right)\frac{\mu_0 n\, i\, d\alpha}{2\sin\alpha}$$

But in right angled triangle AOP, $\left(\frac{a^2}{r^2}\right) = \sin^2\alpha$, therefore-

$$dB = \frac{\mu_0 n\, i\, d\alpha}{2\sin\alpha}\sin^2\alpha = \frac{1}{2}\mu_0 ni \sin\alpha\, d\alpha \qquad .....(34)$$

The magnetic field B at point P due to entire solenoid can be obtained by integrating the above expression eq. (34) between the limits α_1 and α_2, where α_1 and α_2 are the semi-vertical angles subtended at point P by the first and the last turn of the solenoid respectively (Figure 5). Thus-

$$\text{Total magnetic field } B = \int_{\alpha_1}^{\alpha_2} dB = \int_{\alpha_1}^{\alpha_2} \frac{1}{2}\mu_0 ni \sin\alpha\, d\alpha$$

$$= \frac{1}{2}\mu_0 ni \int_{\alpha_1}^{\alpha_2} \sin\alpha\, d\alpha = \frac{1}{2}\mu_0 ni[-\cos\alpha]_{\alpha_1}^{\alpha_2}$$

Or

$$B = \frac{1}{2}\mu_0 ni[\cos\alpha_1 - \cos\alpha_2] \qquad .....(35)$$

When point P is well inside a very long solenoid, then $\alpha_1 \approx 0$ and $\alpha_2 \approx 180^0$ so that $\cos\alpha_1 \approx 1$ and $\cos\alpha_2 \approx -1$. Therefore, equation (35) becomes-

$$B = \frac{1}{2}\mu_0 ni[1-(-1)] = \frac{1}{2}\mu_0 ni[2]$$

Or $$B = \mu_0 ni \qquad .....(36)$$

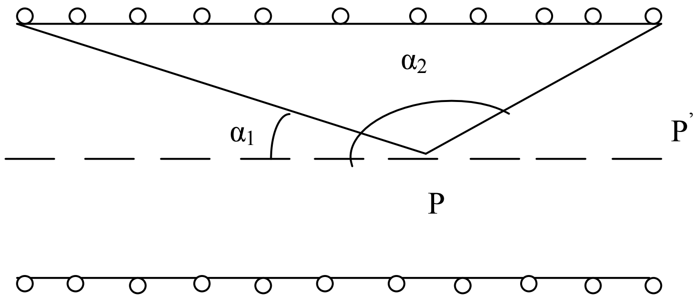


Figure 5

At the end of the last turn, at P', $\alpha_1 = 0$ and $\alpha_2 = 90^0$, therefore from equation (35)-

$$B = \frac{1}{2}\mu_0 ni \qquad .....(37)$$

At the end of the first turn, $\alpha_1 = 90^0$ and $\alpha_2 = 180^0$, therefore from equation (35), you get-

$$B = \frac{1}{2}\mu_0 ni \qquad .....(38)$$

Thus, the magnetic field at the ends of a long solenoid is half of that at the centre. If the solenoid is sufficiently long, the magnetic field within it, except near the ends, is uniform. It does not depend upon the length and area of cross-section of the solenoid. As a parallel plate capacitor produces uniform electric field similarly, a solenoid produces a uniform magnetic field. The uniform magnetic field within a long solenoid is parallel to the solenoid axis. Its direction along the axis is given by a curled straight right hand rule.

8.9 TORQUE ON A CURRENT LOOP IN A UNIFORM MAGNETIC FIELD

Let us consider a rectangular wire loop PQRS, of length l and width b, carrying a current i be suspended in a uniform magnetic field $\vec{B}$ as shown in figure 6. Each side of the current loop experiences a magnetic force in the magnetic field.

The magnitude of the force acting on side PQ, $F_1 = i\,B\,l \sin\theta$

Since the vertical side of loop PQ is always perpendicular to the magnetic field $\vec{B}$. Therefore, $\theta = 90^0$. Thus the force $F_1 = i\,B\,l \sin 90^0 = i\,B\,l$

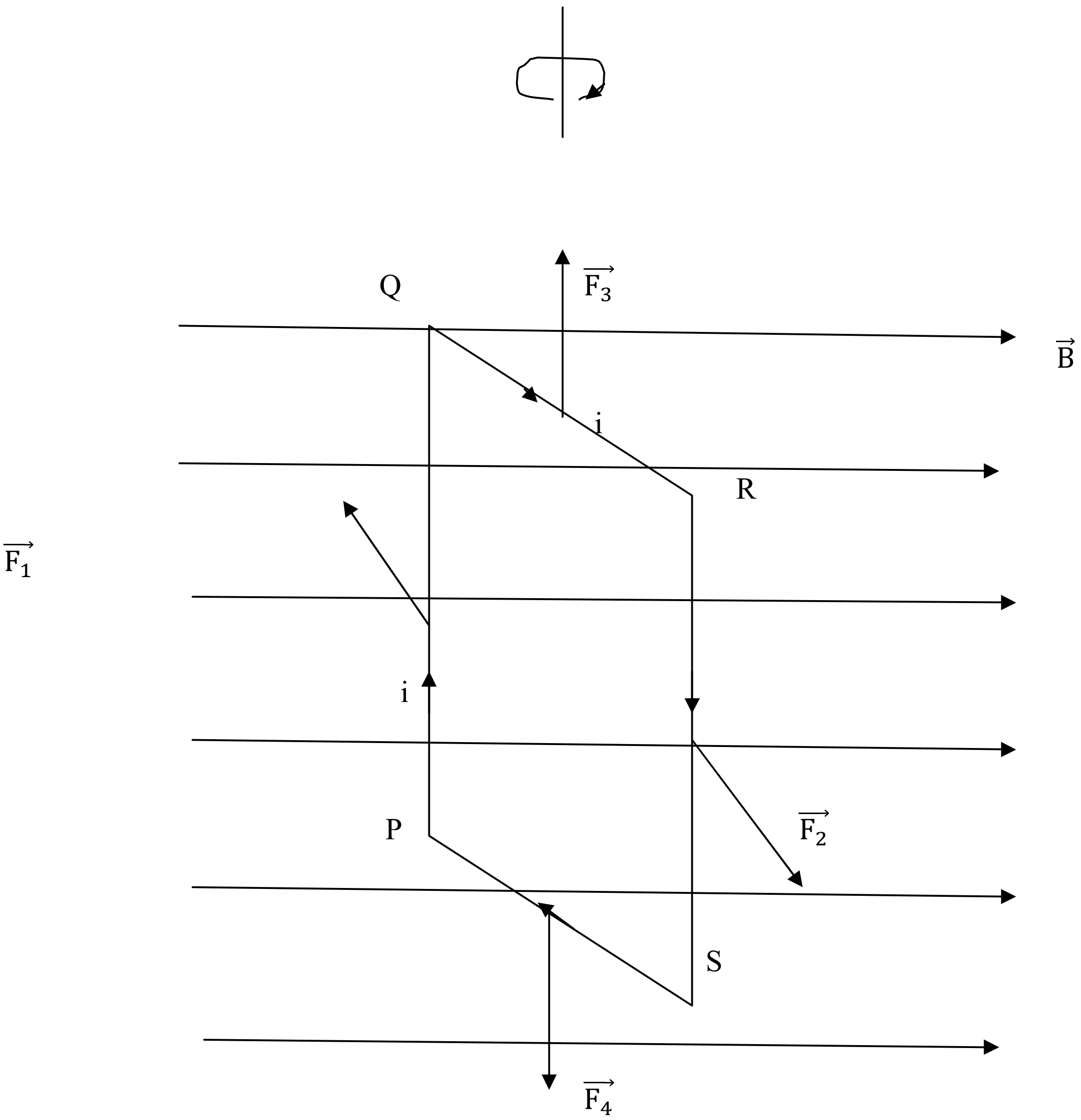


Figure 6

Similarly, the magnitude of magnetic force acting on side RS, $F_2 = i\,B\,l \sin\theta$

Since the vertical side of loop RS is always perpendicular to the magnetic field $\vec{B}$. Therefore, $\theta = 90^0$. Thus the force $F_2 = i\,B\,l \sin 90^0 = i\,B\,l$

Thus, the magnitudes of magnetic forces acting on sides PQ and RS of loop are equal i.e.

$$F_1 = F_2 = i\,B\,l \qquad \text{.....(39)}$$

By Fleming's left hand rule, both forces $\overrightarrow{F_1}$ and $\overrightarrow{F_1}$ are perpendicular to the page directed away from the reader and towards the reader respectively. Obviously, the both forces $\overrightarrow{F_1}$ and $\overrightarrow{F_1}$ are equal, parallel and opposite having different lines of action. These forces form a deflecting couple which tends to rotate the loop clockwise.

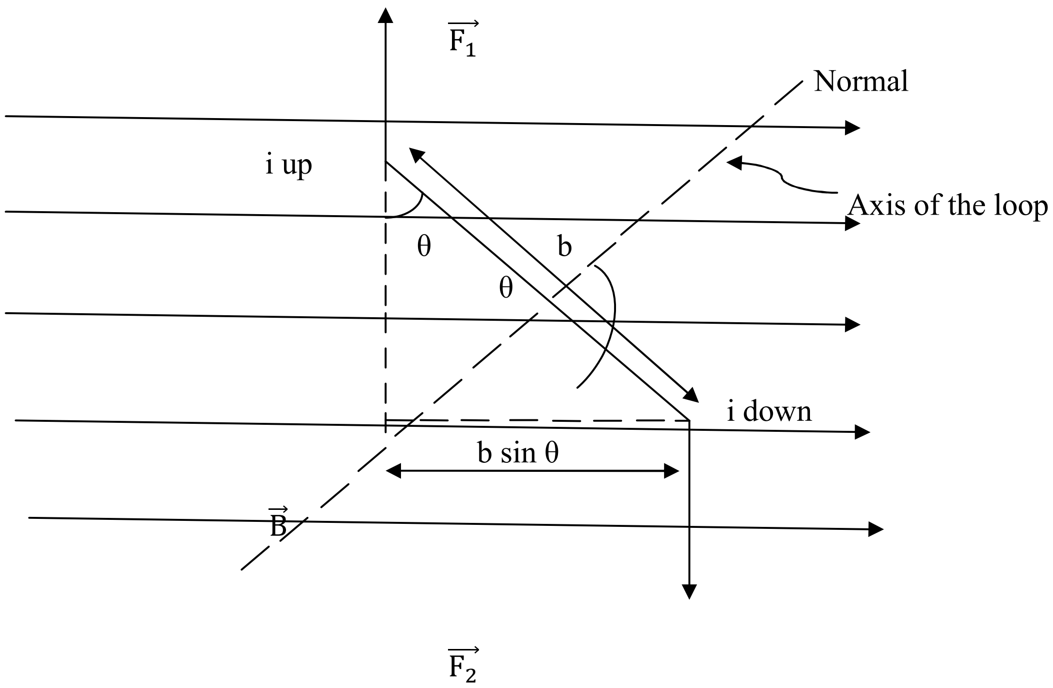


Figure 7

Let us suppose that at any time, the axis of the loop (normal to the plane of the loop) makes an angle θ with the direction of the magnetic field $\vec{B}$ (as shown in figure 7). Then, the instantaneous moment of the deflecting couple, (or the torque) acting on the current loop-

τ = magnitude of the force F_1 (or F_2) × perpendicular distance between the line of action of the forces

$= i\,B\,l \times b \sin\theta = i\,B\,(l \times b)\sin\theta$

But $l \times b$ = Area of the current loop = A (say), therefore-

$$\tau = i\,B\,A\sin\theta \qquad \ldots..(40)$$

The magnetic force acting on the side QR of the loop, $F_3 = i\,B\,b$

Similarly, the magnetic force exerted on the side PS of the current loop, $F_4 = i\,B\,b$

These forces $\overrightarrow{F_3}$ and $\overrightarrow{F_4}$ acting on the sides QR and PS of the current loop are equal and opposite to each other but their line of action is the same. Hence, they cancel each other and do not form a couple. Thus, the net force on the current loop is zero. Only the torque given by equation (40) acts on it. This torque deflects the current loop to a position in which the axis of the current loop is parallel to the magnetic field. In this position, $\theta = 0^0$, therefore the torque becomes zero. This torque τ = i B A sin θ acts on every turn of the current loop. Therefore, if the loop is a close wound coil having N turns, the torque acting on the entire loop is-

$$\tau = N\, i\, B\, A \sin\theta$$

or

$$\tau = N\, i\, A\, B \sin\theta \quad \text{.....(41)}$$

Obviously, the unit of torque is Newton-meter.

But term N i A is defined as the magnitude of the dipole moment $\overrightarrow{M}$ of the coil. Thus-

$$M = N\, i\, A \quad \text{.....(42)}$$

Therefore, equation (41) becomes-

$$\tau = M\, B \sin\theta \quad \text{.....(43)}$$

In vector form-

$$\vec{\tau} = \overrightarrow{M} \times \overrightarrow{B} \quad \text{.....(44)}$$

This is the required expression. It is the basis to the theory of a moving coil galvanometer.This expression holds for closed loops of any shape, rectangular, circular or otherwise.

Example 4: Two similar circular coils of wire having a radius of 70 mm and 60 turns have a common axis and are 18 cm apart. Find the strength of the magnetic field at a point midway between them on their common axis, when a current of 100 mA is passed through them.

Solution: Given, a = 70 mm = 7 cm = 0.07 meter, n = 60, i = 100 mAmp = 0.10 amp, x = 18cm/2 = 9 cm = 0.09 meter and $\mu_0 = 4\pi \times 10^{-7}$ Wb/Amp-meter

The magnetic field due to either of the circular coils at a point on the axis distant x from the centre is-

$$B = \frac{\mu_0 n i a^2}{2(a^2+x^2)^{\frac{3}{2}}}$$

$$= \frac{4\pi\times10^{-7}\times60\times0.10\times(0.07)^2}{2(0.07^2+0.09^2)^{\frac{3}{2}}}$$

$$= 2.5 \times 10^{-5} \text{ Tesla}$$

Example 5: A 30 turns circular coil of diameter 16 cm carries a current of 6 amp. It is suspended vertically in a uniform horizontal magnetic field of 1 Tesla such that the magnetic field lines make an angle of 30^0 with the plane of the coil. Estimate the magnitude of the counter torque needed to be applied to prevent the coil from turning.

Solution: Given, N = 30, radius of the coil r = 16 cm/2 = 8 cm = 0.08 meter, i = 6 amp, B = 1 Tesla, $\theta = 90^0 - 30^0 = 60^0$

Area of the coil $A = \pi r^2 = 3.14 \times (0.08)^2 = 0.0201$ meter^2

Torque $\tau = N\, i\, A\, B \sin\theta = 30 \times 6 \times 0.0201 \times 1 \times \sin 60^0 = 3.13$ Newton-meter

Therefore, a counter torque of 3.13 Newton-meter should be applied.

Self Assessment Question (SAQ) 8: A long solenoid of length 100 cm and radius of cross section 1.5 cm, has five layers of windings of 750 turns each. If the solenoid carries a current of 650 Amp, compute the magnetic field at the centre of the solenoid.

8.10 SUMMARY

In this unit, you have learned about curl and divergence of magnetic field vector, vector potential, magnetic flux and derived the expressions for magnetic field induction for circular and solenoidal currents.We have defined the magnetic flux linked with that plane as the dot (scalar) product of magnetic field ($\vec{B}$) and the area of the plane ($\vec{A}$). In this unit, you have also studied the torque acting on a current loop in a uniform magnetic field and learned how the forces acting on two sides of a loop placed in a uniform magnetic field, form a deflecting couple. To make the concepts of clear, many solved examples are given in the unit. To check your progress, self assessment questions (SAQs) are given place to place.

8.11 GLOSSARY

Steady - stable

Flow- stream, current

Divergence- deviation, departure

Magnetic flux – the surface integral of the magnetic field over that surface

Magnetic flux density – a vector which specifies the magnitude and direction of magnetic field at a point

8.12 TERMINAL QUESTIONS

1. Prove that the curl of $\vec{B}$ is equal toμ_0 times current density.

2. How will you define the curl of a vector?

3. Establish the condition that the magnetic field is solenoidal..

4. Give the significance of divergence.

5. Define vector potential.

6. Give the importance of vector potential.

7. A rectangular coil of size 0.5 meter×0.10 meter and 100 turns is placed perpendicular to a magnetic field of 0.01 Wb/meter^2. Evaluate the change in magnetic flux linked with the coil if it is drawn from the magnetic field.

8. Is magnetic flux a scalar or vector? What about the magnetic flux density?

9. Define magnetic flux. What is its unit?

10.If the plane of a coil is parallel to the magnetic field, then what will be the magnetic flux linked with the coil?

11. Why is $\vec{A}$ called vector potential?

12. Derive an expression for the torque acting on a rectangular coil of area A, carrying a current i, placed in a magnetic field. The angle between the direction of magnetic field and normal to the plane of coil is θ.

13. Establish an expression for the magnetic field at a point on the axis of a circular coil carrying current, and hence at the centre of the coil.

14. Derive the following expression for the magnetic field of a solenoid-

$$B = \frac{1}{2}\mu_0 ni[\cos\alpha_1 - \cos\alpha_2]$$

Where symbols have their usual meanings.

8.13 ANSWERS

Self Assessment Questions (SAQs):

1. (ii) a scalar
2. (i) a vector
3. (iii) divergence of that vector = 0
4. (i) 1 weber = 10^8 maxwell
5. Given, Area of coil A = 200 cm^2 = 200×10^{-4} meter^2 = 2× 10^{-2} meter^2 = 0.02 meter^2, $\theta = 90^0 - 30^0 = 60^0$, Magnetic field B = 0.10 Wb/meter^2

The magnetic flux linked with the coil $\varphi = BA\cos\theta$
$= 0.10\times0.02\times \cos 60^0 = 0.10\times 0.02\times 0.50 = 1\times 10^{-4}$ Wb

6. If the divergence of any vector function in a region is zero, it means that the flux of the vector function entering any element of this region is equal to that leaving it.
7. Since $\varphi = BA$ or $B = \varphi/A$
   Thus the unit of magnetic flux is also expressed in weber/meter^2 (Wb/meter^2). That is why the magnetic field induction B is also called the magnetic flux density.
8. Given, $i = 650$ amp, length $l = 100$ cm = 1 meter, $n = 5 \times 750 = 3750$ turns/meter
   The magnetic field at the centre of the solenoid, $B = \mu_0 ni = 4\pi\times10^{-7} \times3750\times650$
   $= 4\times3.14\times10^{-7}\times3750\times650 = 3.061$ Wb/meter^2

Terminal Questions:

7. Given, Area of the coil $A = 0.5$ meter$\times0.10$ meter $= 5\times10^{-3}$ meter^2, Magnetic field B = 0.01 Wb/meter^2
   Since the coil is placed perpendicular to a magnetic field, therefore $\theta = 0^0$
   Initial magnetic flux linked with the coil $\varphi_1 = BA\cos\theta = BA\cos 0^0 = BA$
   $= 0.01\times5\times10^{-3} = 0.00005$ Wb
   Since the coil is drawn from the magnetic field, no magnetic flux will be linked with the coil. Therefore, final magnetic flux linked with the coil $\varphi_2 = 0$
   Magnetic flux $\Delta\varphi = \varphi_2 - \varphi_1 = 0 - 0.00005 = -0.00005$ Wb $= -5\times10^{-5}$ Wb
8. Magnetic flux is a scalar. Magnetic flux density is a vector.

10. If the plane of a coil is parallel to the magnetic field, then no flux-line will pass through it and the magnetic flux linked with that plane will be zero.

8.14 REFERENCES

1. Electricity and Magnetism, DC Tayal, Himalaya Publishing House, Mumbai

2. Electric and Magnetism, Satya Prakash, Pragati Prakashan, Meerut

3. Nootan Physics, Part-II, Kumar Mittal, Nageen Prakashan Pvt. Ltd., Meerut

8.15 SUGGESTED READINGS

1. Modern Physics, Beiser, Tata McGraw Hill

2. Fundamentals of Physics, David Halliday, Robert Resnick, Jearl Walker, John Wiley & Sons

3. Engineering Physics, S.K. Gupta, Krishna Prakashan Media (P) Ltd., Meerut

4. Electricity and magnetism with electronics, K.K. Tewari, S. Chand Publishing, New Delhi

UNIT 9 MAGNETIC DIAPOLE, TORQUE ON BAR MAGNET POENTIAL ENERGY STORED OF A MAGNETIC DIAPOLE, BALLISTIC GALVANOMETER

Structure

9.1 INTRODUCTION

Dear learners, in the previous unit, you have learnt about curl and divergence of magnetic field, magnetic flux and established the expressions for magnetic field for circular and solenoidal currents. You have also calculated the torque on a current loop in a uniform magnetic field. According to the modern view, the magnetic properties of a substance are endorsed to the electronic motions i.e. orbital motion and spin motion, in the atoms of the substance. Due to these motions, each atom is equivalent to a tiny current loop and produces magnetic field. In the unmagnetised state of the substance the current loops are oriented at random so that the magnetic fields mutually cancel. When the substance is magnetized by some process, all current loops are aligned with their planes parallel to one another and currents circulating in the same direction. Hence a resultant magnetic field is produced. In the present unit, you will study about current loop as a magnetic dipole and torque acting on a bar magnet in a uniform magnetic field. You will also study about potential energy of a magnetic dipole in a magnetic field. You will also learn about Ballistic galvanometer, its function and characteristics.

9.2 OBJECTIVES

After studying this unit, you should be able to-

- understand magnetic dipole
- understand torque on a bar magnet
- calculate the torque on a bar magnet and solve problems
- calculate the potential energy of a magnetic dipole in a magnetic field
- understand ballistic galvanometer

9.3 CURRENT LOOP AS A MAGNETIC DIPOLE

You know that a current carrying solenoid or a coil or a current loop behaves like a bar magnet. A bar magnet having north and south poles at its ends is a magnetic dipole and therefore, a current loop is also a magnetic dipole. You can calculate the magnetic moment of a current loop. In the previous unit, you have learnt that the magnetic field due to a circular current loop of radius a and having n turns at *a* point on its axis, distant x from the centre of the loop, is given by-

$$B = \frac{\mu_0 nia^2}{2(a^2+x^2)^{\frac{3}{2}}} \qquad .....(1)$$

The direction of this magnetic field is along the axis of the loop i.e. perpendicular to the plane of the loop.

For axial points far from the loop, we have x≫a, then the above expression reduces to-

$$B = \frac{\mu_0 nia^2}{2x^3} \quad .....(2)$$

Multiplying by π in numerator and denominator in R.H.S., we get-

$$B = \frac{\mu_0 \pi nia^2}{\pi 2x^3}$$

$$= \frac{\mu_0}{2\pi}\frac{\pi a^2 ni}{x^3} = \frac{\mu_0}{2\pi}\frac{Ani}{x^3} \quad \text{(since } \pi a^2 = A\text{, area of the loop)}$$

Or

$$B = \frac{\mu_0}{2\pi}\frac{niA}{x^3} \quad .....(3)$$

The quantity niA is called magnetic dipole moment $\vec{M}$ of the current loop. Thus-

$$M = niA \quad .....(4)$$

The unit of magnetic dipole moment is ampere-meter^2.

In vector form, we can write-

$$\vec{M} = ni\vec{A} \quad .....(5)$$

The direction of magnetic dipole moment $\vec{M}$ is the same as the direction of the area vector $\vec{A}$ of the current loop. Thus, equation(3), for the magnetic field due to a current loop at a distant axial point can be written as-

$$\vec{B} = \frac{\mu_0}{2\pi}\frac{\vec{M}}{x^3} \quad .....(6)$$

Thus, you see that $\vec{B}$ and $\vec{M}$ have the same direction.

9.4 TORQUE ON A BAR MAGNET IN A UNIFORM MAGNETIC FIELD

You observe that when a bar magnet is suspended in a uniform magnetic field,it sets itself with its axis parallel to the magnetic field. It means that the magnet positioned in the magnetic field experiences a torque which rotates the magnet to a position in which the axis of the magnet is parallel to the magnetic field. A current loop in a magnetic field shows the same behavior. The current loop also experiences a torque which tends to rotate the loop to a position in which the axis of the loop is parallel to the magnetic field. The bar magnet and current loop, both are magnetic dipoles.

According to the modern views regarding magnetism, each atom of the magnet is a small current loop and all these current loops are aligned in the same direction. In a magnetic field, the sum of the torques on these small loops is the torque acting on the magnet.

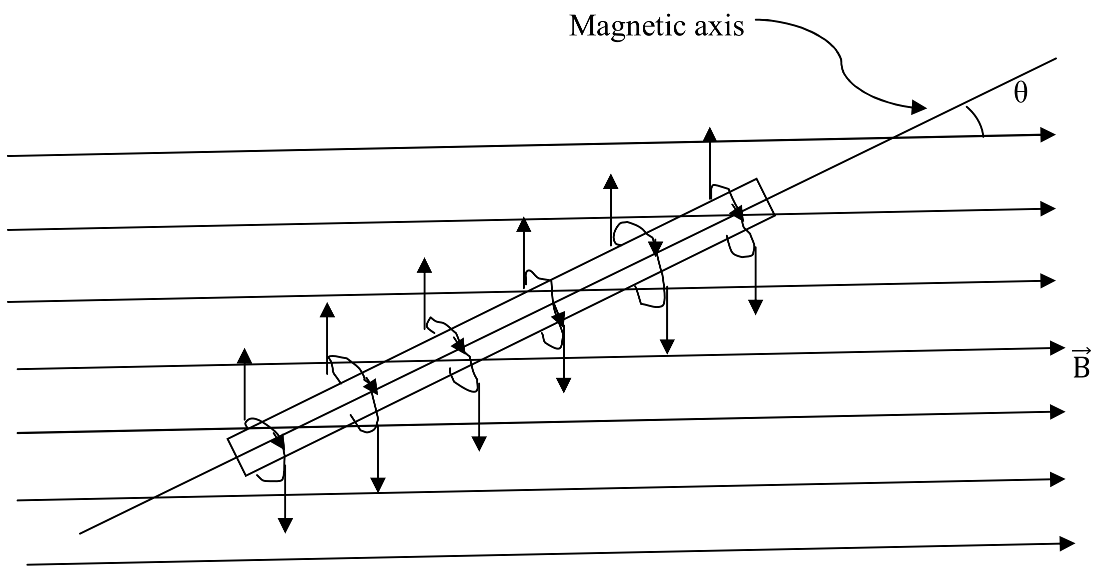


Figure 1

You have learnt that the magnitude of the torque exerted on a current loop positioned in a magnetic field $\vec{B}$ if its axis makes an angle θ with the direction of $\vec{B}$ is given by-

$$\tau = i\,A\,B\sin\theta \qquad .....(7)$$

Here, A is the area of the current loop.

If there are N current loops in a bar magnet, then the torque acting on the entire magnet is-

$$\tau = N\,i\,A\,B\sin\theta \qquad .....(8)$$

You have read that the quantity N i A is defined as the magnitude of the magnetic dipole moment $\vec{M}$ of all the N current loops or of the bar magnet i.e.

$$M = Ni\,A \qquad .....(9)$$

Therefore, equation (8) takes the form as-

$$\tau = M\,B\sin\theta \qquad .....(10)$$

Here θ is the angle between the vectors $\vec{M}$ and $\vec{B}$. In vector, the above expression (10) can be written as-

$\vec{\tau} = \vec{M} \times \vec{B}$ …..(11)

The magnetic moment $\vec{M}$ is directed along the axis of the bar magnet.

We can compare this torque with the torque exerted by an electric field ($\vec{E}$) on an electric dipole which is given as-

$\vec{\tau} = \vec{p} \times \vec{E}$ …..(12)

Here $\vec{p}$ is the electric dipole moment.

There is a difference between these two torques. The torque on a magnetic dipole situated in a magnetic field is an independent physical quantity. One cannot suppose that it is made up of two parallel, equal and opposite forces acting on the magnetic poles. But on the other hand, the torque on an electric dipole is made of two parallel, equal and opposite forces acting on the electric charges of the dipole.

If the axis of the magnetic dipole be perpendicular to the magnetic field $\vec{B}$, then the torque exerted on it will be maximum. Thus,

$$\tau_{max} = M\,B \sin 90^0$$

$$= M\,B \qquad \text{…..(13)}$$

Or $M = \tau_{max} / B$

If B = 1 (unit), then $M = \tau_{max}$

Thus, the magnetic moment of a magnetic dipole is equal to the torque acting on the dipole if it is placed perpendicular to a uniform unit magnetic field.

The SI unit of magnetic dipole moment is Joule/Tesla or ampere-meter^2.

9.5 POTENTIAL ENERGY STORED OF A MAGNETIC DIPOLE

You have read that if a magnetic dipole (bar magnet, current loop etc.) is placed in an external uniform magnetic field, then it is acted upon by a torque which tends to align the magnetic dipole in the direction of the magnetic field. Therefore, work must be done to change the orientation of the magnetic dipole against the torque. It means that the magnetic dipole has magnetic potential energy depending on its orientation in the magnetic field. Let us evaluate this energy.

Let us consider a magnetic dipole of magnetic dipole moment $\vec{M}$placed at an angle θ with the direction of a uniform magnetic field$\vec{B}$. Then, the magnitude of the torque exerted on the magnetic dipole is given as-

$$\tau = M\,B\sin\theta \qquad \text{.....(14)}$$

Now, let the magnetic dipole is rotated through an infinitesimally small angle dθ against the torque, then the amount of work done for this act-

$$dW = \tau\, d\theta \qquad \text{.....(15)}$$

Putting for τ from equation (14) into equation (15), we get-

$$dW = (M\,B\sin\theta)\, d\theta \qquad \text{.....(16)}$$

If the magnetic dipole is rotated from an initial orientation θ_1 to a final orientation θ_2, then the total work needed will be-

$$W = \int_{\theta_1}^{\theta_2} dW$$

$$= \int_{\theta_1}^{\theta_2} (M\,B\sin\theta)d\theta$$

$$= MB\int_{\theta_1}^{\theta_2} (\sin\theta)d\theta = MB\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$= -\,MB\,(\cos\theta_2 - \cos\theta_1)$$

Or $$W = MB\,(\cos\theta_1 - \cos\theta_2) \qquad \text{.....(17)}$$

This work is stored in the form of potential energy U of the dipole in the new orientation θ_2. Therefore,

$$U = MB\,(\cos\theta_1 - \cos\theta_2) \qquad \text{.....(18)}$$

Now let us assume the potential energy of the magnetic dipole to be zero for its any arbitrary orientation. Let us suppose potential energy U is equal to zero when the axis of the dipole makes an angle $\theta = 90^0$ with the direction of magnetic field. Thus, taking $\theta_1 = 90^0$ and $\theta_2 = \theta$, then the expression (18) becomes-

$$U_\theta = MB\,(\cos 90^0 - \cos\theta)$$

$$= MB\,(0 - \cos\theta)$$

Or $$U_\theta = -\,MB\cos\theta \qquad \text{.....(19)}$$

In vector notation, $$U = -\,\vec{M}\,.\,\vec{B} \qquad \text{.....(20 a)}$$

If $\theta = 0^0$, then the potential energy of dipole $U_\theta = -\,MB\cos 0^0$

Or $U_0 = -M\,B$

This is the minimum potential energy that a magnetic dipole can have. Thus, you see that a magnetic dipole has minimum potential energy when $\vec{M}$ and $\vec{B}$ are parallel.

When $\theta = 180^0$, then the potential energy of dipole $U_{180} = - MB \cos 180^0$

$$= M B$$

This is the maximum potential energy that a magnetic dipole can have. In this way, you see that a magnetic dipole has maximum potential energy when $\vec{M}$ and $\vec{B}$ are antiparallel.

The difference in energy between these two orientations is given as-

$$\Delta U = U_{180} - U_0$$

$$= M B - (- MB)$$

$$= 2 MB$$

This much work must be done by an external agent to turn a magnetic dipole through 180^0, starting when it is lined up with the magnetic field.

Example 1: A current of 6 Amp is flowing in a plane circular coil of radius 2 cm having 200 turns. The coil is placed in a uniform magnetic field of 0.2 Wb/meter^2. If the coil is free to rotate, what orientations would correspond to its (a) stable equilibrium, (b) unstable equilibrium? Calculate the potential energy of the coil in these two cases.

Solution: Given, i = 6 Amp, r = 2 cm = 0.02 meter, N = 200, B = 0.2 Wb/meter^2

The area of coil $A = \pi r^2 = 3.14 \times (0.02)^2 = 0.001256 = 1.256 \times 10^{-3}$ meter^2

The magnetic moment of the coil, $M = N i A$

$$= 200 \times 6 \times 1.256 \times 10^{-3}$$

$$= 1507.2 \times 10^{-3} = 1.5072 \text{ Amp-meter}^2$$

The potential energy of the coil when placed in a uniform magnetic field is given by-

$U_\theta = - MB \cos\theta$

(a) In the case of stable equilibrium, the coil will orient itself so as to have a minimum (i.e. maximum negative) potential energy and this corresponds to $\theta = 0^0$ i.e. the axis of the coil will be parallel to the magnetic field i.e. $\vec{M}$ parallel to $\vec{B}$. In this case, the potential energy will be-

$U_0 = - MB \cos 0^0$

$= - M B$

$$= - 1.5072 \times 0.2 = - 0.30144 \text{ Joule}$$

(b) In the case of unstable equilibrium, the coil will have maximum potential energy. This will be so when $\theta = 180^0$ i.e. $\vec{M}$ anti- parallel to $\vec{B}$. The potential energy will be-

$$U_{180} = - MB \cos 180^0 = M B$$

$$= 0.30144 \text{ Joule}$$

Example 2:A short bar magnetof magnetic moment 0.50 Joule/Tesla is held with its axis at 30^0 with a uniform external magnetic field of 0.15 Tesla. Find the magnitude of the torque exerted on the magnet by the magnetic field.

Solution: Given, M = 0.50 Joule/Tesla, $\theta = 30^0$, B = 0.15 Tesla

The torque exerted on the bar magnet-

$$\tau = M B \sin \theta$$

$$= 0.50 \times 0.15 \times \sin 30^0$$

$$= 0.50 \times 0.15 \times 0.50 = 0.0375 \text{ Joule}$$

Self Assessment Question (SAQ) 1: When is the magnetic dipole in stable and unstable equilibrium?

Self Assessment Question (SAQ) 2:A bar magnetof magnetic moment 1.5 Joule/Tesla is set aligned with the direction of a uniform magnetic field of 0.22 Tesla.

(a) Compute the work required to turn the magnet so as to align its magnetic moment (i) normal to the magnetic field and (ii) opposite to the magnetic field direction.
(b) Also find the torques on the bar magnet in the two cases.

Self Assessment Question (SAQ) 3:A couple of moment 1.5×10^{-5} Newton-meter is needed to keep a magnetic dipole perpendicular to a magnetic field of 6×10^{-4} Wb/meter^2. Evaluate the magnetic moment of the dipole.

Self Assessment Question (SAQ) 4: Choose the correct option-

The SI unit of magnetic dipole moment is-

(a) Amp-meter^2 (b) Amp/meter (c) Tesla meter/Amp (d) none of these

9.6 BALLISTIC GALVANOMETER

It is also known as moving coil ballistic galvanometer. Moving coil ballistic galvanometer is a specially designed galvanometer for the measurement of the total quantity of charge passed through it for a short duration. The ordinary galvanometer measures current.

A moving coil ballistic galvanometer consists of a coil of large moment of inertia and large number of turns of insulated fine copper wire wound on a non-conducting frame such as bamboo or ivory. The coil is suspended by means of a thin phosphor bronze strip between the cylindrical pole pieces of a permanent magnet. The lower end of it is attached to a spring of phosphor-bronze wire. A concave mirror is rigidly attached to the phosphor-bronze strip to record the deflection of the coil by a lamp and scale arrangement. A soft iron core is kept symmetrically within the coil without touching it. The whole arrangement is enclosed in a metallic case provided with a glass window on the front side and leveling screws at the base.

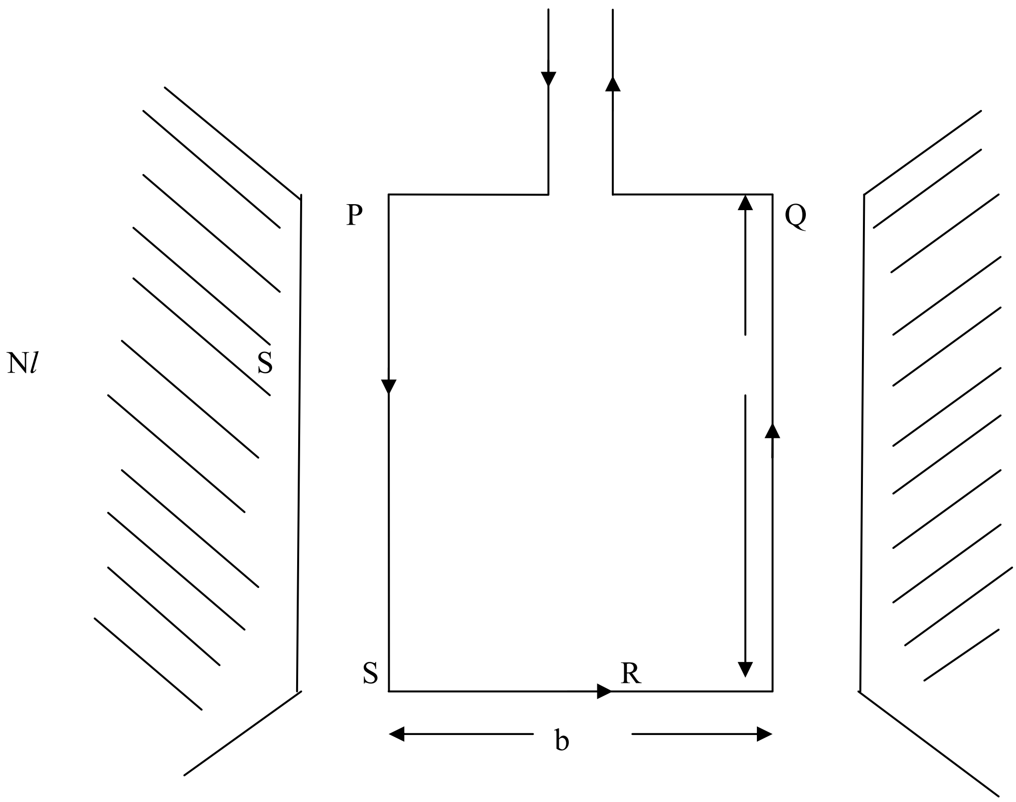


Figure 2

Since the suspension is thin, long and in the form of a strip, therefore the torsional constant C is small. You know that the period of oscillation of a moving coil ballistic galvanometer is given by-

$T = 2\pi\sqrt{\frac{I}{C}}$ …..(20 b)

Since the coil has a large moment of inertia I and its suspension has small torsional constant C, therefore, its period of oscillation [as given by equation (20)] is quite large. Further, as the coil is wound on a non-conducting frame, its electromagnetic damping is reduced. The damping due to viscosity of air is still present, but it is usually small. Now let us discuss the theory of a moving coil ballistic galvanometer.

Theory

Let i be the electric current flowing in the coil at any moment as shown in figure 2. Let N be the number of turns in the coil, l the length of its each vertical side, b its breadth and B the strength of the radial magnetic field in which the coil is suspended.

The magnitude of the mechanical force acting on each of the two vertical sides PS and QR at that moment = $N\, i\, B\, l \sin 90^0 = N\, i\, B\, l$

The force acting on the sides PQ and SR = $N\, i\, B\, l \sin 0^0$ 0

i.e. the force acting on the sides PQ and SR will be zero because these sides are parallel to the magnetic field.

According to Fleming's left hand rule, the forces acting on the vertical sides PS and QR are opposite and perpendicular to the plane of the coil. The magnitudes of these forces are equal. These equal and opposite forces form a couple.

The moment of this couple = magnitude of force × perpendicular distance between the line of action of the forces

= $(NiBl) \times b = N\, i\, B\, (l \times b)$

= $N\, i\, B\, A$

(since $l \times b = A$, the area of the coil)

Let us consider that this couple acts on the coil for an infinitesimal time dt, then the angular impulse given to the coil = couple × time = N i B A × dt

Therefore, the total angular impulse given to the coil in time t = $\int_0^t (N\, i\, B\, A)\, dt$

$$= N\,B\,A \int_0^t i\,dt = N\,B\,A\,q \qquad .....(21)$$

Since $\int_0^t i\,dt = q$, the total charge that has passed through the moving coil galvanometer in time t.

This impulse produces an angular momentum in the coil due to which the coil rotates.

Let ω_0 is the initial angular velocity of the coil and I, the moment of inertia of the coil about the axis of rotation.

The angular momentum produced in the coil due to the angular impulse = I ω_0 .....(22)

From equations (21) and (22), we have-

$$N\,B\,A\,q = I\,\omega_0 \qquad .....(23)$$

When the coil rotates due to the angular momentum and therefore, the suspension wire twists due to which a restoring couple is developed in the suspension. The restoring couple of suspension brings the coil to the position of rest momentarily. Then the coil swings back to its mean position as the suspension unwinds and due to its inertia, the coil does not come to the rest in its mean position but moves in opposite direction. Therefore, the suspension wire twists in opposite direction and again a restoring couple is developed in the coil which tends to bring the coil to its mean position. The process continues and thus the coil oscillates in the horizontal plane about the axis of suspension.

Evidently, the kinetic energy of the coil at start is $\frac{1}{2}I\omega_0^2$. At the moment when the coil comes to the position of rest momentarily, the angle of rotation θ_m is maximum and the kinetic energy of the coil is zero. If the damping is negligible then the energy is entirely used for doing work in twisting the suspension against the restoring couple.

Let C be the restoring couple per unit twist in the suspension, then the couple for a twist θ = C θ

Therefore, the work done for an additional small twist = (C θ) dθ

The work done against the restoring couple = $\int_0^{\theta_m} (C\,\theta)\,d\theta$

$$= C \int_0^{\theta_m} \theta\,d\theta = \frac{1}{2}C\theta_m^2$$

Therefore, $\frac{1}{2}I\omega_0^2 = \frac{1}{2}C\theta_m^2$

Or $\omega_0 = \theta_m\sqrt{\frac{C}{I}}$ .....(24)

Now, putting for ω_0 from equation (23) in equation (24), we get-

$$N\,B\,A\,q\,/\,I = \theta_m\sqrt{\frac{C}{I}}$$

Or $$q = \frac{C}{NBA}\sqrt{\frac{I}{C}}\,\theta_m \qquad .....(25)$$

The time period of the moving system-

$$T = 2\pi\sqrt{\frac{I}{C}}$$

Or $$\sqrt{\frac{I}{C}} = \frac{T}{2\pi} \qquad .....(26)$$

Putting for $\sqrt{\frac{I}{C}}$ from equation (26) in equation (25), we get-

$$q = \frac{C}{NBA}\frac{T}{2\pi}\,\theta_m \qquad .....(27)$$

This is the relation between the charge q flowing through the ballistic galvanometer and the maximum throw θ_m of the coil.

The equation (27) can be written as-

$$q = K\,\theta_m \qquad .....(28)$$

where $$K = \frac{C}{NBA}\frac{T}{2\pi} \qquad .....(29)$$

Here K is known as ballistic constant of the moving coil galvanometer.

Obviously, $$q \propto \theta_0 \qquad .....(30)$$

Thus, when momentary current is passed through the ballistic galvanometer, the total charge passed through the galvanometer is proportional to the maximum angular deflection θ_m of the coil.

The quantity $\frac{\theta_m}{q}$ is called charge sensitivity Q_s i.e. the charge sensitivity of the ballistic galvanometer is defined as the deflection per unit charge i.e.

$$\text{Charge sensitivity} = \frac{\text{deflection}}{\text{charge}}$$

Thus, $$Q_s = \frac{\theta_m}{q} = \frac{NBA}{C}\frac{2\pi}{T} \qquad .....(31)$$

The current sensitivity of a moving coil ballistic galvanometer is defined as the deflection, as read from the scale per unit current i.e.

$$\text{Current sensitivity} = \frac{\text{deflection}}{\text{current}}$$

Thus, Current sensitivity $I_s = \frac{\theta}{i} = \frac{NBA}{C}$ …..(32)

Obviously, charge sensitivity $= \frac{2\pi}{T} \times$ current sensitivity

Hence, the charge sensitivity of a moving coil ballistic galvanometer is $\frac{2\pi}{T}$ times the current sensitivity.

Voltage sensitivity under any given conditions is the deflection per unit voltage and is consequently, current sensitivity divided by the resistance i.e.

$$\text{Voltage sensitivity} = \frac{\text{current sensitivity}}{\text{resistance}}$$

The resistance is that of entire circuit and not that of the instrument alone.

Figure of merit of ballistic galvanometer is the current which will produce a deflection of one scale division, when we use lamp and scale arrangement, it is the current which will produce a deflection of 1 mm on a metre scale one metre away from the galvanometer mirror.

For equilibrium, deflecting couple = restoring couple

$\frac{C}{NBA}$ N i A B = C θ

9.6.1 Correction for Damping

While deriving the above relations, we have assumed that the damping in the coil is zero and the entire energy of the coil is used for twisting the suspension through an angle θ_m. But in fact, the deflection of the coil goes on decreasing due to the damping produced by viscosity of the air and the electromagnetic damping produced by the motion of the coil in the magnetic field of the permanent magnet (Figure 3). Therefore, when charge is passed through the ballistic galvanometer, you observe that the throw of the coil is smaller than its true value θ_m which would have been observed if the damping were entirely absent. Thus, a correction is necessary.

Let θ_1, θ_2, θ_3, θ_4 ………… etc. be the first, second, third, fourth, …… etc. successive throws in continuously decreasing order on either sides of the rest position of the coil θ_1, θ_3, …… are on one side of the rest position of the coil and θ_2, θ_4, ………are on the other side of the rest position of the coil. It is found that the ratio of any two successive throws is constant i.e.

$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = \ldots\ldots.. = d = e^{\lambda}$ …..(33)

Here d is a constant and its logarithm to the base e i.e. $\log_e d$ is called the logarithmic decrement per half cycle and is represented by λ i.e.

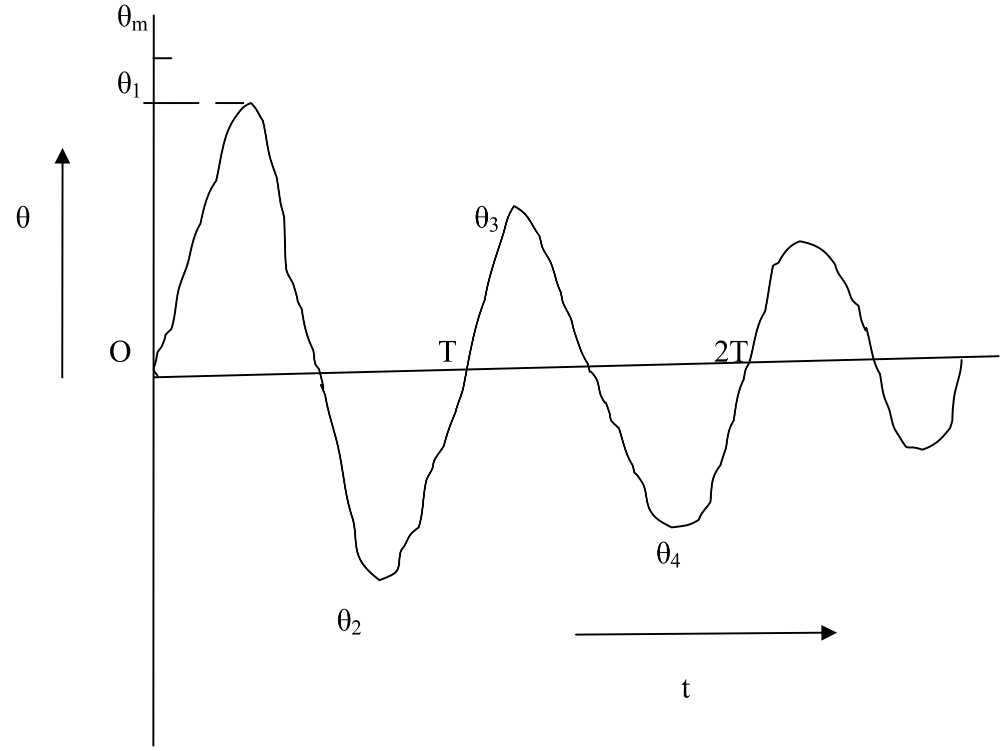


Figure 3

$$\log_e d = \lambda$$

or $$d = e^{\lambda}$$

The decrement in a complete cycle is given as-

$\frac{\theta_1}{\theta_3} = \frac{\theta_1}{\theta_2}\frac{\theta_2}{\theta_3} = e^{2\lambda}$

Obviously, the logarithmic decrement in a quarter cycle will be $\lambda/2$. To calculate the true value of the throw θ_m in the presence of damping, throw θ, is observed after the coil completes a quarter of a vibration. Therefore, the decrement is given by-

$$\frac{\theta_m}{\theta_1} = e^{\lambda/2}$$

Or $$\theta_m = \theta_1 e^{\lambda/2} = \theta_1 \left[1 + \frac{\lambda}{2} + \frac{\left(\frac{\lambda}{2}\right)^2}{2!} + \frac{\left(\frac{\lambda}{3}\right)^3}{3!} + \ldots\ldots\ldots.\right]$$

As λ is small, the terms containing λ^2, λ^3 etc. may be neglected,therefore-

$$\theta_m = \theta_1 (1 + \lambda/2) \qquad \ldots..(34)$$

Putting for θ_0 in equation (27), we get-

$$q = \frac{C}{NBA}\frac{T}{2\pi}\theta_1 (1 + \lambda/2) \qquad \ldots..(35)$$

This is the relation between the charge passed through the galvanometer and the first throw observed.

In actual practice, the value of λ is found by observing first throw θ_1 and eleventh throw θ_{11}.

$$\frac{\theta_1}{\theta_{11}} = \frac{\theta_1}{\theta_2}\frac{\theta_2}{\theta_3}\frac{\theta_3}{\theta_4}\frac{\theta_4}{\theta_5}\frac{\theta_5}{\theta_6}\frac{\theta_6}{\theta_7}\frac{\theta_7}{\theta_8}\frac{\theta_8}{\theta_9}\frac{\theta_9}{\theta_{10}}\frac{\theta_{10}}{\theta_{11}} = e^{10\lambda}$$

Or $$10\,\lambda = \log_e \frac{\theta_1}{\theta_{11}}$$

Or $$\lambda = \frac{1}{10}\log_e \frac{\theta_1}{\theta_{11}}$$

$$= \frac{2.3026}{10}\log_{10} \frac{\theta_1}{\theta_{11}} \qquad \ldots..(36)$$

9.6.2 Conditions for a Moving Coil Galvanometer to be Ballistic

A moving coil galvanometer is said to be ballistic if its coil makes a large number of oscillations before coming to rest, after the entire charge passes through it. The conditions for a moving coil galvanometer to be ballistic are as follows-

(i) The period of oscillation should be large. This is possible if moment of inertia (I) of the coil is large and torsional rigidity (C) of suspension is small.

(ii) The damping is kept least. This can be achieved by winding the coil on a non-conducting frame.

9.6.3 Conditions for a Moving Coil Galvanometer to be Dead Beat

A moving coil galvanometer is said to be dead beat if its coil returns to its rest position quickly without making any oscillation, after being deflected. The conditions for a moving coil galvanometer to be dead beat are given below-

(i) The period of oscillation should be small. This is possible if moment of inertia (I) of the coil is small and torsional rigidity (C) of suspension is large.

(ii) The damping is kept large. This can be achieved by winding the coil on a conducting frame and a soft iron core is kept between the pole pieces of permanent magnet.

9.7 SUMMARY

In this unit, you have learnt about torque acting on a bar magnet in a uniform magnetic field, magnetic dipole and potential energy stored of a magnetic dipole.You have learnt that acurrent carrying solenoid or a coil or a current loop behaves like a bar magnet. A bar magnet having north and south poles at its ends is a magnetic dipole and therefore, a current loop is also a magnetic dipole. Expressions for torque acting on a bar magnet, energy stored in a magnetic field and ballistic constant have been established. You have studied about correction for damping and the reason for this correction. You have also studied the conditions for a moving coil galvanometer to be ballistic and dead beat.You have also learnt about current sensitivity, charge sensitivity and voltage sensitivity of ballistic galvanometer.To present the clear understanding and to make the concepts of the unit clear, many solved examples are given in the unit. To check your progress, self assessment questions (SAQs) are given place to place.

9.8 GLOSSARY

Magnetized – pulled towards you, caught the attention of

Aligned- brought into line

Electromagnetic damping- Damping due to induced currents in the moving system during its motion in the permanent magnetic field

Angular impulse- the time integral of the torque applied to a system, usually when applied for a short time

9.9 TERMINAL QUESTIONS

1. Explain magnetic dipole moment of a bar magnet.

2. Obtain an expression for the torque acting on a magnetic dipole (bar magnet) placed in a uniform magnetic field. Give its unit.

3. What is the difference between the torque acting on a magnetic dipole and an electric dipole?

4. Prove that the potential energy of a magnetic dipole in a uniform magnetic field is given by-

$U = - \vec{M} \cdot \vec{B}$

where symbols have their usual meanings.

5. Give an example of magnetic dipole.

6. Why is a current loop considered a magnetic dipole?

7. In hydrogen atom, the electron revolves round the nucleus 6.1×10^{12} times per second in an orbit of radius 0.53 A^0. Estimate its equivalent magnetic moment.

8. A bar magnet of magnetic moment 0.8 Joule/Tesla placed with its axis at 45^0 with a uniform external magnetic field experiences a torque of magnitude 0.062 Joule. Find the strength of the magnetic field.

9. Describe the theory of a moving coil galvanometer. Establish the expression for ballistic constant of the galvanometer.

10. Why is a correction for damping of a moving coil galvanometer.

11. What are the conditions that a moving coil galvanometer is ballistic?

12. Prove that the total charge passed through the galvanometer is directly proportional to the maximum angular deflection of the coil, on passing the momentary current through the ballistic galvanometer

13. Explain-

(i) Current sensitivity of a ballistic galvanometer

(ii) Charge sensitivity of a ballistic galvanometer

(iii) Voltage sensitivity of ballistic galvanometer

9.10 ANSWERS

Self Assessment Questions (SAQs):

1. A magnetic dipole is in stable equilibrium when $\vec{M}$ is parallel to $\vec{B}$ and in unstable equilibrium when anti-parallel.

2. Given, M = 1.5 Joule/Tesla, B = 0.22 Tesla

(a) The potential energy of a magnet of magnet moment $\vec{M}$ placed in a magnetic field $\vec{B}$ is given by-

$$U_\theta = - MB \cos\theta$$

The potential energies when $\theta = 0^0$, $\theta = 90^0$ and $\theta = 180^0$ are respectively-

$$U_0 = - MB \cos 0^0 = - MB$$

$$U_{90^0} = - MB \cos 90^0 = 0$$

and $$U_{180^0} = - MB \cos 180^0 = -MB(-1) = + M B$$

Therefore, the work required in case (i) is-

$$W = U_{90^0} - U_0 = 0 - (-M B)$$

$$= + MB$$

$$= 1.5 \times 0.22$$

$$= 0.33 \text{ Joule}$$

In case (ii), the work required is-

$$W = U_{180^0} - U_0$$

$$= M B - (-M B)$$

$$= 2 M B = 2 \times 1.5 \times 0.22 = 0.66 \text{ Joule}$$

(b) The torque in case (i) is-

$$\tau = M B \sin\theta$$

$$= 1.5 \times 0.22 \times \sin 90^0$$

$$= 0.33 \text{ Newton-meter}$$

And that in case (ii) is-

$\tau = M B \sin\theta$

$$= 1.5 \times 0.22 \times \sin 180^0 = 0$$

3. Given, $\tau = 1.5 \times 10^{-5}$ Newton-meter, $B = 6 \times 10^{-4}$ Wb/meter^2, $\theta = 90^0$

We know that-

$$\tau = M B \sin\theta$$

or $$M = \frac{\tau}{B\sin\theta} = \frac{1.5\times10^{-5}}{6\times10^{-4}\times\sin 90^0}$$

$$= 0.25\times 10^{-1} = 0.025 \text{ Amp-meter}^2$$

4. (a) Amp-meter^2

Terminal Questions:

5. A bar magnet having north and south poles

6. A bar magnet (which is a magnetic dipole) suspended in a uniform magnetic field experiences a torque and therefore, it sets with its axis parallel to the magnetic field. A current loop also experiences a torque in a magnetic field due to which it sets with its axis parallel to the magnetic field.

7. Given radius of the orbit = 0.53 A^0 = 0.53×10^{-10} meter, No. of revolutions per second = 6.1×10^{12}

The electron revolving in an orbit is equivalent to a current loop. The magnitude of the current is-

i = charge passing per second through any point in the orbit

= charge on electron × number of revolutions per second

= $(1.6 \times 10^{-19}$ coulomb$) \times (6.1 \times 10^{12}$ /sec$)$

= 9.76×10^{-7} Amp

The magnetic moment of the equivalent current loop is-

$$M = N\, i\, A$$

Here N is the number of turns and A is the area of the loop. Here N = 1, i = 9.76×10^{-7} Amp, A = area of the loop = $\pi r^2 = 3.14 \times (0.53 \times 10^{-10})^2 = 0.882 \times 10^{-20}$ meter^2

Therefore, M = $1 \times 9.76 \times 10^{-7} \times 0.882 \times 10^{-20} = 8.608 \times 10^{-27}$ Amp-meter^2

8. Given, M = 0.8 Joule/Tesla, $\theta = 45^0$, τ = 0.062 Joule

We know that-

$$\tau = M B \sin\theta$$

or $$B = \frac{\tau}{M\sin\theta} = \frac{0.062}{0.8\times\sin 45^0} = 1.095 \text{ Tesla}$$

9.11 REFERENCES

1. Electricity and Magnetism, DC Tayal, Himalaya Publishing House, Mumbai

2. Electric and Magnetism, Satya Prakash, Pragati Prakashan, Meerut

3. Nootan Physics, Part-II, Kumar Mittal, Nageen Prakashan Pvt. Ltd., Meerut

9.12 SUGGESTED READINGS

1. Modern Physics, Beiser, Tata McGraw Hill

2. Fundamentals of Physics, David Halliday, Robert Resnick, Jearl Walker, John Wiley & Sons

3. Engineering Physics, S.K. Gupta, Krishna Prakashan Media (P) Ltd., Meerut

4. Electricity and magnetism with electronics, K.K. Tewari, S. Chand Publishing, New Delhi

5. University Physics, Francis W. Sears, Mark W. Zemansky, Hugh D. Young, Narosa Publishing House, New Delhi

UNIT 10 MAGNETISM INTENSITY OF MAGNETIZATION

Structure

10.1 INTRODUCTION

In the previous unit,you have learnt about torque acting on a bar magnet in a uniform magnetic field, magnetic dipole, energy stored for a dipole in a magnetic field and ballistic galvanometer.The magnetic properties of a substance are explained in terms of tiny current loops within the substance. These current loops occur due to motion of electrons within atom. You know that an atom consists of positively charged nucleus, surrounded by a cloud of electrons. These electrons circulate about the nucleus in definite orbits and also spin about their own axes. These moving electrons are equivalent to tiny current loops and produce magnetic fields. In an unmagnetised material, the current loops are oriented randomly; therefore, the magnetic fields produced by them are cancelled. The magnetization process consists of aligning these loops such that the magnetic moment produced by them is parallel to the magnetizing field and hence a resultant magnetic field is created. Whenever any material is placed in a magnetic field, the elementary current loops tend to get aligned parallel or antiparallel to field direction and the material is said to be magnetized. The magnetic field at any point is the resultant of the original magnetic field and the field set up due to alignment of current loops. In the present unit, you will learn about flux density in a magnetic material and some important terms used in magnetism such as magnetic induction, intensity of magnetization, magnetic intensity etc. You will also study the classification of magnetic materials on the basis of relative magnetic permeability.

10.2 OBJECTIVES

After studying this unit, you should be able to-

- understand magnetic induction
- understand Intensity of magnetization
- calculate the magnetic intensity

10.3 MAGNETIC INDUCTION

You know that when a piece of any substance is placed in an external magnetic field, the substance becomes magnetized. The magnetism produced in this way in the substance is called induced magnetism and this phenomenon is called magnetic induction.

Let us consider an iron bar placed in a uniform magnetic field with its length parallel to the magnetic lines of force as shown in figure (1). The bar is magnetized by induction, with a south pole induced on the left end where lines of force enter the bar and a north pole induced on the right end where lines of force leave the bar. The magnetized bar produces its own magnetic field. Its lines of force are in the same direction as those of the original magnetic field inside the bar but in opposite direction outside the bar. This results in a concentration of the lines of force within the bar as shown in figure (2). The magnetic flux density within the bar is increased whereas it becomes quite weak at certain places outside the bar.

The magnetic lines of force inside the magnetized bar are called magnetic lines of induction.

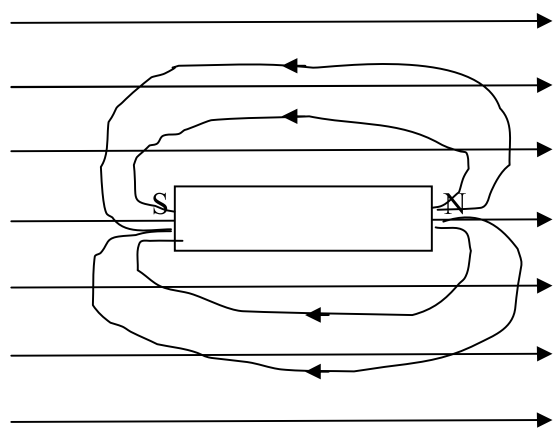


Figure 1

The number of magnetic lines of induction inside a magnetized substance crossing unit area normal to their direction is called the magnitude of magnetic induction or magnetic flux density inside the substance. It is represented by B. Magnetic induction is a vector quantity whose direction at any point is the direction of magnetic line of induction at that point.

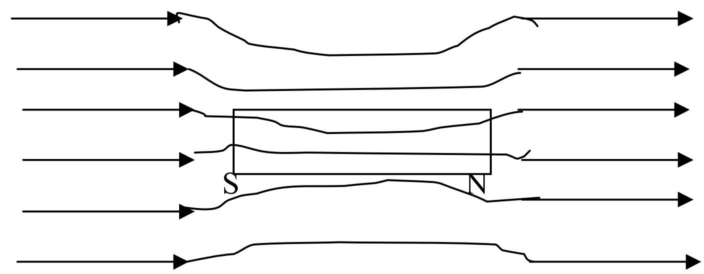


Figure 2

In vector form, it is written as $\vec{B}$. The SI unit of magnetic induction is Tesla (T) or Weber/meter^2 (Wb/m^2) or Newton/ (amp-meter). Gauss is the CGS unit of magnetic induction.

10.4 INTENSITY OF MAGNETIZATION

The intensity of magnetization of a magnetized substance represents the extent to which the substance is magnetized. It is also known as simply magnetization.

It is defined as the magnetic moment(μ_M) per unit volume of the magnetized substance. It is denoted by M. Therefore,

$$M= \frac{\mu_M}{V} \quad .....(1)$$

It is also vector. In vector form, intensity of magnetization is written as $\vec{M}$.The SI unit of intensity of magnetization is ampere/meter.

In case of bar magnet, if m be the pole-strength of the magnet, 2*l* its magnetic length and A its area of cross-section, then-

$$M = \frac{\mu_M}{V} = \frac{m\times 2l}{A\times 2l}$$

$$= \frac{m}{A} \quad .....(2)$$

Thus, magnetization may also be defined as pole strength per unit area of cross-section.

10.5 MAGNETIC INTENSITY

Magnetic intensity is also known as magnetic field strength. When a substance is placed in an external magnetic field, it becomes magnetized. The actual magnetic field inside the substance is the sum of the external magnetic field and the field due to its magnetization. The ability of the magnetizing field to magnetize the substance is expressed by means of a vector $\vec{H}$, called the magnetic intensity of the field.

The magnetic intensity is defined through the vector relation-

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \quad .....(3)$$

where $\vec{B}$ is the magnetic field induction inside the substance and $\vec{M}$ is the intensity of magnetization. μ_0 is the permeability of free space.

The SI unit of $\vec{H}$ is same as of $\vec{M}$ which is ampere/meter. Oersted is the CGS unit of magnetic intensity.

10.6 MAGNETIC PERMEABILITY

It is denoted by μ. The magnetic permeability of a substance is a measure of its conduction of magnetic lines of force through it.

The magnetic permeability is defined as the ratio of the magnetic induction $\vec{B}$ inside the magnetized substance to the magnetic intensity $\vec{H}$ of the magnetizing field, i.e.

$$\mu = \frac{\vec{B}}{\vec{H}} \qquad \text{.....(4)}$$

Numerically, it is written as- $\mu = \frac{B}{H}$ .....(5)

Its SI unit is Weber/ (ampere-meter) or Newton/ampere^2.

10.7 RELATIVE MAGNETIC PERMEABILITY

The relative magnetic permeability of a substance is the ratio of the magnetic permeability μ of the substance to the permeability of free space μ_0, i.e.

$$\mu_r = \frac{\mu}{\mu_0} \qquad \text{.....(6)}$$

It is a dimensionless quantity. It is equal to 1 for vacuum.

The relative permeability of a substance is also defined as the ratio of the magnetic flux density B in the substance when placed in a magnetic field and the flux density B_0 in vacuum in the same field i.e.

$$\mu_r = \frac{B}{B_0} \qquad \text{.....(7)}$$

Now we can classify the substances in terms of μ_r as-

$\mu_r < 1$ diamagnetic

$\mu_r > 1$ paramagnetic

$\mu_r >> 1$ ferromagnetic

10.8 RELATION BETWEEN B, H AND M

Let us consider the material in the form of a Rowland ring having a toroidal winding of N turns around it. When a current i is passed through the winding, the material ring is magnetized along its circumferential length. The current i is the real current which magnetizes the ring.

Figure (3) shows the section of the magnetized ring. The small circles represent the current loops. Now except at the periphery i.e. outer circle, every portion of each loop is adjacent to another loop in which the current at the point of contact is in opposite direction. Hence net current inside is zero.

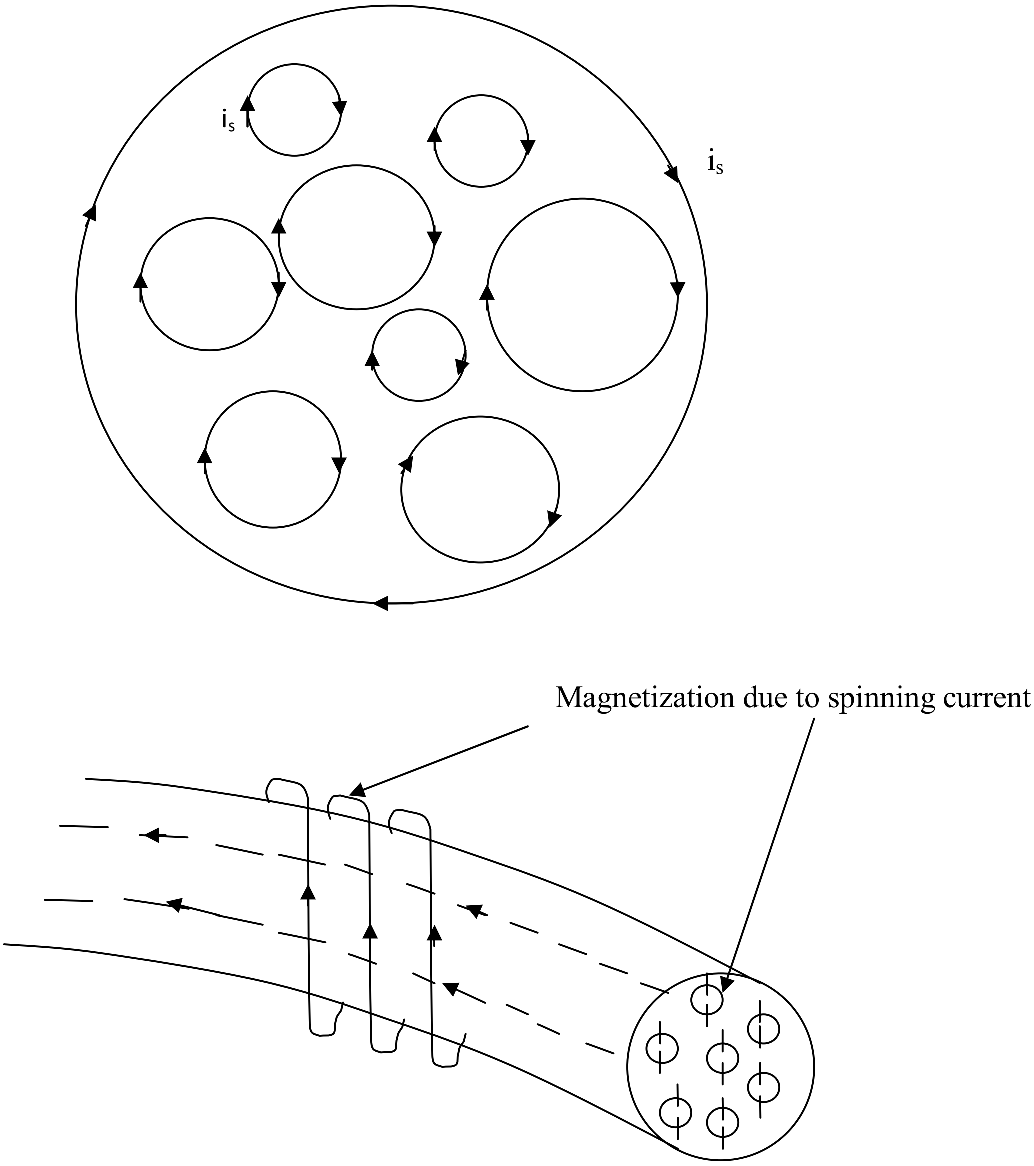


Figure 3

The current in the outer loop remains uncancelled. Thus the whole network of electronic currents within the material can be replaced by a current i_s circulating around the surface of the ring. This current is called Amperian surface current.

Let A be the cross-sectional area and *l* the circumferential length of the ring.

The volume of the ring = A *l*

This ring behaves as a large dipole of magnetic moment i_s A.

Therefore, the magnetization = $\frac{\text{Magnetic moment}}{\text{Volume}} = \frac{i_s A}{Al}$

$$= \frac{i_s}{l} = M \qquad .....(8)$$

Thus, the magnetization equals the Amperian surface current per unit length. This is also known as magnetizing current.

Now, the magnetic flux density B within the material of the ring arises due to free current i and due to Amperian surface current i_s.

Therefore,
$$B = \mu_0 \left(\frac{N\,i}{l} + \frac{i_s}{l}\right) \qquad .....(9)$$

Putting $\frac{i_s}{l} = M$ from equation (8) in equation (9), we get-

$$B = \mu_0 \left(\frac{Ni}{l} + M\right)$$

Or
$$\frac{B}{\mu_0} - M = \frac{N\,i}{l} \qquad .....(10)$$

The quantity $\left(\frac{B}{\mu_0} - M\right)$ is of significance in magnetism and is known as magnetic intensity or magnetic field intensity (H).

Therefore,
$$H = \frac{B}{\mu_0} - M$$

Or
$$B = \mu_0 (H + M) \qquad .....(11)$$

Since B, M and H are vectors, therefore, in vector forms-

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) \qquad .....(12)$$

and
$$\vec{H} = \frac{N\,i}{l} = n\,i$$

where n is the number of turns per unit length.

In this way, the value of H depends only on the free current I and is independent of the material.

If the Rowland ring is empty, then $\vec{M} = 0$. The flux density in vacuum is –

$\vec{B_0} = \mu_0 \vec{H}$ .....(13)

The equation (12) is the relation between three magnetic vectors $\vec{B}$, $\vec{H}$ and $\vec{M}$.

Example 1: The horizontal component of the flux density of the earth's magnetic field is 1.7×10^{-5} Wb/meter^2. What is the horizontal component of the magnetic intensity?

Solution: Given $B_0 = 1.7 \times 10^{-5}$ Wb/meter^2, $\mu_0 = 1.26 \times 10^{-6}$ H/m

We know that-

$$B = \mu_0 H$$

Or $$H = \frac{B}{\mu_0} = \frac{1.7 \times 10^{-5}}{1.26 \times 10^{-6}} = 13.5 \text{ Ampere/meter}$$

Example 2: A bar magnet has a coercivity of 4×10^3 Amp/meter. It is desired to demagnetise it by inserting it inside a solenoid 12 cm long and having 60 turns. What current should be sent through the solenoid?

Solution: The bar magnet requires a magnetic intensity $H = 4 \times 10^3$ Amp/meter to become demagnetised.

$$n = \text{number of turns per unit length} = 60/\ (12\times10^{-2}) = 500$$

Let i be the current carried by the solenoid to produce the magnetic intensity, then-

$$H = n\ i = (N/l)\ i$$

Or $$i = H / n = 4 \times 10^3/500 = 8 \text{ Amp}$$

Self Assessment Question (SAQ) 1: A current of 2 Amp is passed through a winding of 20 turns per cm. If the magnetic induction is 1.2 Wb/meter^2, calculate the intensity of magnetic field and the intensity of magnetization.

Self Assessment Question (SAQ) 2: Choose the correct option-

The unit of intensity of magnetization is-

(a) Amp/meter^2 (b) weber/meter^2 (c) Amp/meter (d) Amp× meter

Self Assessment Question (SAQ) 3: Choose the correct option-

The relationship between three magnetic vectors is-

(a) $B = \mu_0 (H + M)$ (b) $H = \frac{B}{\mu_0} + M$ (c) $H = \frac{B}{\mu_0} \times M$ (d) none of these

Self Assessment Question (SAQ) 4: Choose the correct option-

The magnetic permeability of a substance is a measure of -

(a) its conduction of magnetic lines of force through it

(b) its conduction of electric lines of force through it

(c) its conduction of electricity through it

(d) none of these

Self Assessment Question (SAQ) 5: Choose the correct option-

The magnetic permeability of vacuum, in SI units, is-

(a) 1 (b) infinite (c) zero (d) $4\pi\times10^{-7}$

Self Assessment Question (SAQ) 6: Choose the correct option-

Magnetism in substances is caused by-

(a) orbital motion of electrons only (b) spin motion of electrons only

(c) due to spin and orbital motion of electrons both (d) none of these

10.9 SUMMARY

In this unit, you have learnt about magnetic induction, intensity of magnetization, magnetic intensity, magnetic permeability and relative magnetic permeability. The intensity of magnetization of a magnetized substance represents the extent to which the substance is magnetized. It is also known as simply magnetization. It is defined as the magnetic moment(μ_M) per unit volume of the magnetized substance. You have also defined the magnetic intensity which is also known as magnetic field strength. When a substance is placed in an external magnetic field, it becomes magnetized. The magnetic permeability is defined as the ratio of the magnetic induction $\vec{B}$ inside the magnetized substance to the magnetic intensity $\vec{H}$ of the magnetizing field. You have also classified the magnetic materials on the basis of relative magnetic permeability. In the present unit, you have established the relation between three magnetic vectors as $\vec{B} = \mu_0(\vec{H} + \vec{M})$. To present the clear understanding of the unit, solved examples are given in the unit. To check your progress, self assessment questions (SAQs) are given in the unit.

10.10 GLOSSARY

Magnetic- attractive, compelling

Magnetic field-the region surrounding a magnet in which the force of the magnet can be detected

Induction- brining on, stimulation, initiation

10.11 TERMINAL QUESTIONS

1. Define magnetic induction and intensity of magnetization.

2. Write notes on-

   (a) Magnetic Intensity (b) Magnetic Permeability (c) Relative Magnetic Permeability

3. Establish the relation among three magnetic vectors-

$\vec{B} = \mu_0(\vec{H} + \vec{M})$

4. An iron rod of volume 10^{-4} meter^3 and relative permeability 1000 is placed inside a long solenoid wound with 5 turns per cm. If a current of 0.5 Amp is passed through the solenoid, find the magnetic moment of the rod.

5. A material core has 1000 turns/meter of wire wound uniformly upon it which carries a current of 2 Amp. The flux density in the material is 1 Wb/meter^2. Calculate the magnetizing force and magnetization of the material. What would be the relative permeability of the core? ($\mu_0 = 4\pi\times10^{-7}$ Wb/Amp-meter

10.12 ANSWERS

Self Assessment Questions (SAQs):

1. Given i = 0.2 Amp, n = number of turns per unit length = 20 turns/cm = $20/(1\times10^{-2})$ = 2000, B = 1.2 Wb/meter^2

We know-

Intensity of magnetic field H = n i = 2000 × 0.2 = 400 Amp/meter

Again we know- $H = \frac{B}{\mu_0} - M$

Or $M = \frac{B}{\mu_0} - H = \frac{1.2}{1.26\times10^{-6}} - 400 = 9.5\times10^5 - 400$

$= 950000 - 400 = 949600$ Amp/meter

2. (c) Amp/meter

3. (a) $B = \mu_0 (H + M)$

4. (a) its conduction of magnetic lines of force through it

5. (d) $4\pi\times10^{-7}$

6. (c) due to spin and orbital motion of electrons both

Terminal Questions:

4. Given $V = 10^{-4}$ meter^3, $\mu_r = 1000$, n = 5 turns/cm = 500 turns/meter, i = 0.5 Amp

We know $H = n\,i = 500\times0.5 = 250$ Amp/meter

Again, $B = \mu H = (\mu_r \mu_0) H$ (since $\mu_r = \frac{\mu}{\mu_0}$)

$= (1000\times1.26 \times 10^{-6})\times 250 = 0.315$ weber/meter^2

We know that- $B = \mu_0 (H + M)$

Or $M = \frac{B}{\mu_0} - H = 0.315/1.26 \times 10^{-6} - 250$

$= 250\times10^3 - 250 = 2.49\times10^5$ Amp/meter

Using $M = \frac{\mu_M}{V}$

Or $\mu_M = M \times V = 2.49\times10^5 \times 10^{-4} = 24.9$ Amp/meter^2

5. Given n = 1000 turns/meter, i = 2 Amp, B = 1 Wb/meter^2, $\mu_0 = 4\pi\times10^{-7}$ Wb/Amp-meter

Magnetizing force $H = n\,i = 1000\times 2 = 2000$ Amp/meter

We know that $B = \mu_0 (H + M)$

Or Magnetization $M = \frac{B}{\mu_0} - H = \frac{1}{4\pi\times10^{-7}} - 2000 = 7.94\times 10^5$ Amp/meter

Relative permeability $\mu_r = \frac{\mu}{\mu_0} = \frac{B/H}{\mu_0} = \frac{B}{\mu_0 H} = \frac{1}{4\pi\times10^{-7}\times2000} = 397$

10.13 REFERENCES

1. Electricity and Magnetism, DC Tayal, Himalaya Publishing House, Mumbai

2. Electric and Magnetism, Satya Prakash, Pragati Prakashan, Meerut

3. Nootan Physics, Part-II, Kumar Mittal, Nageen Prakashan Pvt. Ltd., Meerut

4. Engineering Physics, Gupta and Kumar, Dhanpat Rai & Co.(P) Ltd, Delhi

10.14 SUGGESTED READINGS

1. Modern Physics, Beiser, Tata McGraw Hill

2. Fundamentals of Physics, David Halliday, Robert Resnick, Jearl Walker, John Wiley & Sons

3. Engineering Physics, S.K. Gupta, Krishna Prakashan Media (P) Ltd., Meerut

4. Electricity and magnetism with electronics, K.K. Tewari, S. Chand Publishing, New Delhi

5. University Physics, Francis W. Sears, Mark W. Zemansky, Hugh D. Young, Narosa Publishing House, New Delhi

UNIT 11 MAGNETIC SUBSTANCES, CURIE'S LAW AND HYSTERESIS IN A FERROMAGNETIC MATERIALS

Structure

11.1 INTRODUCTION

In the previous unit, you have learnt about magnetic induction, intensity of magnetization, magnetic intensity, magnetic permeability and relative magnetic permeability. In that unit, you have established the relation between three magnetic vectors as $\vec{B} = \mu_0(\vec{H} + \vec{M})$.You have also classified the magnetic materials on the basis of relative magnetic permeability. In the present unit, you will learn about magnetic susceptibility and establish the relation between relative permeability and magnetic susceptibility. In the unit, you will classify the substances according to their magnetic behaviour. You will also know that the magnetic susceptibility and the magnetic permeability of the substance are not constant but vary with magnetic field strength and also depend upon the past history of the substance and study the hysteresis and the uses of hysteresis curve.

11.2 OBJECTIVES

After studying this unit, you should be able to-

- understand magnetic susceptibility
- understand Curie's law
- understand the magnetic substances
- understand hysteresis and calculate the energy loss due to hysteresis
- know the applications of hysteresis curve

11.3 MAGNETIC SUSCEPTIBILITY

Magnetic susceptibility is a measure of how easily a substance is magnetized in a magnetizing field. For some types of magnetic materials like paramagnetic and diamagnetic substances, the magnetization (intensity of magnetization) $\vec{M}$ is directly proportional to the magnetic intensity $\vec{H}$ of the magnetizing field i.e.

$\vec{M} \propto \vec{H}$

Or $$\vec{M} = \chi_m \vec{H} \qquad .....(1)$$

where χ_m is a constant called the magnetic susceptibility of the substance. It may be defined as the ratio of the intensity of magnetization to the magnetic intensity of the magnetizing field i.e.

$$\chi_m = \frac{M}{H} \qquad .....(2)$$

χ_m is a pure number and is unit less. Its value for vacuum is zero as there can be no magnetization in vacuum. We can classify the substances in terms of χ_m as follows-

χ_m = + ve, substance is paramagnetic

χ_m = -ve, substance is diamagnetic

χ_m = +ve and very large, substance is ferromagnetic

However, for them, the magnetization $\vec{M}$ is not accurately proportional to $\vec{H}$ and therefore, χ_m is not strictly constant.

11.4 RELATION BETWEEN RELATIVE PERMEABILITY AND MAGNETIC SUSCEPTIBILITY

You know that when a substance is kept in a magnetizing field, it becomes magnetized. The total magnetic flux density B within the substance is the flux density that would have been produced by the magnetizing field in vacuum plus the flux density due to the magnetization of the substance. If M be the intensity of magnetization of the substance, then, we know the relation (in magnitude)-

$$B = \mu_0 (H + M) \qquad .....(3)$$

where H is the magnetic intensity.

But $\chi_m = \frac{M}{H}$

Or $M = \chi_m H$

Putting for M in the above expression (3), we get-

$$B = \mu_0 (H + \chi_m H)$$

or $B = \mu_0 H (1+\chi_m)$ .....(4)

Again we have $B = \mu H$

Now, substituting the value of B in equation (4), we get-

$\mu H = \mu_0 H (1+\chi_m)$

or $\mu = \mu_0 (1+\chi_m)$

or $\frac{\mu}{\mu_0} = 1 + \chi_m$ .....(5)

Since $\frac{\mu}{\mu_0} = \mu_r$, the relative permeability, therefore-

$$\mu_r = 1 + \chi_m \qquad .....(6)$$

This is the relation between relative permeability and magnetic susceptibility.

11.5 MAGNETIC SUBSTANCES

All substances, solids, liquids and gases, show magnetic properties. You can classify these substances on the basis of their magnetic behaviour-

11.5.1 Diamagnetic Substances

Some substances, when are placed in a magnetic field, are softly magnetized opposite to the direction of the magnetizing field. These substances when brought close to a pole of a powerful magnet, are somewhat repelled away from the magnet. They are called diamagnetic substances and their magnetism is called the diamagnetism. Bismuth, zinc, copper, lead, gold, silver, water, hydrogen, sodium chloride, nitrogen, mercury etc. are the examples of diamagnetic substances.

Properties

Diamagnetic substances have the following properties-

1. These substances have negative magnetic susceptibility.
2. The flux density in a diamagnetic substance placed in a magnetising field is slightly less than that in the free space.
3. The relative permeability of these substances is less than 1 i.e. $\mu_r < 1$ for diamagnetic substances.
4. The susceptibility of diamagnetic substances is independent of temperature.
5. A diamagnetic gas, when allowed to ascend in between the poles of a magnet, spreads across the magnetic field.
6. If a diamagnetic solution is poured into a U- tube and one arm of this U-tube is placed between the poles of a strong magnet, the level of the solution in that arm is depressed as shown in figure (1).

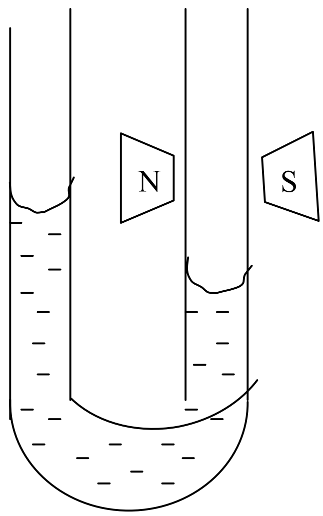


Figure 1

7. In a non-uniform magnetic field, a diamagnetic substance tends to move from the stronger to the weaker part of the magnetic field. If we take a diamagnetic liquid in a watch glass placed on two magnetic poles very near to each other, then the liquid is depressed in the middle, where the magnetic field is strongest. Now, if the distance between the poles is increased, the liquid rises in the middle, because now the magnetic field is strongest near the poles (Figure 2).

Figure 2

8. When a rod of diamagnetic material is suspended freely between two magnetic poles, then its axis becomes perpendicular to the magnetic field. The poles produced on the two sides of the rod are similar to the nearer magnetic poles (Figure 3).

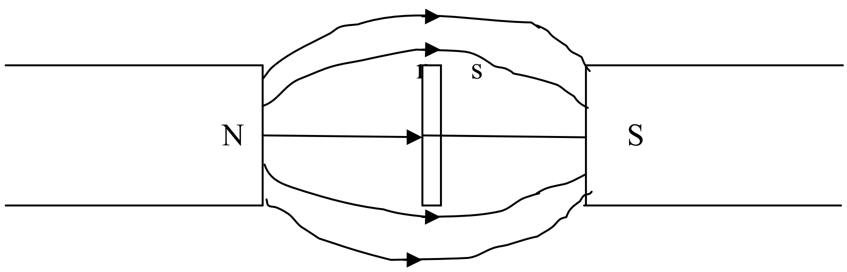


Figure 3

11.5.2 Paramagnetic Substances

Some substances, when are placed in a magnetic field, are softly magnetized in the direction of the magnetising field. These substances, when brought close to a pole of a powerful magnet, are attracted towards the magnet. These substances are called paramagnetic substances and their magnetism is called paramagnetism. Aluminium, antimony, copper chloride, sodium, platinum, manganese, liquid oxygen, solutions of salts of iron and nickel etc. are the examples of paramagnetic substances.

Properties

Paramagnetic substances have the following properties-

1. These substances have positive magnetic susceptibility.
2. The flux density in a paramagnetic substance placed in a magnetising field is slightly greater than that in the free space.
3. The relative permeability of these substances is greater than 1 i.e. $\mu_r > 1$ for paramagnetic substances.
4. The susceptibility of paramagnetic substances varies inversely as the kelvin temperature of the substance i.e.

$$\chi_m \alpha \frac{1}{T}$$

   This is known as Curie's law.
5. A paramagnetic gas, when allowed to ascend in between the poles of a magnet, spreads along the magnetic field.
6. If a paramagnetic solution is poured into a U- tube and one arm of this U-tube is placed between the poles of a strong magnet, the level of the solution in that arm rises as shown in figure (4).

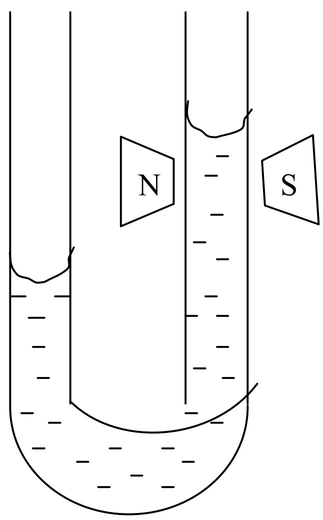


Figure 4

7. In a non-uniform magnetic field, a paramagnetic substance tends to move from the weaker to the stronger part of the magnetic field. If we take a paramagnetic liquid in a watch glass placed on two magnetic poles very near to each other, then the liquid rises in the middle, where the magnetic field is strongest. Now, if the distance between the poles is increased, the liquid depresses in the middle and rises near the edges, because now the magnetic field is strongest near the poles (Figure 5).

Figure 5

8. When a rod of paramagnetic material is suspended freely between two magnetic poles, then its axis becomes parallel to the magnetic field. The poles produced at the ends of the rod are opposite to the nearer magnetic poles (Figure 6).

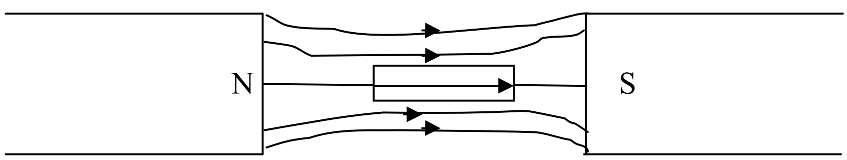


Figure 6

11.5.3 Ferromagnetic Substances

Some substances, when placed in a magnetic field, are strongly magnetised in the direction of the magnetising field. These materials are attracted fast towards a magnet when brought close to either of the poles of the magnet. These are called ferromagnetic substances and their magnetism is called ferromagnetism. Iron,cobalt, nickel, magnetite etc. are some ferromagnetic substances.

Properties

The ferromagnetic substances have the following properties-

1. These substances have positive and very large magnetic susceptibility.

2. The relative permeability of these substances is very-very greater than 1 i.e. $\mu_r >> 1$ for ferromagnetic substances.

3. These substances show all the properties of paramagnetic substances to a much high degree.

4. Ferromagnetism decreases with increase in temperature. If you heat a ferromagnetic substance, then at a definite temperature the ferromagnetic property of the substance suddenly disappears and the substance becomes paramagnetic. The temperature above which a ferromagnetic substance becomes paramagnetic is known as Curie temperature of the substance. The Curie temperature of iron is 770^0 C and that of nickel is 358^0 C.

   You should know that as a matter of fact, every substance is diamagnetic. In those substances which are paramagnetic or ferromagnetic, the diamagnetic property is masked by the stronger paramagnetic or ferromagnetic properties.

11.6 CURIE'S LAW

In 1895, Curie discovered experimentally that the magnetization or intensity of magnetization of a paramagnetic substance is directly proportional to the magnetic intensity H of the magnetising field and inversely proportional to the Kelvin temperature T i.e.

$$M \,\alpha\, \frac{H}{T}$$

Or

$$M = C\,\frac{H}{T} \qquad .....(7)$$

where C is a constant. This equation is known as Curie's law and the constant C is called the Curie constant. The law, however, holds so long the ratio $\frac{H}{T}$ does not become too large.

M cannot increase without limit. It approaches a maximum value corresponding to the complete alignment of all the atomic magnets contained in the substance.

You can express Curie's law in an alternative form. You know that the magnetic susceptibility χ_m is defined as-

$$\chi_m = \frac{M}{H}$$

Putting the value of M from equation (7) in the above equation, we get-

$$\chi_m = \frac{C\frac{H}{T}}{H} = \frac{C}{T}$$

or $$\chi_m \propto \frac{1}{T}$$

i.e. the magnetic susceptibility is inversely proportional to Kelvin temperature.This is known as the Curie's law.

11.7 HYSTERESIS

As you know that for ferromagnetic substances the magnetic flux density B is not a linear function of magnetic intensity H because in such cases, the relative magnetic permeability is not constant but is a function of H. In other words, we can say that there is no unique value of relative magnetic permeability for a particular ferromagnetic substance. The relationship between magnetic flux density B and corresponding magnetic intensity H for such a material initially unmagnetised is represented by a typical curve as shown in figure (7), known as the magnetization curve or B-H curve.

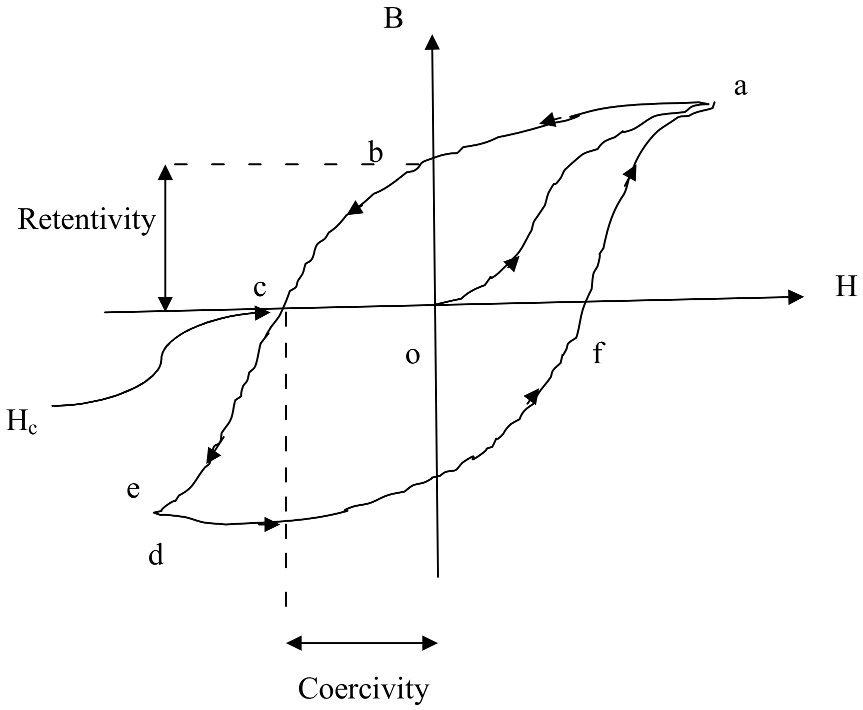


Figure 7

Figure (7) represents the variation in B with variation in H. The point O represents the initial unmagnetised state of the substance (B =0) and a zero magnetic intensity (H =0). As H is increased, B increases non-uniformly along curved path oa. At a, the substance acquires a state of magnetic saturation. Any further increase in H does not produce any increase in B. Now the value of B becomes practically constant.

If now the magnetising magnetic field H is decreased, the magnetic flux density B of the substance also decreases following a new path ab, not the original path ao. Thus B lags behind H. When H becomes zero, B still has a value equal to ob. The magnetic flux density in the substance is seen to depend upon not on the magnetic intensity alone but on the magnetic history of the substance as well. At point b, the specimen has become a permanent magnet since magnetization is still present even though the magnetising field H has been cut off. The magnetization remaining in the substance when the magnetising field is reduced to zero is called the 'residual magnetism'. The power of retaining this magnetization is called the 'retentivity' or the 'remanence' of the substance. In this way, the retentivity of a substance is a measure of the magnetization remaining in the substance when the magnetising field is removed. In the above figure, ob represents the retentivity of the substance.

If now the magnetising field H is increased in the reverse direction, the magnetic flux density B decreases along path bc, still lagging behind H, until it becomes zero at point c where H equals oc. This value of H is denoted by H_c. This value oc of the magnetising field is called the 'coercive force' or 'coercivity' of the substance. Thus, the coercivity of a substance is a measure of the reverse magnetising field required to destroy the residual magnetism of the substance. When we increase H beyond oc, the substance is increasingly magnetised in the opposite direction along cd and a reverse induction is set up in the substance which quickly attains the saturation value. At point d, the substance is again magnetically saturated.

By taking H back from its maximum negative value, through zero, to its original maximum positive value, a symmetrical curve defa is obtained. At point e where the substance is magnetised in the absence of any external magnetising field, it is said to be a permanent magnet.

In this way, we found that the magnetization and also the magnetic flux density B always lags behind the magnetising field H. The lagging of B behind H is called 'hysteresis'. The closed curve or loop, abcdefa which represents a cycle of magnetization of the substance is known as the 'hysteresis curve or loop' of the substance. On repeating the process, the same closed curve is traced again but the portion oa is never obtained.

11.7.1 Energy Loss due to Hysteresis

According to molecular theory of magnetization, the molecules of magnetised or unmagnetised magnetic substance are themselves complete magnets. When we apply a magnetised field, the molecular magnets align themselves in the direction of the field. During this process, work is done by the magnetising field in turning the molecular magnets against the mutual attractive forces. This energy required to magnetise a substance is not completely recovered when the magnetising field is turned off, since the magnetization does not become zero. The specimen retains some magnetization because some of the molecular magnets remain aligned in the new formation due to the group forces. To destroy them out completely, a coercive force in the reverse direction has to be applied. In this way, there is a loss of energy in taking a magnet through a cycle of magnetization. This loss of energy or heat is called 'hysteresis loop'. Now let us calculate this loss of energy.

Let us consider a magnetic material having n molecular magnets per unit volume. Let m be the magnetic moment of each elementary magnet and θ the angle which its axis makes with the direction of magnetising field H, then magnetic moment per unit volume parallel to the magnetic field is-

$$\mu_M = \Sigma\ m \cos\theta \qquad .....(8)$$

The magnetic moment per unit volume perpendicular to the magnetising field is $\Sigma\ m \sin\theta$ and this is equal to zero since there can be no magnetization perpendicular to H.

Now, the torque due to the magnetising field acting on the dipole of moment m when it is inclined at an angle θ to the field is-

$$\tau = \mu_0\ m\ H \sin\theta \qquad .....(9)$$

and the work done when it moves through a small angle from θ to θ + dθ = - τ dθ

$$= - \mu_0\ m\ H \sin\theta\ d\theta$$

Here minus sign comes in because the work has to be done against the magnetic field in increasing θ by dθ.

Hence, the work done per unit volume of the material $dW = - \mu_0\ m\ H \sin\theta\ d\theta$ .....(10)

As θ increases by dθ, the intensity of magnetization M also increases by dM obtained from equation (8) as-

$$dM = d\ (\Sigma\ m \cos\theta)$$

$$= - \Sigma\ m \sin\theta\ d\theta \qquad .....(11)$$

From equations (10) and (11), we get-

$$dW = \mu_0 H\ dM \qquad .....(12)$$

Thus, the work done by the magnetising field per unit volume of the material for completing a cycle is-

$$W = \oint dW$$

$$= \oint \mu_0 H\ dM = \mu_0 \oint H\ dM$$

$= \mu_0 \times$Area of M-H loop .....(13)

Since we know that-

$B = \mu_0 (H + M)$

Or $$dB = \mu_0 (dH + dM)$$

Or $$dM = \frac{dB}{\mu_0} - dH \qquad .....(14)$$

Putting for dM in equation (13), we get-

$$W = \mu_0 \oint H\left(\frac{dB}{\mu_0} - dH\right)$$

$$= \mu_0 \oint H \frac{dB}{\mu_0} - \mu_0 \oint H\, dH$$

$$= \oint H\, dB - \mu_0 \oint H\, dH \quad .....(15)$$

But $\oint H\, dH = 0$, because the plot of H against H is a straight line and the area enclosed by it is zero. Thus equation (15) gives-

$$W = \oint H\, dB$$

$$= \text{Area of B-H loop} \qquad .....(16)$$

Thus, the work done per unit volume of the material per cycle is equal to the area of μ_0 times the area of M-H loop or the area of B-H loop. The unit of this work is Joule/meter^3 per cycle and is dissipated in the form of heat.

11.7.2 Uses of Hysteresis Curve

Importance of hysteresis curve

By using the hysteresis curve of various ferromagnetic materials, we can select the material which gives minimum hysteresis curve when put to cycle of magnetization. From hysteresis curve, an idea of the magnetic properties like susceptibility, permeability, retentivity, coercivity of a ferromagnetic material can be made.

The choices of a magnetic material for the construction of a permanent magnets, electromagnets, cores of transformer and magnetic shielding can be decided from the hysteresis curve of the sample.

(i) Permanent magnets- The materials used for permanent magnets must have the following characteristics-

a) high retentivity so that the magnet may cause strong magnetic field

b) high coercivity so that the magnetization is not wiped out by strong external fields, mechanical ill-treatment and temperature changes. The loss due to hysteresis is immaterial because the magnet in this case is never put to cyclic changes.

According to these considerations, steel is better for permanent magnets than soft iron.

(ii) Electromagnets- The material used for cores of electromagnets must have-

a) maximum flux density with comparatively small magnetising field

b) high initial permeability

c) low hysteresis

d) low coercivity

e) high retentivity

Considering these facts, soft iron is an ideal material for electromagnets.

(iii) Transformer cores, Telephone diaphragms, Armature of dynamos and motors and cores of choke-In these cases, the material is subjected to cyclic changes of magnetization. Their material, therefore, must have the following characteristics-

a) low hysteresis loss

b) high initial permeability

c) high specific resistance

Therefore, soft iron is a good material for these purposes.

(iv) Magnetic shielding- The magnetic material used for magnetic shield must have high saturation induction and very low coercivity.

Example 1:A substance has magnetic susceptibility equal to 2. Calculate the relative permeability.

Solution: Given, $\chi_m = 2$

We know-

$$\mu_r = 1 + \chi_m = 1 + 2 = 3$$

Example 2: The relative permeability for a material is 3. What will be its magnetic susceptibility?

Solution: Given $\mu_r = 3$

We know that-

$\mu_r = 1 + \chi_m$

or $\chi_m = \mu_r - 1 = 3-1 = 2$

Self Assessment Question (SAQ) 1: Choose the correct option-

Diamagnetic substance when placed in a magnetic field is-

(a) weakly attracted (b) strongly attracted (c) repelled (d) none of these

Self Assessment Question (SAQ) 2: Choose the correct option-

The magnetic susceptibility of a diamagnetic material is-

(a) large and positive (b) large and negative (c) small and positive (d) small and negative

Self Assessment Question (SAQ) 3: Choose the correct option-

Which one represents the Curie's law-

(a) $\chi_m \, \alpha \, \frac{1}{T}$ (b) $\chi_m \, \alpha \, \frac{B}{T}$ (c) $\chi_m \, \alpha \, \frac{1}{T^2}$ (d) $\chi_m \, \alpha \, \frac{1}{T^3}$

11.8 SUMMARY

In the present unit, you have learnt about magnetic susceptibility, different types of magnetic materials and Curie's law. The magnetic susceptibility is defined as the ratio of the intensity of magnetization to the magnetic intensity of the magnetizing field. According to Curie's law, the magnetic susceptibility of a magnetic material is inversely proportional to Kelvin temperature. You have also study about hysteresis, energy loss due to hysteresis and the applications of hysteresis loop. In the unit, you have calculated the energy loss due to hysteresis and proved that the work done per unit volume of the material per cycle is equal to the area of μ_0 times the area of M-H loop or the area of B-H loop. To present the clear understanding of the unit, some solved examples are given in the unit. To check your progress, self assessment questions (SAQs) are also given in the unit.

11.9 GLOSSARY

Align- line up, bring into line, arrange in a line

Hysteresis- When a ferromagnetic substance is first magnetized by external applied field H and then demagnetized, then the flux density lags behind the field H. This is called hysteresis.

Magnetization- The magnetic state of any substance is described by a quantity

11.10 TERMINAL QUESTIONS

1. Define magnetic susceptibility.

2. Define relative permeability.

3. What is hysteresis? What does the area of hysteresis curve represent?

4. What do you mean by retentivity and coercivity? Explain.

5. How will you classify the substances on the basis of magnetic susceptibility?

6. Establish the relation between relative permeability and magnetic susceptibility.

7. Discuss the classification of substances on the basis of their magnetic behaviour.

8. Discuss the properties of dia, para and ferromagnetic materials.

9. Explain Curie's law.

10. What are the importances of hysteresis curve? Explain.

11. Steel is better for permanent magnet than soft iron. Why?

12. Why soft iron is an ideal material for electromagnet?

13. The relative permeability for a material is 6. What will be its magnetic susceptibility?

11.11 ANSWERS

Self Assessment Questions (SAQs):

1. (c) repelled
2. (d) small and negative
3. (a) $\chi_m \propto \frac{1}{T}$

Terminal Questions:

13. Given $\mu_r = 6$

We know that-

$\mu_r = 1 + \chi_m$

or $\chi_m = \mu_r - 1 = 6-1 = 5$

11.12 REFERENCES

1. Electricity and Magnetism, Sehgal, Chopra, Sehgal, Sultan Chand & Sons, New Delhi

2. Engineering Physics, Gupta and Kumar, Dhanpat Rai & Co.(P) Ltd, Delhi

3. Nootan Physics, Part-II, Kumar Mittal, Nageen Prakashan Pvt. Ltd., Meerut

4. Electric and Magnetism, Satya Prakash, Pragati Prakashan, Meerut

5. Physics Glossary, Devendra Kumar, Arihant Prakashan, Meerut

11.13 SUGGESTED READINGS

1. Modern Physics, Beiser, Tata McGraw Hill

2. Engineering Physics, S.K. Gupta, Krishna Prakashan Media (P) Ltd., Meerut

3. Electricity and magnetism with electronics, K.K. Tewari, S. Chand Publishing, New Delhi

4. University Physics, Francis W. Sears, Mark W. Zemansky, Hugh D. Young, Narosa Publishing House, New Delhi

5. Physics A Fundamental Glossary, Commission for Scientific and Technical Terminology, Department of Higher Education, Government of India.

UNIT 12 CURRENT DENSITY, EQUATION OFCONTINUITY,ELECTRIC CONDUCTIVITY, WIEDEMANN-FRENZ LAW

Structure

12.1 INTRODUCTION

In this unit we shall discuss the dynamics of charges. When the charges are stationary, an electrostatic field and static potential are developed in their vicinity. If the charges are placed in a region of non-uniform potential, they start to move and a current is set up. In conductors the electrons in the outermost orbits are relatively loosely bound to their respective atoms. When these conductors are placed in an electric field, a force starts to act on these free electrons. The direction of the force on positive charges is along the direction of the field and on negative charges is opposite to the field. The free charges start to move under the action of this force. The flow of free charges in a conductor constitutes electric current.

12.2OBJECTIVES

When you finish your study of this unit you should be able to

- define electric current, its units and the conditions necessary for flow of current.
- tell what are resistance, conductance, resistivity and conductivity. What are their symbols and what units are used to measure them? On what factors these quantities depend.
- describe the relationship between voltage (potential difference),current and resistancein a simple circuit(Ohm's Law).Physical significance of Ohm’s law and its vector form.
- define drift velocity and current density.
- state equation of continuity and its physical significance
- discuss the Wiedemann-Franz law and its drawbacks.
- solve numerical problems involving the values of voltage (potential difference), current, resistance, drift velocity, current density, resistivity, conductivity etc.

12.3 ELECTRIC CURRENT

In an electric circuit the charge is often carried by moving electrons. It can also be carried by ions in electrolyte. The rate at which charge flows past a point in a circuit is called the current. The current is a physical quantity that can be measured and expressed numerically. The current in a circuit at any instant can be measured by determining the quantity of charge passing per second through the cross-section of the wire at that instant.

If the rate of flow of charge is independent of time (i.e. steady) and q charge flows through the circuit in time t then current is given by

$$i = \frac{q}{t} \qquad \text{------------- (1)}$$

If the rate of flow of charge varies with time then the instantaneous current is given by

$$i = \frac{dq}{dt} \text{ ------------ } (2)$$

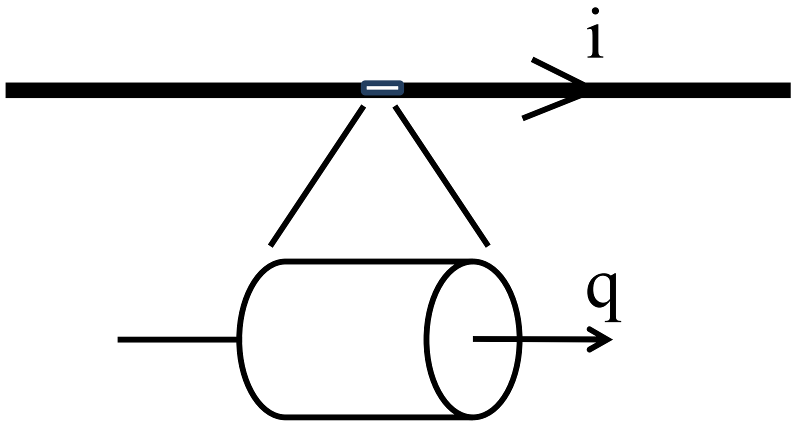


Figure 1: Charge q is passing through the cross-section of wire in time t, i.e., i = q/t

If the charge is measured in Coulomb and time in seconds then the unit of current is Ampere. Ampere is often shortened to *Amp(amp)* and is abbreviated by the unit symbol *A*. Thus a current of 1 ampere means that there is 1 Coulomb of charge passing through the cross-section of wire every 1 second.

i.e., 1 ampere = 1 Coulomb/1 second

Electric current is a scalar quantity as it does not follow the law of vector addition. The arrows used in the electric circuits represent the direction of flow of positive charge.

Example 1: 1.0 mm long cross section of wire is isolated and 10 C of charge is allowed to pass through it in 10 s.Determine the value of current through it.

Solution: The current is given by, $I = \frac{q}{t} = \frac{10\ C}{10\ s}$=0.5amp.

Example 2: In a discharge tube, the $1.1\text{x}10^{19}$ electrons move parallel to the tube while $3\text{x}10^{18}$ positive helium ions move in a direction opposite to that of electrons through the cross section per second. Find the magnitude and direction of the current.

Solution:The current due to positive ions will be along the same direction as that of the motion of these ions. The direction of current due to the motion of electrons will be opposite to the direction of motion of electrons. Thus in the given problem, the contribution of current from positive ions and the electrons will be along the same direction and hence will be added.

The charge on each electron, $q_1 = 1.6\text{x}10^{-19}$ C.

Number of electrons, $n_1 = 1.1 \text{x} 10^{19}$

The current due to electrons, $I_1 = \frac{n_1 q_1}{t_1}$

$$= \frac{1.1 \times 10^{19} \times 1.6 \times 10^{-19}}{1} = 1.76 \text{ A}$$

The charge on each helium ion, $q_2 = 1.6 \text{x} 10^{-19}$ C.

The number of helium ions, $n_2 = 3 \text{x} 10^{18}$.

The current due to helium ions, $I_2 = \frac{n_2 q_2}{t_2}$

$$= \frac{3 \times 10^{18} \times 1.6 \times 10^{-19}}{1} = 0.48 \text{ A}$$

Thus the total current through discharge tube,

$$I = I_1 + I_2 = 1.76 + 0.48 = 2.24 \text{ A}$$

The direction of the current will be along the direction of the movement of positive helium ions.

Self Assessment Question (SAQ) 1: A current of 1.6 amp exist in 10 ohm resistance for 1 minutes. How many electrons pass through any cross-section of the resistance in this time?

Self Assessment Question (SAQ) 2: Whether the current is a scalar quantity or vector quantity. Justify your answer.

Self Assessment Question (SAQ) 3: In the absence of any external field the free electrons in metallic conductor do not contribute anything towards current density. Justify.

12.4 DRIFT VELOCITY

You should have learnt by now that in a conductor the outermost orbit electrons in the atom are loosely bound and almost free to move from one place to another within the conductor because of the available thermal energy. In the absence of any field the motion of these free electrons is random just like the gas molecules in a vessel. They are, therefore, also called electron gas.

In a metal when free electrons leave their atoms, the metal is left with positive ions. These electrons, during their motion, collide again and again with the positive ions and continuously change their directions of motion. Thus their velocities (due to thermal agitation) are randomly distributed and consequently the average velocity is zero. In other words we can say that the net transport of charge in any particular direction is zero and therefore, the electric current on account of thermal motion of free electrons is zero.

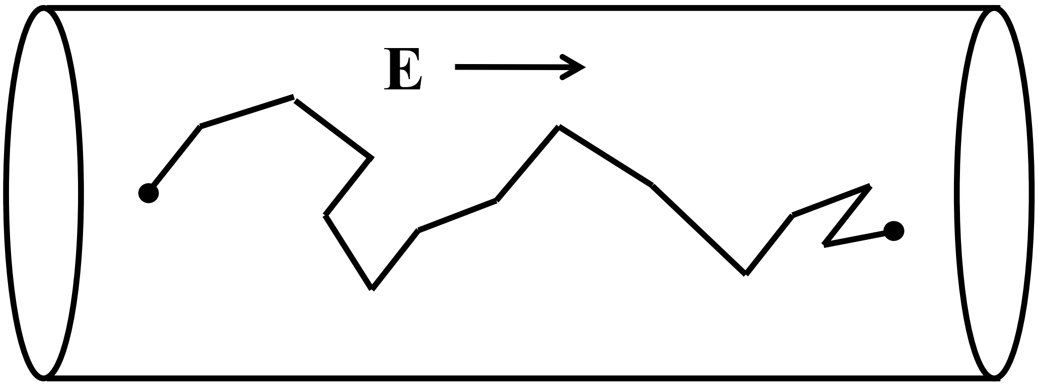


Figure 2: Drift motion of charge carriers in the direction of a field

Now if a potential difference is set upbetween the two ends of a conductor, say by connecting a battery in the circuit, then the free electrons experience a force of $-e\vec{E}$. Where e is the electronic charge and $\vec{E}$ is the electric field developed. Negative sign indicates that the force on electrons is in the direction opposite to that of field direction. Due to this force the free electrons (charge carriers) are accelerated and in the way interact with the other free electrons and positive ions present in the conductor. In each collision they loss their energy and again accelerate by the field present. So we can think of backward force acting on the electrons during their motion. This force is called collision drag.

The overall effect of applying potential difference between two ends of a conductor is that it gives a small constant velocity to charge carriers along the length of the conductor. This is known as drift velocity. Thus the average velocity with which the charge carriers move, under the action of electric field, is known as drift velocity. The drift velocity is usually represented as v_d.

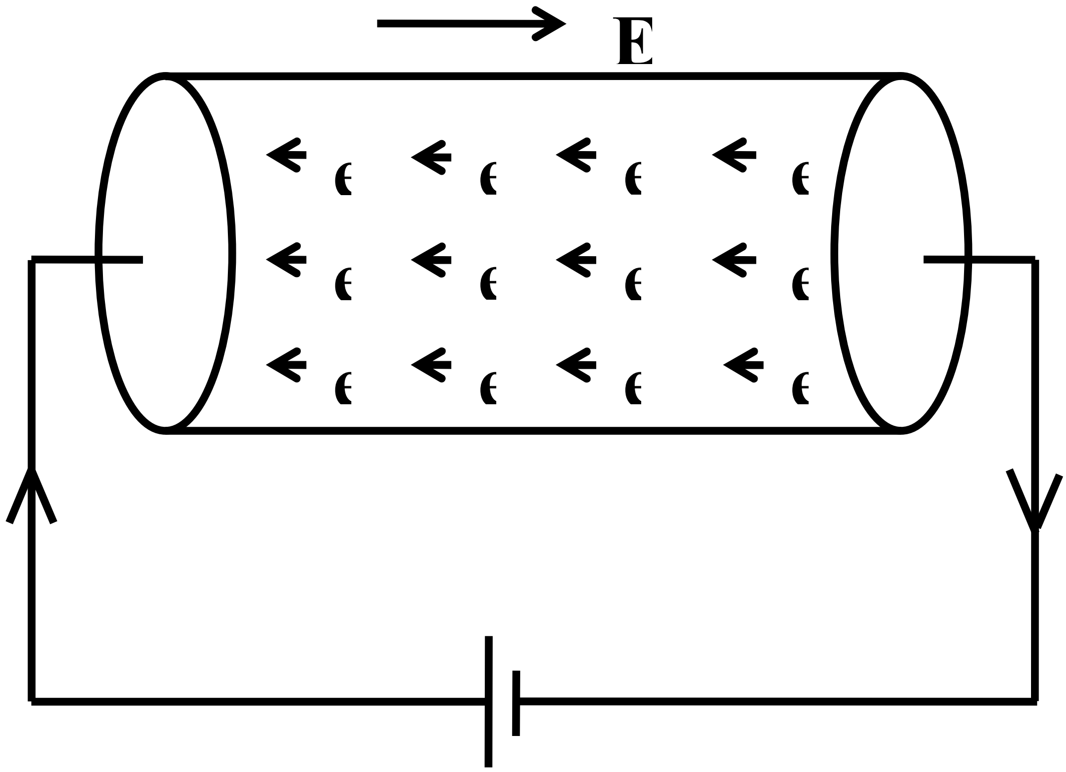


Figure 3: Motion of the charge carries (electrons) in a conductor on theapplication of electric field and the current in the circuit

12.5 CURRENT DENSITY

If a current is flowing in the conductor then the current per unit area of it, when the area is taken along a direction normal to the current, is known as current density. Let us consider a current flowing through a conductor of length l and uniform cross-sectional area A. suppose this current is due to the motion of the electrons only. These electrons will possess the average drift velocity v_d in a direction opposite to that of applied field. The value of v_d for one second, in fact, gives the distance travelled by the electrons in one second. Therefore, the volume of the cylinder around the path traversed by electron in one second is given by

$$dV = \overrightarrow{v_d}.\overrightarrow{ds} \text{------------ (3)}$$

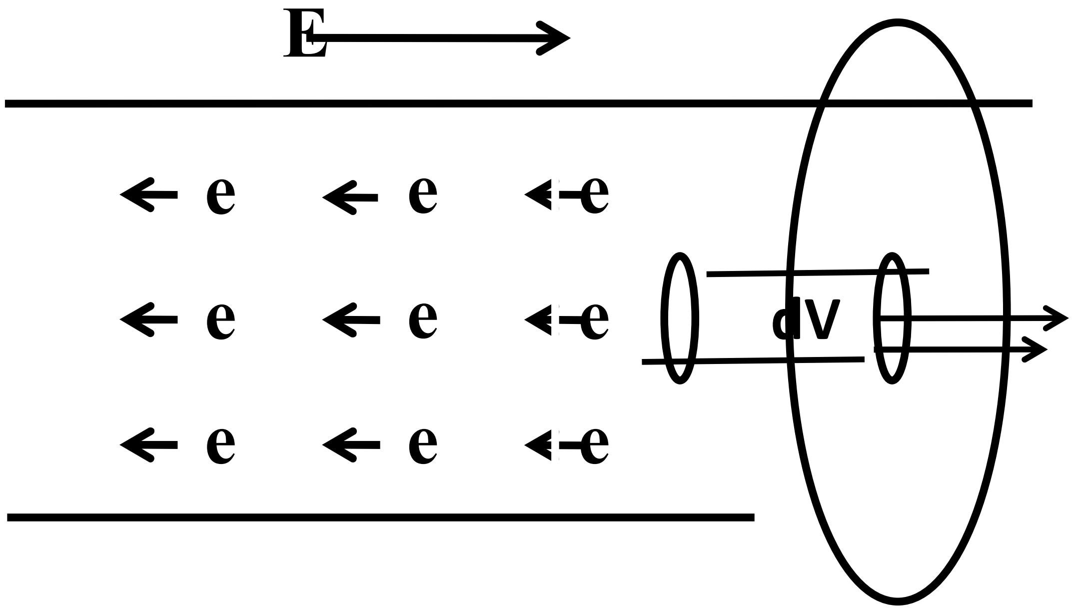


Figure 4: The small volume element of volume $\overrightarrow{v_d}$. $\overrightarrow{ds}$

If N is the number of charge carriers (electrons of charge e) per unit volume then the charge passing through the area $\overrightarrow{ds}$ in one second is

$$dq = N\,e\,(\overrightarrow{v_d}.\overrightarrow{ds}) \text{------------ (4)}$$

But charge passing per second is nothing but the current, hence

$$dI = N\,e\,(\overrightarrow{v_d}.\overrightarrow{ds}) \text{------------ (5)}$$

Here, the quantity $N\,e\ \overrightarrow{v_d}$ is a vector, called the current density. The direction of the current density at a point is that along which a positive charge carrier would move if placed at that point. The current density is represented by $\vec{J}$ and has the same direction as that of drift velocity.

i.e., $\vec{J} = N\,e\ \overrightarrow{v_d}$ ------------ (6)

From equations (5) and (6) we can write down

$dI = \vec{J}.\overrightarrow{ds}$------------ (7)

If we take a small element *dI*through a small area *ds* around a point and if *ds* is normal to *dI* then current density at that point is given by

$$\vec{J} = \frac{dI}{ds}\hat{n} \text{------------ (8)}$$

Where unit vector $\hat{n}$ represents the direction of current.

If $ds_\perp$ is the area element perpendicular to the current at a point then, current density may also be defined as

$$\vec{J} = \frac{dI}{ds_\perp} \text{------------ (9)}$$

From equations (5) and (7) we can write an expression for the total current through a total surface *S*, using surface integral, as

$$I = \int dI = \int_S \vec{J}.d\vec{S}$$

$$I = \int_S Ne(\vec{v}_d.d\vec{S}) \text{------------ (10)}$$

Here integral sign with S represent the integration over the entire closed surface taken into consideration (surface integral).

Example 3: An aluminium wire whose radius is 0.05 cm is welded end to end to a copper wire with a diameter of 0.068 cm. the composite wire carries a current of 5.0 ampere. What is the current density in each wire?

Solution: The cross-sectional area of aluminium wire is

$$A_1 = \pi r^2 = 3.14 \text{ x } (0.05)^2 = 0.00785 \text{ cm}^2$$

Therefore the current density in it is

$$J = \frac{I}{A} = \frac{5}{0.00785} = 636.94 \; amp/cm^2$$

The cross-sectional area of copper wire is

$$A_2 = \pi r^2 = 3.14 \text{ x } (0.034)^2$$

$= 0.00363 \text{ cm}^2$

Therefore the current density in it

$$J = \frac{I}{A} = \frac{5}{0.00363}$$

$$= 1377.41\ amp/cm^2$$

Example 4: A current of 1.0 ampere is flowing through a copper wire. If the area of cross-section of wire be 0.01 cm^2, find out the drift velocity of electrons in it. Assume one free electron per atom of copper and number of copper atoms in one cm^3 is 8.0 x 10^{22}.

Solution: The expression for drift velocity is

$$v_d = \frac{J}{n\,e} = \frac{I}{n\,e\,A} \qquad (\because J = i/A)$$

It is given that I = 1.0 ampere, n = 8.0 x 10^{22} cm^{-3} = 8.0 x 10^{28} $metre^{-3}$, e = 1.6 x 10^{-19} C, and A = 0.01 cm^2 = 1.0 x 10^{-6} $metre^2$

$$\therefore \qquad v_d = \frac{1.0}{(8.0 \times 10^{28}) x (1.6 \times 10^{-19}) \times (1.0 \times 10^{-6})}$$

$$= 7.81 \text{ x } 10^{-4} \text{ m/s.}$$

Example 5: In a region 40% of electrons have a drift velocity 2.0 cm/s along the direction of negative X-axis while the rest are moving with a drift velocity of 3.0 cm/s along positive direction of X-axis. Find the current density in the region assuming 5× 10^{22} electrons per c.c. present in the region.

Solution: We know that the direction of current is taken opposite to the motion of the electrons. Thus due to the electrons motion along negative X-axis the current will flow along positive X-axis and vice-versa.

∴ Current density along positive X-axis

$J_+ = N\,e\,v_d = \frac{40}{100} \times 5 \times 10^{22} \times 1.6 \times 10^{-19} \times 2.0$

$= 6.4 \times 10^3$ amp/cm^2

Similarly the current along negative X-axis

$J_- = N\,e\,v_d = \frac{60}{100} \times 5 \times 10^{22} \times 1.6 \times 10^{-19} \times 3.0$

$= 14.4 \times 10^3$ amp/cm^2

Thus the net current density will be along negative X-axis and its value will be

$J = J_- \sim J_+ = 14.4 \times 10^3 - 6.4 \times 10^3$

$= 8.0 \times 10^3$ amp/cm^2

Self Assessment Question (SAQ) 4: A copper wire of diameter 0.1626 cm is welded end to end with an aluminium wire of same radius. The composite wire carries a current of 20 amp. Calculate the current density in the wire and drift velocity of electrons in copper wire by assuming one free electron per atom in copper. The molecular weight of copper is 64, its density is 9.0 g/cc and the Avogadro number is 6 × 10^{23}.

Self Assessment Question (SAQ) 5: A current of 1.0 amp is flowing through an aluminium wire of cross-sectional area 10^{-6} m^2. There are 10^{22} free electrons per cm^3. The resistivity of aluminium is 1.6×10^{-3} ohm-m. Find the average speed of electrons in aluminium and the electric field within the wire.

Self Assessment Question (SAQ) 6. A copper wire of diameter 1.0 mm carries a charge of 90 coulombs in 75 minutes. It contains $5.8\text{x}10^{22}$ free electrons per cm^3. Find the current in the wire andthe drift velocity of electrons.

Self Assessment Question (SAQ) 7: A copper wire of cross-section 10^{-4} m^2 carrying a current of 1.5 amp. If each atom contributes one free electron, calculate the drift velocity of free electrons. The atomic weight and density of copper are 63 and 9 g/cc respectively.

12.6 EQUATION OF CONTINUITY

By now you have understood that how the drift velocity of charge carriers is responsible for current and how the current is related with current density. We can say that the amount of electric charge at any point can only change by the amount of electric current flowing into or out of that point.

The continuity equation in physics describes the transport of some quantity. This equation tells us that any physical quantity (like energy) can move by a continuous flow. It cannot be teleported from one place to another. The equation of continuity in electric field is a relation between volume charge density ρ and current density vector $\vec{J}$, this equation is given by

$$\vec{\nabla}.\vec{J} + \frac{\partial \rho}{\partial t} = 0 \text{------------ (11)}$$

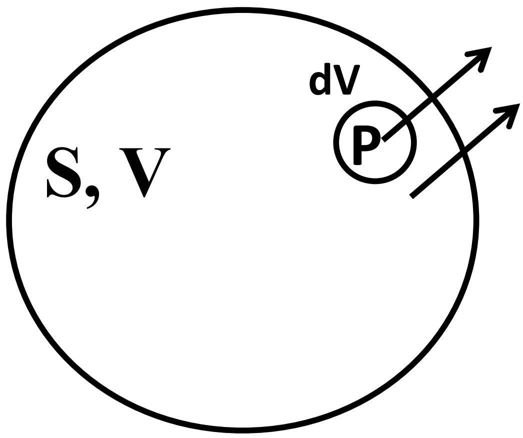


Figure 5: A closed surface S enclosing a volume V and small area element ds enclosing a volumedV. Vectors $\vec{J}$ and $\overrightarrow{\boldsymbol{ds}}$ represents the area vector and current density directions

In order to prove this equation, we consider a closed surface S enclosing a region of volume Vand take an area element $\overrightarrow{ds}$ at point P of this surface. If $\vec{J}$ is the current density vector at thatpoint then charge flowing out from area element $\overrightarrow{ds}$ per second (current) is given by equation (7) as

$dI = \vec{J}.\overrightarrow{ds}$------------ (12)

Thus the total charge crossing the surface S per second (current) is given by equation (10) as

$I = \int dI = \int_S \vec{J}.d\vec{S}$ ------------ (13)

If the current density ($\vec{J}$) remains unchanged with time everywhere then the current is said to be steady or stationary. Taking a case when current is not steady. The total charge enclosed by the closed surface, in terms of volume charge densityρ, is given by

$$q = \iiint \rho \, dV \qquad \text{------------ (14)}$$

Where triple integral sign represents the integral over the entire volume i.e., the volume integral. Thus time rate of decrease of charge within the surface is given by

$$-\frac{dq}{dt} = I = -\frac{\partial}{\partial t}\iiint \rho \, dV \qquad \text{------------ (15)}$$

From the conservation of charge we know that the total charge crossing the surface per second will be equal to the rate of decrease of charge in the volume enclosed by that surface. Thus from equations (13) and (15), we have

$$I = \int_S \vec{J}.d\vec{S} = -\frac{\partial}{\partial t}\iiint \rho dV = -\iiint \frac{\partial \rho}{\partial t} dV \qquad \text{------------ (16)}$$

From Gauss divergence theorem the surface integral of equation (16) may be converted into volume integral as

$$I = \int_S \vec{J}.d\vec{S} = \iiint (\vec{\nabla}.\vec{J})\, dV \qquad \text{------------ (17)}$$

Using equation (17) into (16), we obtain,

$$\iiint (\vec{\nabla}.\vec{J})\, dV = -\iiint \frac{\partial \rho}{\partial t} dV \qquad \text{------------ (18)}$$

or $$\vec{\nabla}.\vec{J} = -\frac{\partial \rho}{\partial t} \text{------------ (19)}$$

or $$\vec{\nabla}.\vec{J} + \frac{\partial \rho}{\partial t} = 0 \text{------------ (20)}$$

or $$div\vec{J} + \frac{\partial \rho}{\partial t} = 0 \text{------------ (21)}$$

This equation is known as equation of continuity and represents the conservation of charge. The first term $div\vec{J}$ in this equation represents the net outward flow of electric current per unit area from the closed surface S while the second term $\frac{\partial \rho}{\partial t}$ gives the rate of change of charge per unit volume.

12.7 RESISTIVITY AND CONDUCTIVITY

On the basis of his experimental observations George Simon Ohm (1787-1854) find out that the current passing through a conductor is directly proportional to the potential difference applied across its two ends, provided the physical conditions (temperature etc.) remain the same. This is known as Ohm's law. Thus if I is the current flowing through the conductor when potential difference between its two ends is V then from Ohm's law we have

$$I \propto V$$

or $$I = k\, V \text{------------ (22)}$$

Where the proportionality constant k is known as the conductance of the conductor. Alternately we can say that the potential difference developed across the ends of a conductor is proportional to the current flowing through it, i.e.,

$$V \propto I$$

$V = R\,I$------------ (23)

Where the constant of proportionality R is called the resistance of the conductor. From equations (22) and (23), we have,

$k = \frac{1}{R}$------------ (24)

Thus the conductance is reciprocal of resistance. The SI unit of resistance is 'ohm' and therefore that of conductance is $(\text{ohm})^{-1}$ or 'mho'. From equation (23) we can say that if on the application of 1 volt potential difference between the two ends of a linear conductor a current of 1 amp flows through it then its resistance will be 1 ohm. The conductors obeying Ohm's law are called Ohmic conductors or linear conductors and not obeying Ohm's law are called non-Ohmic conductors. The Ohmic conductors show a linear variation of I with V while non-Ohmic conductors show a non-linear variation.

At a given temperature the resistance of an Ohmic conductor is directly proportional to its length l and inversely proportional to its area of cross-section A, i.e.,

$R \propto \frac{l}{A}$------------ (25)

or $R = \rho \frac{l}{A}$------------ (26)

The constant of proportionality ρ is known as the specific resistance or resistivity of the material of the conductor. Thus resistivity

$\rho = R\frac{A}{l}$------------ (27)

The SI unit of resistivity is Ohm-metre. Resistivity is the property of the material. It is independent of the shape and size of the conductor but depends on the nature and temperature of the material. From equation (27) we can say that the resistivity of the material of conductor of unit length and unit area of cross-section is equal to its resistance. The reciprocal of resistivity (or specific resistance) is known as electrical conductivity (or specific conductance) and is represented by symbol σ. Thus

$\sigma = \frac{1}{\rho} = \frac{l}{R\,A}$------------ (28)

The SI unit of electrical conductivity is (Ohm-metre)$^{-1}$ or mho/metre.

Now for a linear conductor of homogeneous material (Ohmic medium) if I is the current flowing through it, J is current density, A is the area of cross-section and V is the potential difference applied across its ends, then electric field inside the conductor has magnitude

$E = \frac{V}{l}$------------ (29)

or $V = E\, l$------------ (30)

and current in terms of current density may be written as

$I = J\, A$------------ (31)

From equations (23), (30) and (31) we can write

$E\, l = R\, J\, A$ ------------ (32)

or $J = \frac{l}{R\, A}\, E$------------ (33)

Using equation (28) in (33), we get,

$J = \sigma\, E$ ------------ (34)

Thus for homogeneous material the current density *J* at any point is proportional to electric field strength *E* as long as the field is low. In vector notation it can be written as

$\vec{J} = \sigma\, \vec{E}$ ------------ (35)

This is known as vector form of Ohm's law. The electrical conductivity σ may also be defined as the ratio of current density *J* to electric field strength *E*, i.e.,

$\sigma = \frac{|\vec{J}|}{|\vec{E}|}$------------ (36)

The SI unit of σ is $\frac{ampere/metre^2}{volt/metre}$= (Ohm-m)$^{-1}$ or Siemens per metre (S-m^{-1}). Again since resistivity (ρ) is reciprocal to the conductivity (σ), it may also be defined as

$\rho = \frac{1}{\sigma} = \frac{|\vec{E}|}{|\vec{J}|}$ ------------ (37)

The resistivity of a good conductor increases with temperature. The conductivity is the reciprocal of resistivity, therefore, decreases with increasing temperature. At very low temperature it becomes very large and at temperatures near absolute zero, the conductors become superconducting.

Example 6: For a current of 1.0 ampere through a copper wire of length 10 m and diameter 0.08 mm, the resistance of the wire is 32.85 ohm. Calculate the resistivity of wire and potential difference between its two ends.

Solution: The resistivity of copper wire is given by

$$\rho = \frac{R\,A}{l} = \frac{R\,\pi\,r^2}{l}$$

Given that, $R = 32.85\ ohm$, Radius (r) = $\frac{0.08}{2}\,mm = 0.04 \times 10^{-3}$ m, l = 10 m

$$\therefore \qquad \rho = \frac{32.85\,ohm \times 3.14 \times (0.04 \times 10^{-3})^2 m^2}{10\,m}$$

$$= 1.65 \times 10^{-8}\,ohm - m$$

The potential difference between two ends of the wire,

$$V = R\,I = 32.85\ ohm \times 1\ amp = 32.85\ volts$$

Example 7: A metallic wire carries a current of 1 amp. Its area of cross section is 0.1 cm^2 and the resistivity of metal is 1.7×10^{-7} ohm-m. Calculate the electric field strength in the copper and the potential difference between two points 20 m apart along the length of this conductor.

Solution: The current density in terms of electric field strength is given by

$$J = \sigma E$$

or $$E = \frac{J}{\sigma} = \frac{I/A}{1/\rho} = \frac{\rho\,I}{A}$$

It is given that $\rho = 1.7\times 10^{-7}$ ohm-m, I = 1.0 amp, A = 0.1 cm^2 = 0.1×10^{-4} m^2 and l =20 m

$$\therefore \qquad E = \frac{1.7\times10^{-7}\times1.0}{0.1\times10^{-4}} = 1.7\times10^{-2}\ \text{volt/m}$$

The required potential difference

$V = E\,l = 1.7\times10^{-2}\times 20 = 0.34$ volt

Self Assessment Question (SAQ) 8: An aluminium wire of cross-section 1 cm^2 carries a current of 5.0 amp. What will be the value of electric field within the conductor. Also find the value of potential drop across its 2.0 km length. Resistivity of aluminium is 1.7×10^{-6} ohm-cm.

Self Assessment Question (SAQ) 9: A conductor of uniform cross-sectional area is 130 cm long. It has a voltage of 1.3 volt across its ends and a current density of 6.65×10^5 Am^{-2}. What is the conductivity of its material?

12.7.1 Conductivity- an atomic view

Due to the electric field $\vec{E}$ inside the conductor a force $q\vec{E}$ starts acting on charge carriers of charge q each. If m is the mass of charge carrier then its acceleration will be $q\vec{E}/m$. During their motion in the electric field, the charge carriers suffer collisions in the way and again accelerated. Just after a collision the velocity of charge carrier can be assumed to be zero. It then speed up with an acceleration $q\vec{E}/m$and therefore, from equation of motion, its velocity after time *t* is given by

$$\vec{v} = 0 + \frac{q\,\vec{E}}{m}\,t \text{------------ (38)}$$

If the charge carriers are electrons with negative charge then the average drift velocity is

$$\overrightarrow{v_d} = \frac{0+v}{2} = -\frac{1}{2}\frac{e\,\vec{E}}{m}\,\tau \text{------------ (39)}$$

Where τ is the relaxation time (the time between two successive collisions. The expression for current density due to flow of electrons is obtained from equation (6) by replacing charge *e* with charge of electron –*e* as

$$\vec{J} = -N\,e\,\overrightarrow{v_d} \text{------------ (40)}$$

Substituting the value of $\overrightarrow{v_d}$ from equation (39) into (40), we get,

$$\vec{J} = \frac{N\,e^2\,\tau}{2\,m}\vec{E} \text{------------ (41)}$$

Comparing equations (35) and (41), we can write

$$\sigma = \frac{N\,e^2\,\tau}{2\,m} \qquad \text{------------ (42)}$$

For a given sample, *N*, *e* and *m* are constant quantities independent of$\vec{E}$. If time is also constant then

$$\sigma = \frac{\vec{J}}{\vec{E}} = \frac{N\,e^2\,\tau}{2\,m} = \text{constant} \qquad \text{------------ (43)}$$

Since in a conductor there are large numbers of charge carriers, the total charge density is given by

$\vec{J} = \sum_i \frac{N_i {e_i}^2 \tau_i}{2\, m_i} \vec{E}$ ------------ (44)

Here N_i, e_i, τ_i and m_i represent the number density, charge, relaxation time and mass of one type (say i^{th}) of charge carrier.

12.7.2 Wiedemann-Franz law

The rate of heat transfer from one portion to another within the material depends upon the temperature gradient and thermal conductivity of the material. Thermal conductivity of metals is quite high. The metals which are good thermal conductors are generally good electrical conductors.

At a given temperature the electrical and thermal conductivities of a metal are proportional to each other. On increasing the temperature, the thermal conductivity increases while the electrical conductivity decreases. This behavior is depicted by Wiedemann-Franze law. This law is named after Gustav Wiedemann and Rudolph Franz, who in 1853 reported that the ratio of thermal conductivity (K) to electrical conductivity (σ) has almost the same value for different metals at the same temperature. The proportionality of $K/\sigma$with temperature was discovered by Ludvig Lorentz in 1872. Thus mathematical formof Wiedemann-Franz law can be written as

$\frac{K}{\sigma} \propto T$------------ (45)

or $\frac{K}{\sigma} = L\, T$ ------------ (46)

where K is thermal conductivity, σ is electrical conductivity and the constant of proportionality L is called the Lorentz number. The relationship between two conductivities is based on the fact that the heat and electrical transport both involve the free electrons in the metal.

On increasing the temperature, the average velocity of the carriers increases. This increases the forward transport of energy (thermal) and hence thermal conductivity increases. The electrical conductivity decreases with increasing temperature as the collisions divert the electrons from forward transport of charge. Thus the ratio of thermal conductivity to electrical conductivity varies with the square of average velocity. If we consider the electron gas model then the thermal conductivity of a Fermi gas is given by

$K = \frac{\pi^2}{3} . \frac{N\, k_B^2\, T}{m\, v_F^2} v_F . l = \frac{\pi^2\, N\, k_B^2\, T\, \tau}{3\, m}$------------ (47)

Where k_B is Boltzmann constant, v_F is the velocity at Fermi surface, N is electron concentration, l is mean free path (the path between two successive collisions) and τ is the relaxation time (the average time between two successive collisions) and therefore we have, $l = v_F . \tau$. Again we know that electrical conductivity is given by equation (42) as

$\sigma = \frac{N\,e^2\,\tau}{2\,m}$ ------------ (48)

Therefore Lorentz number

$L = \frac{K}{\sigma\,T} = \frac{2}{3}\pi^2(\frac{k_B}{e})^2$------------ (49)

The value of *L* calculated by this formula is in good agreement with the experimental results. Experiments performed for measurement of electrical and thermal conductivities show that the value of *L* is not exactly the same for all materials. In the book entitled 'Introduction To Solid State Physics' 5th edition, New York: Wiley 1976, p.178 by Charles Kittel, some values of *L* are given; ranging from $L = 2.23\text{x}10^{-8}$ for copper at 0^0 C to $3.2\text{x}10^{-8}$ W Ω K for tungsten at 100^0 C. Rosenberg noted that Wiedemann-Franz law is generally valid for high temperatures and low temperatures but may not hold true at intermediate temperatures.

Example 8: Considering electron gas model, calculate the average time between two successive collisions of an electron with positive ions in copper. The electron concentration is 10^{28} per m^3 and resistivity of copper is 1.7×10^{-7} ohm-m.

Solution: It is given that

$N = 10^{28}$ electrons per m^3, and $\rho = 1.7 \times 10^{-7}\Omega$-m

Thus conductivity, $\sigma = \frac{1}{\rho} = \frac{1}{1.7\times10^{-7}}$

$$= 5.88 \times 10^6(\Omega - \text{m})^{-1}$$

In electron gas model, the conductivity is given by

$$\sigma = \frac{N\,e^2\,\tau}{2\,m}$$

$$\therefore \tau = \frac{2\,m\,\sigma}{N\,e^2} = \frac{2 \times (9.1 \times 10^{-31}) \times 5.88 \times 10^6}{10^{28} \times (1.6 \times 10^{-19})^2}$$

$= 4.18 \times 10^{-14} s.$

Self Assessment Question (SAQ) 10: What will be the conductivity in a copper wire having electron concentration 10^{22} per cm^3. The relaxation time of electrons is 4.18×10^{-14} s. The charge and mass of electron are 1.6×10^{-19} C and 9.1×10^{-31} kg respectively.

12.8 SUMMARY

In this unit you have studied about the motion of the charge in an electric field and its consequences. To discuss the physics involved, various physical quantities like electric current, drift velocity, current density, resistivity, conductivity etc are explained and equation of conductivity and Wiedemann-Franz law are also described in detail. You have learnt that the free charge carriers are responsible for electric conduction in materials. In the absence of any field the free electrons move randomly due to available thermal energy. Thus the net current in any direction is zero. When an electric field is applied a force starts acting on them and electrons acquire a net velocity along a specified direction. Due to this drift of electrons a conduction current is set up. The current density of this current is given by$\vec{J} = N\,e\,\vec{v_d}$. Conductivity, the reciprocal of resistivity, is defined as $\sigma = \frac{1}{\rho} = \frac{l}{R\,A}$.The vector form of Newton's is given by the equation $\vec{J} = \sigma\,\vec{E}$ andthe resistivity in terms of electric field strength and current density is defined as$\rho = \frac{1}{\sigma} = \frac{|\vec{E}|}{|\vec{J}|}$. At atomic level the conductivity equation modifies to $\sigma = \frac{N\,e^2\,\tau}{2\,m}$. The conservation of charge in space is given equation of continuity $div\vec{J} + \frac{\partial\rho}{\partial t} = 0$. The Wiedemann-Franz law gives the proportionality relation between thermal conductivity and electrical conductivity as$\frac{K}{\sigma} = L\,T$.The understanding of the solved examples given in the unit provide reader an easy grasp of the subject and the reader can check his or her progress by going through self assessment questions.

12.9 GLOSSARY

Vicinity – nearness or closeness of space; In the vicinity – near.

Constitute – be the components or essence of, make up, form.

Respectively – in the order mentioned, for each separately or in turn.

Significance – importance, noteworthiness, a concealed or real meaning.

Drift – slow movement or variation.

Instantaneous – occurring or done in an instant or instantly (immediately).

Discharge – let go, release.

Assessment –estimate the size or quality or value etc., evaluation

Random – made, done, move etc., without method or conscious choice.

Interaction –the action of atomic and sub atomic particles on each other, reciprocal action or influence.

Drag – pull along (with effort or difficulty).

Traverse – travel or lie across.

Composite –made up of various parts.

Teleport – move at a supposedly by paranormal means.

Conservation –constancy of any quantity (preservation).

Homogeneous – consisting of parts all of the same kind, uniform.

Suffer – undergo, experience or be subjected to (pain, loss, grief, defeat, change etc.).

Successive – following one after another.

Gradient – the rate of rise or fall of temperature pressure etc.

Depict –to describe.

Vary – undergo change (become or be different)

12.10 TERMINAL QUESTIONS

1. Define electric current, drift velocity and current density. Is current density a vector quantity or scalar quantity?

2. A copper conductor of cross sectional area 10^{-4} m^2 carries a current of 200 amp. There are about 8.5×10^{28} free electrons per m^3 and the resistivity of copper is 1.72×10^{-8} ohm-m^{-1}. Find the drift velocity of free electrons, the average electric field strength and the potential difference between two points of the conductor 200 m apart.

3. The conductivity of sulphur is about 10^{-15} mho/m. Find the current density in sulphur when it is subjected to an electric field of 2000 volts/cm.

4. Derive the expression $J = N\ e\ v_d$ for current density.

5. What will be the current in a hydrogen discharge tube if in each second 4×10^{18} electrons and 10^{18} protons move in opposite direction through the cross section of the tube?

6. The conductivity of a copper wire is 3.5×10^{7} mho/m. It carries a current of uniform density 8×10^{5} A/m^2. Find the electric field in the wire and the potential difference per unit length of the wire.

7. Define specific resistance and electrical conductivity. Using Ohm's law derive relation $J = \sigma E$

8. Derive an expression for equation of continuity. What is its physical significance?

9. State Wiedemann-Franz law and write an expression showing the mutual dependence of electrical and thermal conductivities.

12.11 OBJECTIVE TYPE QUESTIONS

Q.1. The specific resistance of a wire depends upon

(a) its length (b) its cross-sectional area

(c) its dimensions (d) its material

Q.2. The resistance of a wire is doubled if

(a) its radius and length both are doubled (b) its radius is doubled and length is halved

(c) its radius is halved and length is doubled (d) its radius and length both are halved

Q.3. The resistance of a wire of uniform diameter *d* and length *l* is *R*. The resistance of another wire of same material but diameter *2d* and length *4l* will be

(a) *2R* (b) *R* (c) *R/2* (d) *R/4*

Q.4. Zero potential difference is applied across a metallic conductor. The mean velocity of free electrons at absolute temperature T is:

(a) proportional to T (b) proportional to $\sqrt{T}$

(c) zero (d) finite but independent of T

Q.5. The equation, J = σE is a form of

(a) Ohm's law (b) Ampere's law

(c) continuity equation (d) Maxwell's equation

Q.6. The continuity equation for steady current gives

(a) $\nabla.\vec{J} = 0$ (b) $\nabla.\vec{J} = -\frac{\partial \rho}{\partial t}$

(c) $\nabla \times \vec{J} = \rho$ (d) $\nabla.\vec{J} = \rho I$

12.12 ANSWERS

Self Assessment Questions (SAQ):

1: The charge passed through the cross-section of the resistance in 2 minutes is

$$q = I\,t = 1.6 \text{ amp} \times (1 \times 60) \text{ sec} = 96 \text{ C}$$

Thus the number of electrons passing through the cross section of resistance in 2 min

$$N = \frac{q}{e} = \frac{96}{1.6 \times 10^{-19}} = 6.0 \times 10^{20}$$

2: Refer Article 13.3.

3: Refer Article 13.4.

4: The area of cross-section of wire is

$$A = \pi r^2 = 3.14 \times \left(\frac{0.1626 \times 10^{-2}}{2}\right)^2$$

$$= 2.075 \times 10^{-6} m^2$$

Thus the current density in wire

$$J = \frac{I}{A} = \frac{20}{2.075 \times 10^{-6}}$$

$$= 9.636 \times 10^{6} A/m^2$$

The density (ρ) of copper is 9.0 g/cc, thus the mass of the copper per unit volume is 9.0 g. Since 64 g (molecular weight, M) of copper contains 6×10^{23} atoms (Avogadro number, N_A), the number of atoms per unit volume, N (number of atoms in 9.0 g) will be

$$N = \frac{\rho N_A}{M} = \frac{9.0 \times 6 \times 10^{23}}{64}$$

$$= 8.437 \times 10^{22} \frac{electrons}{cm^3}$$

$$= 8.437 \times 10^{28}\, electrons/m^3$$

The current density is given by $J = N\,q\,v_d$.Thus

$$v_d = \frac{J}{N\,q} = \frac{9.636 \times 10^{6}}{8.437 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$= 5.668 \times 10^{-4}\, m/s.$

5: The drift velocity

$$v_d = \frac{J}{n\,q} = \frac{I/A}{n\,q} = 6.25 \times 10^{-4}\, m/s$$

We also know that,

$$J = \sigma E$$

or $$E = \frac{J}{\sigma} = J\,\rho = 1.6 \times 10^{-2}\, V/m$$

6: (i) The current in the wire,

$I = \frac{total\ charge}{total\ time} = \frac{90\ C}{75 \times 60\ sec} = 0.02\ A$

The current density, $J = n\,e\,v_d$

or $\frac{I}{A} = n\,e\,v_d \quad (\because J = \frac{I}{A})$

Here $n = 5.8 \text{x} 10^{22}$ electrons/cm^3= $5.8 \text{x} 10^{28}$ m^3, e = $1.6 \text{x} 10^{-19}$C, r = $\frac{1.0 \text{x} 10^{-3}}{2}$ = $0.5 \text{x}\ 10^{-3}$ m

$\therefore A = \pi \times r^2 = 3.14 \text{x}\ (0.5 \text{x}\ 10^{-3})^2$

Thus $$v_d = \frac{I}{n\,e\,A} = \frac{0.02}{(5.8\times10^{28})(1.6\times10^{-19})\times3.14\times(0.5\times10^{-3})^2}$$

$= 2.746 \text{x} 10^{-5}$ m/s

7: Current density $J = \frac{I}{A} = 1.5 \times 10^4 \frac{C}{m^2 s}$

Number of atoms in 63 g = 6.02×10^{23} (Avogadro number)

Thus, the number of atoms in 9 g (number of atoms per cc),

$$N = \frac{6.02 \times 10^{23} \times 9}{63}$$

Drift velocity, $v_d = \frac{J}{N\,e} = 10.9 \times 10^{-5}$ ms^{-1}

8: Electric field

$$E = \frac{J}{\sigma} = J\,\rho = \frac{I}{A}\,\rho$$
$$= 5.0\ \times 1.7\ \times 10^{-6} = 8.5\ \times 10^{-6}\ volt/cm$$

Potential difference, $V = E\,l = 8.5 \times 10^{-6} \times 10^{5} = 0.85$ volt

9: Conductivity

$$\sigma = \frac{J}{E} = \frac{J}{V/l} = \frac{6.65 \times\ 10^{5}}{1.3/1.3}$$
$$= 6.65 \times 10^{5}\ (\text{ohm-m})^{-1}$$

10: The conductivity

$$\sigma = \frac{N\,e^2\,\tau}{2\,m} = 5.88\ \times\ 10^{6}(\Omega - \text{m})^{-1}$$

Terminal Questions:

2. 1.5×10^{-4} m/s, 0.0344 volt, 7. 08 volt

3. 2×10^{-10} A/m^2.

5. 0.88 A.

6. 2.287 volt/m, 2.287 volt

Objective Type Questions:

1. d. 2. d. 3. b. 4.c. 5. a. 6. a.

12.13 REFERENCES

1. Feynman, Richard P. (2005). The Feynman Lectures on Physics. 2 (2nd ed.). Addison-Wesley.

2. Griffiths, David J. (1998), Introduction to Electrodynamics (3rd ed.), Prentice Hall.

3. Arora, C.L. and Hemne, P.S. (2012), Physics For Degree Students, S. Chand and Company Ltd., New Delhi.

4. Pollack, Gerald L. and Stump, Daniel R. (2002), Electromagnetism*,* Addison Wesley.

5. Purcell, Edward Mills (1985), Electricity and Magnetism*,* McGraw-Hill.

6. Reitz, John R., Milford, Frederick J. and Christy, Robert W. (2008), Foundations of Electromagnetic Theory (4th ed.), Addison Wesley.

7. Mahajan A.S. and Rangwala Abbas A., Electricity and Magnetism (1986), Tata McGraw Hill.

12.14 SUGGESTED READINGS

1. Charles Kittel (1976), Introduction to Solid State Physics, 5th edition, Wiley Eastern Limited, New York.

2. Saslow, Wayne M. (2002), Electricity, Magnetism, and Light, Academic Press.

3. I.S. Grant, W.R. Phillips (2008), Electromagnetism (2nd ed.), Manchester Physics, John Wiley & Sons.

4. Maxwell, James Clerk (1873), A Treatise on Electricity and Magnetism, Dover.

5. Sadiku, Matthew N. O. (2006), Elements of Electromagnetics (4th ed.), Oxford University Press.

6. Jackson, J. D. (1999), Classical Electrodynamics (3rd ed.), Wiley.

7. Garg, Anupam (2012), Classical Electromagnetism in a Nutshell, Princeton.

UNIT 13 ALTERNATING CURRENT

Structure

13.1INTRODUCTION

Alternating current is produced by a voltage source whose terminals polarity keeps alternating with time.As a result of constantly reversing polarity of voltage source, the direction of current flow in the circuit also keeps reversing. It is obvious that an alternating voltage source will cause an alternating current in the circuit.In short alternating current is denoted by AC. On the other hand a voltage source, whose polarity remains constant with time, is called DC source and the current produced by this source is called direct current. AC voltage or AC current is sometimes called sinusoidal voltage and sinusoidal current.

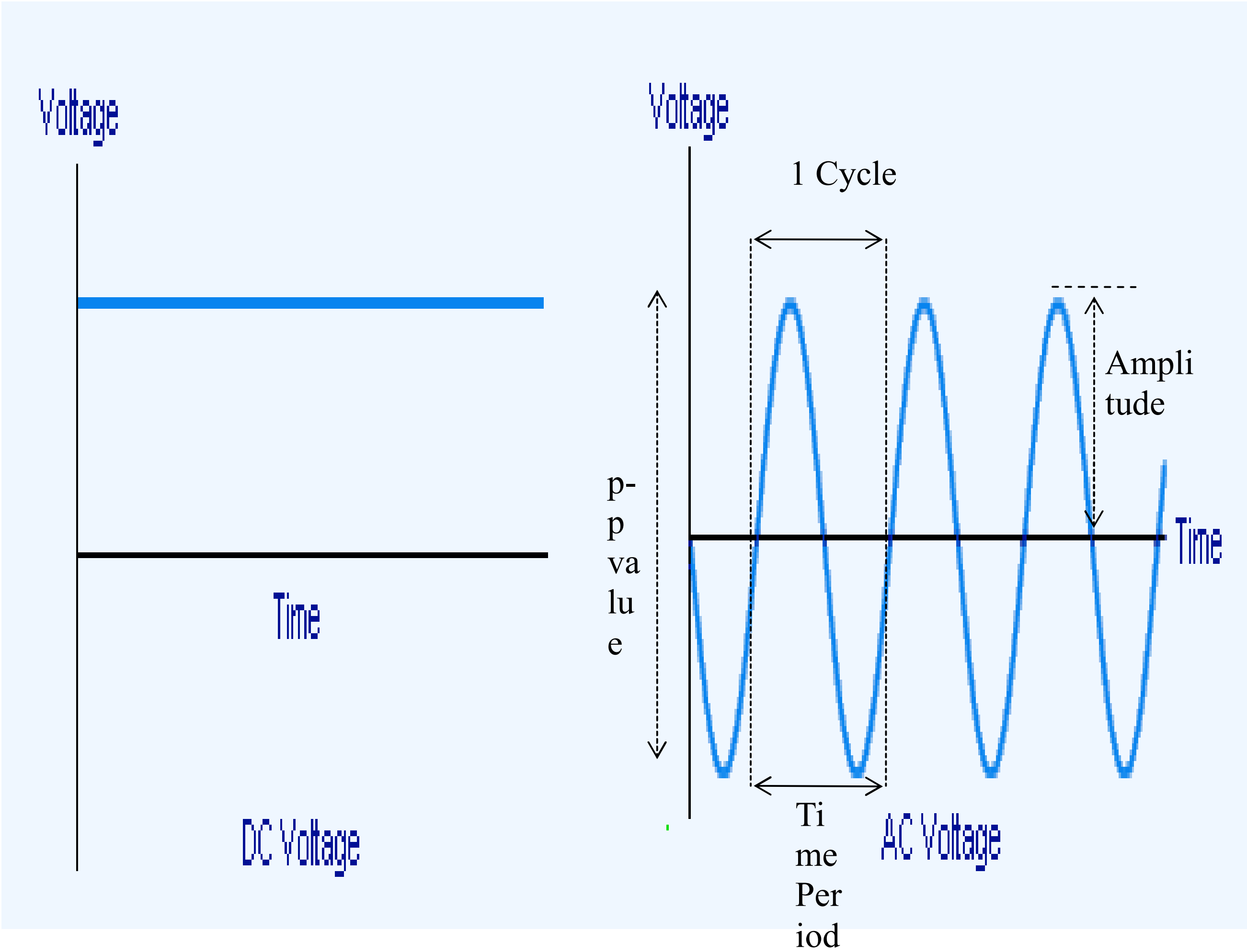


Figure 1

13.2 OBJECTIVE

The purpose of this chapter is to study the behavior of alternating current in different electronic components (resistor, inductor and capacitor). We shall discuss resistance produced by every component called resistance, inductive reactance and capacitive reactance respectively. We shall also discuss the net resistance produced by the combinations of these components called impedance.

13.3 WHAT IS ALTERNATING CURRENT?

Alternating current (AC) is an electric current which periodically reverses direction.

13.3.1 Cycle

One complete set of positive and negative values of an alternating quantity is known as a cycle. A cycle is sometimes specified in terms of angular measure, one complete cycle equal to 2π radians.

Time period-It is the time taken by alternating voltage or current to complete one cycle.

13.3.2Frequency

The number of cycles per second made by alternating voltage or current is called its frequency.
Instantaneous value**-** It is the value of current that exists at any instant of time measured from one reference point, mathematically it is given by $I = I_o sin\ \omega t$

13.3.3Peak value or maximum value

It is the highest valuereached by the current in one cycle. This peak value of current is also called amplitude of the current.
Peak to peak value**-** this is the positive peak and negative peak values usually written as p-p value.

13.3.4Root mean square value

It is also called the effective value. The value of alternating voltage which we read by an instrument is the r.m.s. value of voltage, actually peak value =√2×r.m.s. value.

13.3.5 Average value

It is the arithmetic average of all instantaneous values in one half cycle of the wave. For a sinusoidal wave

$$\text{value average} = 0.673 \times p - p\ value$$

13.4 AC CIRCUIT HAVING PURE RESISTANCE ONLY

When an alternating voltage is applied across a pure ohmic resistance it produces an alternating current through the resistance.

1. Which is in phase with the voltage

2. Whose r.m.s. value is given by $I = V/R$

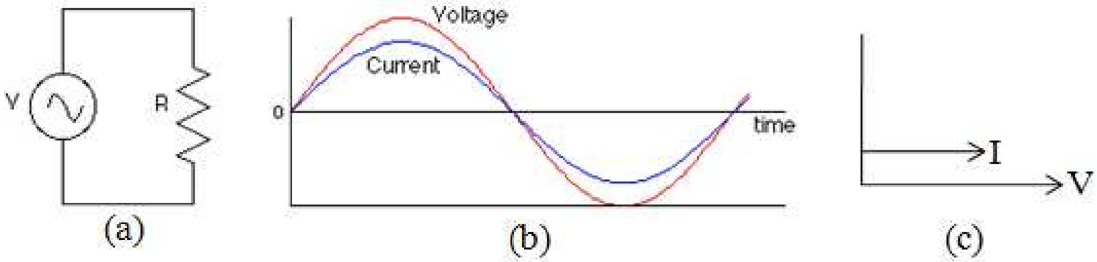


Figure 2

If the expression of applied voltage is

$$V = V_o sin\ \omega t \qquad \text{..........(1)}$$

then the equation of current is

$$I = I_o sin\ \omega t \qquad \text{..........(2)}$$

Comparing equation (1) and (2) it is obvious that in a pure resistor the current is always in the same phase as the applied voltage which is graphically represented in Figure 2(b). The power dissipated in the circuit in the form of heat is I^2R.

13.5 AC CIRCUIT HAVING PURE CAPACITANCE ONLY

When an alternating voltage $V = V_o sin\ \omega t$ is applied across a pure capacitor it produces an alternating current through the circuit whose magnitude is given by

$$I = \frac{V}{X_c}$$

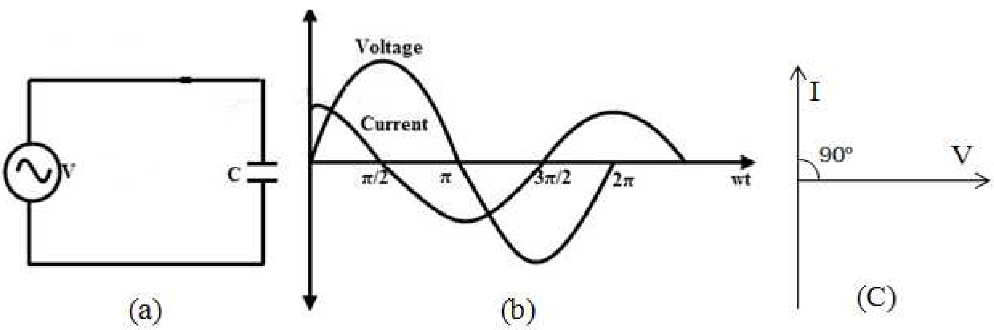


Figure 3

Where $X_c = \frac{1}{\omega C} = \frac{1}{2\,\pi f C}$ called *capacitive reactance*.

For direct current (DC)

$$f = 0, \text{ hence } X_c = \infty$$

The current through the circuit leads the applied voltage by 90°as shown in figure(3b) hence the equation of current is given by

$$I = I_o sin(\omega t + \pi/2) = I_o cos\ \omega t$$

Instantaneous power dissipation in this circuit is given by

$$P = VI \;\; = V_o\ sin\ \omega t\ I_o\ cos\ \omega t$$

$$= V_o I_o sin\ \omega t\ cos\ \omega t$$

Average power dissipated by this circuit is zero because

$$< \sin \omega t \cos \omega t > = \frac{1}{2} < sin2\omega t > = 0.$$

$$=> P = 0$$

13.6 AC THROUGH PURE INDUCTANCE ONLY

When an alternating voltage is applied across a pure inductive coil of inductance L it produces an alternating current through the circuit. As the current in the coil variescontinuously an opposite back voltage is set up in the coil whose magnitude is L $\frac{dI}{dt}$ where I is instantaneous current. The net instantaneous voltage is

$$\mathrm{V_o sin\ \omega t} - \mathrm{L}\,\frac{dI}{dt}$$

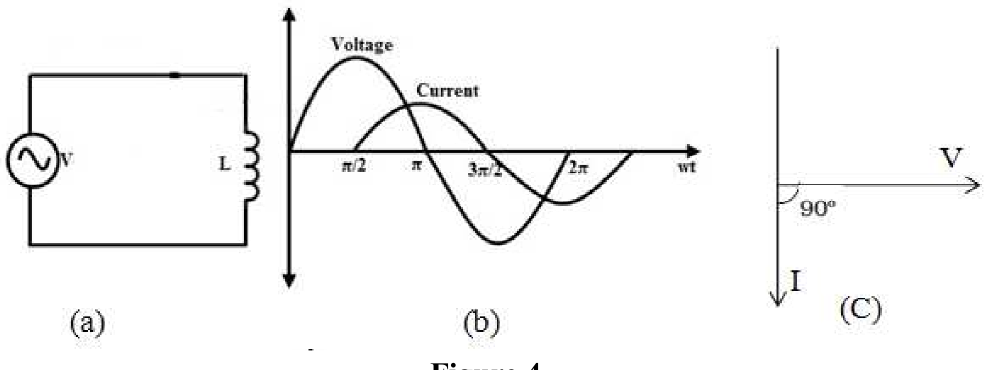


Figure 4

Since there is no resistance in the circuit, hence instantaneous voltage should be zero. Thus

$$\mathrm{V_o sin\ \omega t} - \mathrm{L}\,\frac{dI}{dt} = 0$$

$$V_o \sin \omega t = L\,\frac{dI}{dt} \qquad \text{……………(3)}$$

Solving above equation

$$I = -\frac{Vo}{\omega L}\cos \omega t$$

$$I = \frac{Vo}{\omega L}\sin\left(\omega t - \pi/2\right)$$

$$I = I_o\, sin\,\left(\omega t - \frac{\pi}{2}\right) \qquad \text{……………(4)}$$

where $I_o = \frac{Vo}{\omega L}$ is the maximum current

Comparing the above current equation with voltage equation it is clear that for a pure inductive circuit current lags behind the voltage by π/2 as shown in figure(4b).

Instantaneous power dissipation in this circuit is given by

$$P = VI \;\; = V_{\,o} sin\,\omega t\; I_o\, cos\,\omega t$$

$$P = V_o I_o sin\,\omega t\; cos\,\omega t \qquad \text{…….(5)}$$

Average power dissipated by this circuit is zero because

$$\sin \omega t \cos \omega t = \frac{1}{2}\sin 2\omega t$$

$$< \sin 2\omega t > = 0$$

$$=> P = 0$$

In the expression $I_o = \frac{Vo}{\omega L}$, ωL has the dimension of resistance and it is called inductive reactance and denoted by X_L

thus $$X_L = \omega L = 2\pi f L,$$

when f is in Hertz and L is in Henry then X_L is in ohm.

13.7 AC THROUGH L-R CIRCUIT

When an alternating voltage is applied across a pure inductive coil of inductance L, in series with a resistor R, it produces an alternating current through the circuit. The potential difference arises across L and R is V_L and V_R respectively.

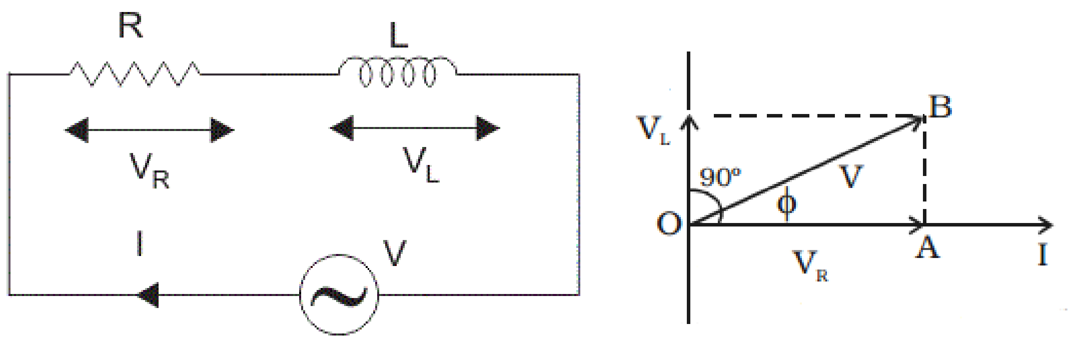


Figure 5

$$V_o \sin \omega t = RI + L\frac{dI}{dt} \qquad ....(6)$$

Suppose the solution of the above equation is

$$I = I_o \sin(\omega t - \phi) \qquad ....(7)$$

where I_o is the peak value of the current and ϕ is the phase angle to be determined. Differentiating equation (7) w.r.t. time, we get

$$dI/dt = I_o\, \omega \cos(\omega t - \phi)$$

substituting this diff. coff. in above equation (6)

$$RI_o \sin(\omega t - \phi) + LI_o\, \omega \cos(\omega t - \phi) = V_o \sin \omega t$$

$$= V_o \sin\{(\omega t - \phi) + \phi\}$$

$$RI_o \sin(\omega t - \phi) + LI_o\, \omega \cos(\omega t - \phi) = V_o\{\sin(\omega t - \phi)\cos\phi - \cos(\omega t - \phi)\sin\phi\}$$

comparing coefficients both side

$$RI_o = V_o \cos\phi \qquad ....(8)$$

$$LI_o\, \omega = V_o \sin\phi ....(9)$$

squaring and adding equ. (8) & (9)

$$(L^2\omega^2 + R^2)I_o^2 = V_o^2$$

$$I_o = \frac{V_o}{\sqrt{(L^2\omega^2 + R^2)}} \qquad ....(10)$$

dividing equ. (9) by (8)

$$\tan \phi = \frac{\omega L}{R} \qquad ....(11)$$

substituting the value of Io in equation (7)

$$I = \frac{V_o}{\sqrt{(L^2\omega^2 + R^2)}} \sin(\omega t - \phi) \qquad ....(12)$$

this is the instantaneous value of current in the circuit and $\frac{V_o}{\sqrt{(L^2\omega^2 + R^2)}}$ is its amplitude. The impedance of the circuit is defined as

$$Z = \frac{V_o}{I_o} = \sqrt{(L^2\omega^2 + R^2)} \qquad ....(13)$$

13.8 AC THROUGH R-C CIRCUIT

When an alternating voltage is applied across a capacitor of capacitance C in series with a resistor R, it produces an alternating current through the circuit. The potential difference arises across C and R is V_C and V_R respectively.

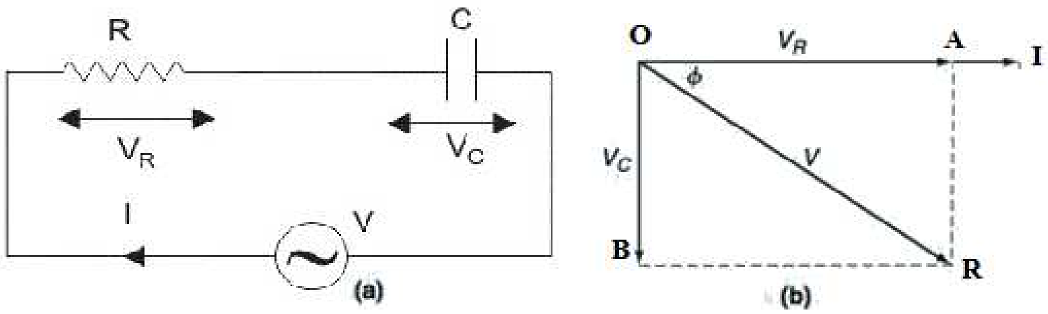


Figure 6

Let q be the charge on the capacitor at any instant and I the current in circuit at that instant. The potential difference across the capacitor at this instant is q/C. The effective potential difference in the circuit is

$$V_o sin\ \omega t - \frac{q}{C}$$

which must be equal to RI.

$$\text{or} \quad RI + q/C = V_o sin\ \omega t$$

differentiating w.r.t. t, we get

$$R\frac{dI}{dt} + \frac{1}{C}\frac{dq}{dt} = V_o\omega \cos \omega t$$

$$but \frac{dq}{dt} = I$$

$$\text{hence} R \frac{dI}{dt} + \frac{I}{C} = V_o \omega \cos \omega t \quad \ldots\ldots\ldots\ldots..(14)$$

The current in circuit varies harmonically with same frequency as the applied alternative potential difference but differing in amplitude and phase. Hence we may assume the solution of equ.(14) as

$$I = I_o sin\,(\omega t - \phi) \ldots\ldots\ldots(15)$$

differentiating equ. (15) w.r.t. t

$$\frac{dI}{dt} = I_o\, \omega \cos\,(\omega t - \phi)$$

substituting the value of I and dI/dt in equ (14)

$$R\, I_o\, \omega \cos\,(\omega t\text{-}\phi) + \frac{Io}{C} \sin\,(\omega t - \phi) = V_o \omega \cos \omega t$$

$$= V_o \omega \cos\,[(\omega t - \phi) + \phi]$$

$$= V_o \omega\,[\cos\,(\omega t - \phi) \cos\phi - \sin\,(\omega t - \phi) \sin\phi]$$

comparing the coefficients of *sin* and *cos* functions both side

$$I_o R \omega = V_o \omega \cos \phi \ldots\ldots\ldots\ldots(16)$$

$$\frac{I_o}{C} = -V_o \omega \sin \phi \quad \ldots\ldots\ldots\ldots(17)$$

squaring and adding equ (16) and (17)

$$I_o^{\,2} \left(R^2 \omega^2 + \frac{1}{C^2}\right) = V_o^{\,2} \omega^2$$

$$I_o^{\,2} \left(R^2 + \frac{1}{\omega^2 C^2}\right) = V_o^{\,2}$$

$$I_o = \frac{V_o}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \quad \ldots\ldots\ldots\ldots(18)$$

dividing equ.(iv) by equ.(iii)

$$\tan\phi = -\frac{1}{R\omega C}$$

substituting the value of I_o in equation (15)

$$I = \frac{V_o}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}\sin(\omega t - \phi)$$

where $\phi = \tan^{-1}\left(-\frac{1}{R\omega C}\right)$, which indicates that phase angle ϕ is negative. Hence the proper way of writing the expression for current is

$$I = \frac{V_o}{\sqrt{R^2+\frac{1}{\omega^2 C^2}}}\sin(\omega t + \phi)\text{............(19)}$$

This is the equation of current in circuit at any instant. Where$I_o = \frac{V_o}{\sqrt{R^2+\frac{1}{\omega^2 C^2}}}$ is the amplitude of the current, $\frac{1}{\omega C}$ is capacitive reactance of capacitor and denoted by X_C. The quantity $\sqrt{R^2 + \frac{1}{\omega^2 C^2}}$ is impedance 'Z' of the circuit. Thus $Z = \sqrt{R^2 + X_C^2}$. Thus it is clear from equation (19) that the current leads the applied potential (emf) by an angle $\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$

Since a pure capacitor consumes no power, the entire power consumption is due to resistor only

$$P = I^2 R = VI\ \cos\phi$$

Vector diagram:

Let Vc and V_R are the magnitude of potential differences across C and R respectively. Then we have

$$V_C = IX_C\ and\ V_R = IR$$

Since V_R is in phase with current while V_C is lags behind I by an angle π/2. These quantities can be represented by a Vector diagram as shown in figure 6(b).

$$V^2 = V_R^2 + V_C^2$$

$$but \quad V = IZ$$

thus

$$I^2Z^2 = I^2R^2 + I^2X_C^2$$

$$Z^2 = R^2 + X_C^2$$

$$Z = \sqrt{R^2 + X_C^2}$$

$$\tan\phi = \frac{V_C}{V_R} = \frac{1}{\omega CR}$$

where ϕ is the angle by which current in the circuit leads the applied potential difference.

13.9 AC THROUGH L-C CIRCUIT

when an alternating voltage, $V = V_o \, sin\, \omega t$, is applied across a series LC circuit then the potential difference across the capacitor at an instant is q/C and potential difference across inductor is $L(dI/dt)$ both are opposite in phase. Since there is no resistance in the circuit hence the effective potential in the circuit is

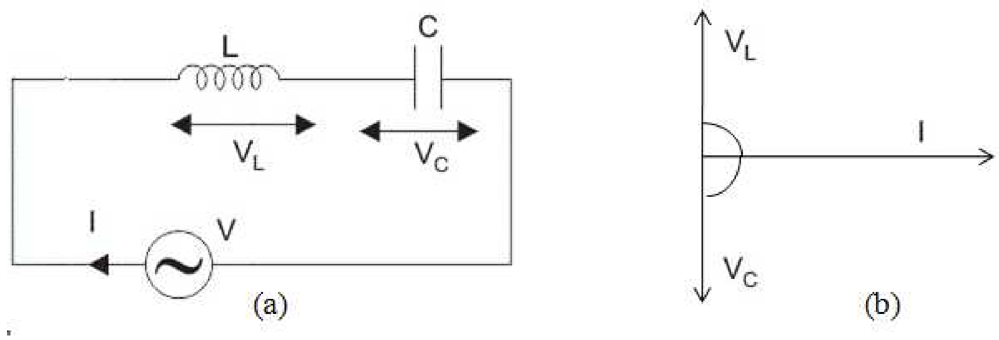


Figure 7

$$V_o \sin\omega t = \frac{q}{C} + L\frac{dI}{dt}$$

differentiating w.r.t. t, we get

$$L\frac{d^2I}{dt^2} + \frac{1}{C}\frac{dq}{dt} = V_o\, \omega\cos\omega t$$

but $\frac{dq}{dt} = I$

hence $L\frac{d^2I}{dt^2}+\frac{I}{C}=V_o\,\omega\cos\omega t$……….(20)

let the solution of above equation in the form

$I=I_o\sin(\omega t-\phi)$………(21)

where I_o and ϕ are constants to be determined. Differentiating equation (21) twice we get

$$\frac{d^2I}{dt^2}=-I_o\,\omega^2\sin(\omega t-\phi)$$

putting this value in equation (20)

$$-LI_o\,\omega^2\sin(\omega t-\phi)+\frac{I_o}{C}\sin(\omega t-\phi)=V_o\,\omega\cos\omega t$$
$$=V_o\,\omega\cos[(\omega t-\phi)+\phi]$$
$$=V_o\,\omega[\cos(\omega t-\phi)\cos\phi-\sin(\omega t-\phi)\sin\phi]$$

This equation should be true for all values of t. hence the coefficients of $\sin(\omega t-\phi)$ and $\cos(\omega t-\phi)$ functions of both sides must be equal

$-LI_o\omega^2+\frac{I_o}{C}=-V_o\,\omega\sin\phi$…………(22)

$0=V_o\,\omega\cos\phi$ …...…….(23)

squaring and adding equ.(22) and (23)

$$\left(-LI_o\omega^2+\frac{I_o}{C}\right)^2=(V_o\omega)^2$$
$$LI_o\omega-\frac{I_o}{C\omega}=V_o$$

$I_o=\frac{V_o}{L\omega-\frac{1}{C\omega}}$ …………(24)

dividing equ (22) by equ (23)

$$\tan \phi = \infty$$

$$\text{or } \phi = \frac{\pi}{2} \qquad \text{...........(25)}$$

substituting the value of $I_o and\ \phi$ in equation (ii) we get

$$I = \left(\frac{V_o}{L\omega \quad - \frac{1}{C\omega}}\right) \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$I = \left(\frac{V_o}{X_L - X_C}\right) \sin\left(\omega t - \frac{\pi}{2}\right) \qquad \text{...........(26)}$$

where X_L is inductive reactance and X_C capacitive reactance.

This is the equation of instantaneous current in circuit which is exactly lagging behind by a factor $\frac{\pi}{2}$ with applied alternating potential or emf, and the current I will be infinite when $X_L = X_C$

$$\text{or} L\omega \quad - \frac{1}{C\omega} = 0$$

$$\omega^2 = \frac{1}{\text{LC}}$$

$$\omega = \frac{1}{\sqrt{\text{LC}}}$$

$$2\pi\text{f} = \frac{1}{\sqrt{\text{LC}}}$$

$$\text{f} = \frac{1}{2\pi\sqrt{\text{LC}}} \qquad \text{..........(27)}$$

This expression represents the natural frequency of the circuit. Hence the amplitude of the current in the circuit is infinite (maximum) when frequency of the applied alternative potential (emf) is exactly equal to the natural frequency of the circuit. This condition is called resonance.

13.10 AC THROUGHL-C-R CIRCUIT

When an alternating voltage, $V = V_o\ sin\ \omega t$, is applied across a series LCR circuit then the potential difference across the capacitor is q/C and potential difference across inductor is

$L(dI/dt)$ both are opposite to the applied voltage. The effective voltage at that instant is therefore

$$V_o \sin \omega t \; - \; q/C \; - \; LdI/dt$$

which must be equal to IR

$$V_o \sin \omega t \; - \; q/C \; - \; LdI/dt \; = \; IR$$

$$\text{or } V_o \sin \omega t \; = \; IR + \; q/C \; + \; LdI/dt$$

$$LdI/dt + IR + \; q/C \;\; = V_o \sin \omega t$$

differentiating w.r.t. to t

$$\mathrm{Ld^2I/dt^2 + RdI/dt} + \frac{1}{C}\mathrm{dq/dt} \; = \mathrm{V_o \; \omega cos \; \omega t}$$

but $dq/dt \; = \; I$

$$\frac{\mathrm{Ld^2I}}{\mathrm{dt^2}} + \mathrm{RdI/dt} + \frac{I}{C} = \mathrm{V_o \omega cos \; \omega t} \quad \text{……….(28)}$$

In the steady state the current alternates with same frequency as the applied voltage but may differ in amplitude and phase. The solution of the above equation in steady state will be in the form

$$I = Io \sin (\omega t - \phi) \quad \text{………(29)}$$

$$\frac{dI}{dt} = I_o \omega \cos (\omega t - \phi)$$

$$d^2I/dt^2 \; = \; -I_o \omega^2 \sin (\omega t - \phi)$$

$$-LI_o\omega^2 \sin (\omega t - \phi) + RI_o \omega \cos (\omega t - \phi) + I_o/C \sin (\omega t - \phi) = \; V_o \; \omega \cos \omega t$$

$$\left(-L\omega^2 + \frac{1}{C}\right) I_o \sin (\omega t - \phi) + RI_o \omega \cos (\omega t - \phi) = \; V_o \; \omega \cos[(\omega t - \phi) + \phi]$$

$$= V_o \; \omega[\cos(\omega t - \phi)\cos\phi - \sin(\omega t - \phi)\sin\phi]$$

comparing the coefficients of $\sin (\omega t - \phi)$ and $\cos (\omega t - \phi)$ both side

$$(-L\omega^2 + 1/C)I_o = \; -V_o \; \omega \; \sin\phi \quad \text{……….(30)}$$

and $RI_o\omega = V_o \; \omega \; \cos\phi$ ………..(31)

dividing equation (30) and (31)

$$\tan\phi = \frac{L\omega^2 - 1/C}{R\,\omega}$$

$$= \frac{L\omega - 1/\omega C}{R} \quad \left(\frac{\text{reactance}}{\text{resistance}}\right)$$

squaring and adding (31) and (31)

$$[(-L\omega^2 + 1/C)^2 + R^2\omega^2]\, I_o^2 = V_o^2\omega^2$$

or $I_o = \frac{V_o}{\sqrt{(1/\omega C - L\omega)^2 + R^2}}$

substituting this value in equation (29)

$$I = \frac{V_o}{\sqrt{(1/\omega C - L\omega)^2 + R^2}} \sin(\omega t - \phi) \text{………….(32)}$$

$$\text{where } \phi = \tan^{-1}\left(\frac{L\omega - 1/\omega C}{R}\right)$$

This is the value of current at any instant. The amplitude of current is

$$I_o = \frac{V_o}{\sqrt{(1/\omega C - L\omega)^2 + R^2}}$$

$L\omega$ is inductive reactance and denoted by X_L, $1/\omega C$ is the resistance produced in the circuit due to capacitor called capacitive reactance and denoted by X_C, hence the quantity

$$\sqrt{\left(\frac{1}{\omega C} - L\omega\right)^2 + R^2} = \sqrt{(X_L - X_C)^2 + R^2}$$

is called impedance of the circuit and represented by Z. The quantity $(X_L - X_C)$ is called the resultant reactance of the circuit which is the difference between inductive reactance and capacitive reactance. Thus

$$Z = \sqrt{(X_L \sim X_C)^2 + R^2}$$

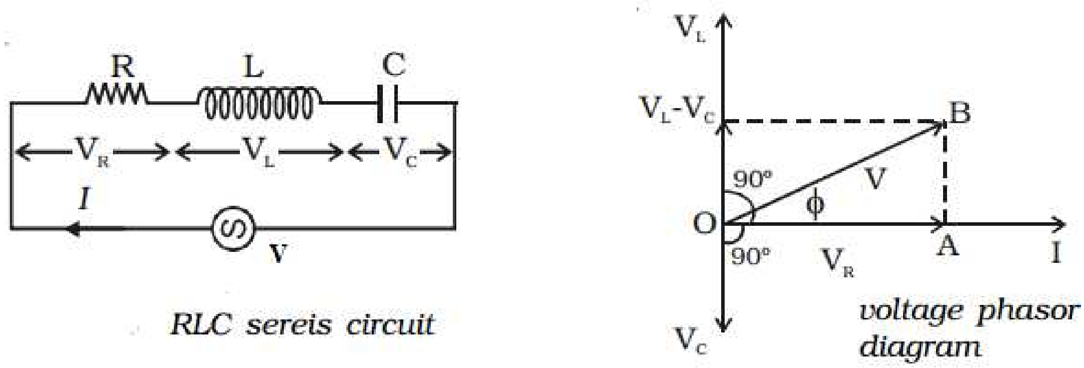


Figure 8

it is clear from equ.(32) that current lags in phase from applied voltage by an angle

$$\phi = \tan^{-1}\left(\frac{L\omega - 1/\omega C}{R}\right)$$

$$\text{or } \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

depending on the values of X_L and X_C, following three cases are arises

1. When $X_L > X_C$, ϕ is positive so the net current in the circuit lags behind the applied voltage.
2. When $X_L < X_C$, ϕ is negative so the net current leads the applied voltage.
3. When $X_L = X_C$, ϕ=0, and the current in the circuit is in phase with applied voltage.

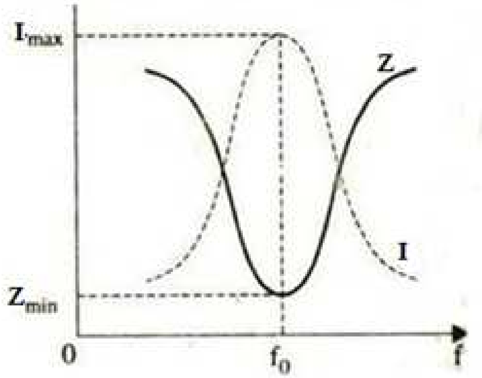


Fig. 9

The last condition when $X_L = X_C$, the impedance of the circuit is becomes minimum Z= R, is called electrical resonance. Hence amplitude of the current is maximum. Thus at resonance X_L=X_C

or $\omega L = \frac{1}{\omega C}$

$$\text{or } \omega = \frac{1}{\sqrt{(LC)}}$$

if f is the frequency of current in the circuit, then ω=2πf

$$2\pi f = \frac{1}{\sqrt{(LC)}}$$

$$f = \frac{1}{2\pi\sqrt{(LC)}} = f_0 \qquad \text{…………..(33)}$$

where f_0 is resonance frequency when reactance of the circuit is zero

13.10.1 LCR series resonant circuit

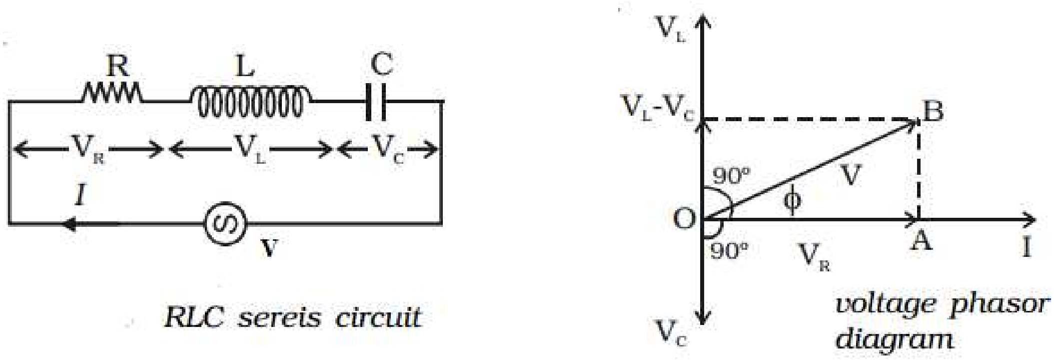


Figure 10

A series LCR circuit has high inductive reactance at high frequency and high capacitive reactance at low frequency. In both cases impedance of the circuit is very high. At some particular frequency impedance becomes minimum i.e. $Z = R$. If the values of R, L and C remains constant and frequency of applied voltage varies continuously from zero the current varies as shown in figure.

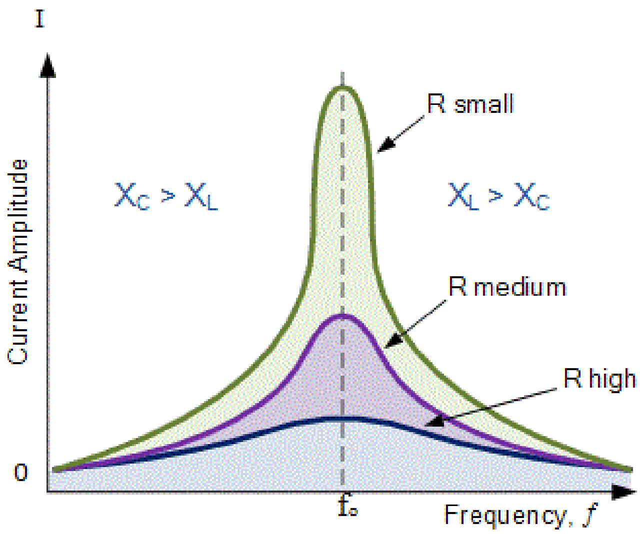


Figure 11

Initially current flows very slowly and increases to a maximum when the frequency increases to the resonance frequency then falls again. At resonance condition current in the circuit depends only on the value of R. In the figure three curves are plotted, for the values of resistance small, medium and large.The resonant current in the circuit is larger for the smaller values of resistance. The resonance is sharper for small value of resistance than for large resistance, however resonant frequency remains unchanged.

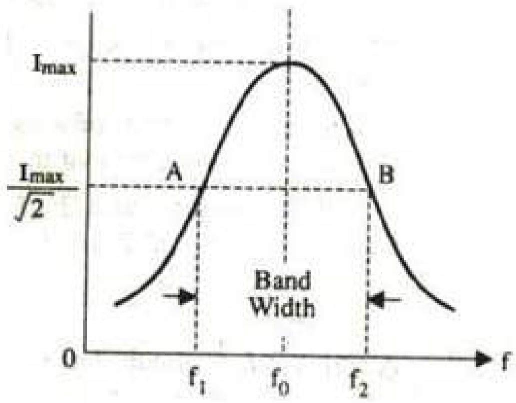


Fig. 12

This circuit is often called acceptor circuit because the impedance of the circuit is minimum at resonance so that it most readily accepts that current out of many currents whose frequency is equal to the natural frequency of the circuit.

From figure we have seen that at resonant frequency the amplitude of oscillating system becomes maximum. If the frequency of applied voltage is increased or decreased the amplitude falls from maximum value. The term sharpness of resonance refers to the rate of fall of amplitude with the change in the applied alternating source on either side of the resonant frequency. Sharpness of resonance is defined by Q factor, which is related to how quickly the energy of the oscillating system decays.

$$sharpness\ of\ resonance = \frac{f_2 - f_1}{f_o}$$

13.10.2 Parallel resonant circuit

When an alternating voltage is applied to a circuit having inductance L and resistance R in parallel with a capacitance C as shown in figure11. The peak value of the current in lower branch of the circuit is given by

$$i_1 = \frac{V_o}{\sqrt{(R^2+X_L{}^2)}} \qquad \text{...........(34)}$$

the peak current in capacitance is given by

$$i_2 = \frac{V_o}{X_c} \qquad \text{..........(35)}$$

current i_1 lags behind V_o by an angle ϕ and the current i_2 leads V_o by 90^o.The impedance if parallel circuit is given by

$$Z = \frac{V_o}{i_o}$$

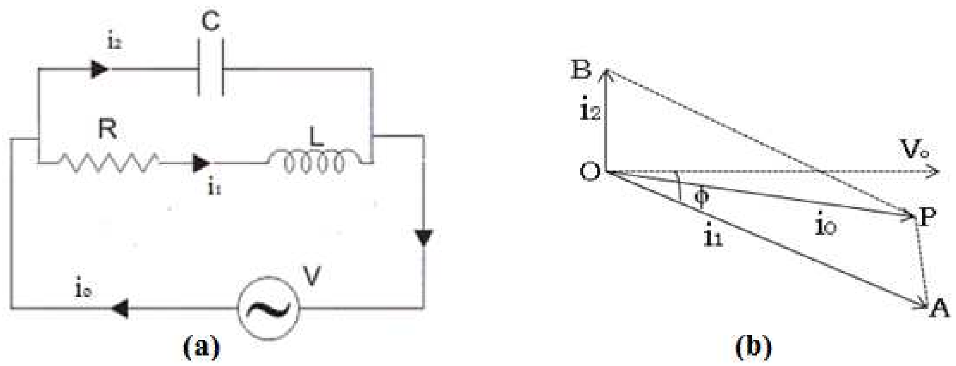


Figure 13

The parallel circuit is said to be in resonance when the current $i_o$is in phase with the applied voltage V_o, and the current in the circuit does not depend on the value of capacitance and inductance circuit behaves like a pure resistance.

from the phase diagram for parallel circuit we can write

$$i_o = i_1 \cos\phi \qquad \text{........(36)}$$

$$i_2 = i_1 \sin \phi \qquad \text{..........(37)}$$

now since $i_1 = \frac{V_o}{\sqrt{(R^2+X_L{}^2)}}$, $i_2 = \frac{V_o}{X_c}$ *and* $\sin \phi = \frac{X_L}{\sqrt{(R^2+X_L{}^2)}}$

substituting these values in equ.(37)

$$\frac{V_o}{X_c} = \frac{V_o}{\sqrt{(R^2+X_L{}^2)}} \frac{X_L}{\sqrt{(R^2+X_L{}^2)}}$$

or

$$X_L X_c = R^2 + X_L{}^2$$

$$\frac{\omega L}{\omega C} = R^2 + \omega^2 L^2$$

$$\frac{1}{LC} = \frac{R^2}{L^2} + \omega^2$$

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \qquad \text{.........(38)}$$

this is the resonant frequency of parallel LCR circuit. Now since frequency is a real quantity hence

$$\frac{1}{LC} > \frac{R^2}{L^2}$$

or $R^2 < \frac{L}{C}$

$$\text{or} R < \sqrt{\frac{L}{C}} \qquad \text{.................(39)}$$

this is the requirement for a parallel circuit to be resonant. When r is very small the value of $\frac{R^2}{L^2}$ is very less in comparison to $\frac{1}{LC}$ the resonant frequency becomes

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

which is same as for the series resonant circuit.

The impedance of the circuit is$Z = \frac{V_o}{i_o}$

now$i_o = i_1 \cos\phi$

$$i_1 = \frac{V_o}{\sqrt{(R^2 + X_L{}^2)}} \; and \cos\phi = \frac{R}{\sqrt{(R^2 + X_L{}^2)}}$$

hence

$$i_o = \frac{V_o}{\sqrt{(R^2 + X_L{}^2)}} \frac{R}{\sqrt{(R^2 + X_L{}^2)}}$$

$$i_o = \frac{V_o R}{R^2 + X_L{}^2}$$

but $R^2 + X_L{}^2 = X_L X_c = \frac{\omega L}{\omega C} = \frac{L}{C}$

hence $i_o = \frac{V_o RC}{L}$ ………..(40)

and the impedance of the circuit is $Z = \frac{L}{CR}$ …………(41)

Now it is clear that impedance of the circuit is very high for small value of resistance R when $R = 0$ no current will be drawn by the circuit from source. Thus at resonance, circuit rejects the current of same frequency as the natural frequency of the circuit. That is why this circuit is called rejecter or filter circuit.

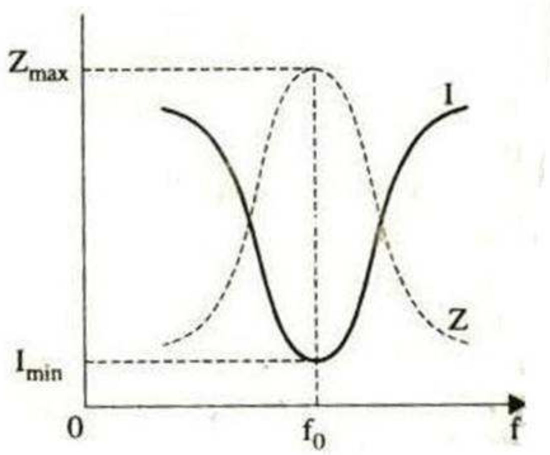


Figure 14

Figure 14 is the frequency response curve for a parallel LCR circuit. Graph shows that the response starts at its maximum value, reaches its minimum value at the resonance frequencywhen $I_{MIN} = I_R$ and then increases again to maximum as *f* becomes very high.

13.10.3 Quality factor of a circuit

Reactive components such as capacitors and inductors are often described with a figure of merit called Q (quality factor). While it can be defined in many ways

The quality factor of an oscillating electric circuit is defined as 2π times the ratio of the energy stored to the energy loss (energy dissipated) per period. i.e.

$$Q = 2\pi \frac{Energy\ stored}{Energy\ loss\ per\ period\ at\ resonance} = 2\pi \frac{E_S}{E_D}$$

Energy loss per period is directly related to the damping. Hence less is the damping higher is the quality factor.

This definition does not specify what type of system is required. Thus, it is quite general. For a LCR series circuit energy stored in the circuit is

$$E_S = \frac{1}{2}LI^2 + \frac{1}{2}CV_C{}^2 \ldots\ldots\ldots\ldots\ldots\ldots(42)$$

for $V_C = A\ sin\ \omega t$the current flowing in the circuit is

$$I = C\ \frac{dV_C}{dt} = \omega CA \cos \omega t$$

The total energy stored in the reactive element is

$$E_S = \frac{1}{2}L\omega^2C^2A^2\cos^2\omega t + \frac{1}{2}CA^2\sin^2\omega t \ldots\ldots\ldots\ldots\ldots\ldots(43)$$

At the resonance frequency where $\omega = \omega_o = \frac{1}{\sqrt{LC}}$, the energy stored in the circuit stored becomes

$$E_S = \frac{1}{2}L\frac{1}{LC}C^2A^2\cos^2\omega t + \frac{1}{2}CA^2\sin^2\omega t$$

$$E_S = \frac{1}{2}CA^2(\cos^2\omega t + \sin^2\omega t)$$

$$E_S = \frac{1}{2}CA^2 \ldots\ldots\ldots\ldots\ldots(44)$$

the energy dissipated per period is equal to the average resistive power dissipated times the oscillation period T

$$E_D = R < I^2 > T$$

$$E_D = \frac{R < I^2 >}{f}$$

$$E_D = R < I^2 > \frac{2\pi}{\omega_o} = R\left(\omega_o^2 \frac{1}{2} C^2 A^2\right)\frac{2\pi}{\omega_o} = 2\pi\left(\frac{1}{2}\frac{RC}{\omega_o L} A^2\right)$$

$$E_D = 2\pi\left(\frac{1}{2}\frac{RC}{\omega_o L} A^2\right) \quad \text{..........(45)}$$

with the help of equ. (44) and (45) putting the value of E_S and E_D in the definition of quality factor

$$Q = 2\pi \frac{E_S}{E_D}$$

$$Q = 2\pi \frac{\frac{1}{2} C A^2}{2\pi\left(\frac{1}{2}\frac{RC}{\omega_o L} A^2\right)}$$

$$Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC} \quad \text{..............(46)}$$

It is obvious that quality factor increases with decreasing R.

The selectivity or Q-factor for a parallel resonance circuit is generally defined as the ratio of the circulating branch currents to the supply current and is given as:

$$Q = \frac{R}{\omega_o L} = \omega_o RC \quad \text{..........(47)}$$

13.11 THE IDEAL TRANSFORMER

A transformer is an electrical device that transfers electrical energy between two or more circuits through electromagnetic induction. A device that changes AC electric power at one voltage level to AC electric power at another voltage level through the action of a magnetic field.

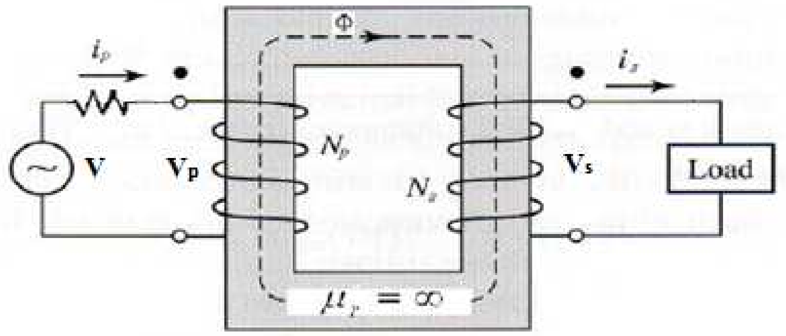


Figure 15

The core type transformer shown in figure 15. It consists two highly inductive coils which are electrically separated and magnetically linked through an iron core. If one coil is connected to source of alternating voltage an alternating magnetic flux is set up in core, which is shown in figure by rectangular shaded part, and this flux is linked with the other coil. Hence an induced alternating voltage is produced in the second coil.If second coil is closed, the current flow in it and so electric energy is transferred from first coil to second coil. The first coil in which electric energy is fed is called primary coil and the other from which energy is drawn out is called secondary coil.

An ideal transformer is a theoretical, linear transformer that is lossless and perfectly coupled; that is, there are no energy losses and flux is completely confined within the magnetic core. Perfect coupling implies infinitely high core magnetic permeability and winding inductances and zero net magnetomotive force.

Assumptions

1. Relative permeability of core material, $\mu_r = \infty$
2. Total magnetic flux linked with primary coil should be linked with secondary coil, i.e. flux loss $= 0$
3. No core loss
4. No winding loss

13.11.1 Voltage relationship

Whether voltage across secondary coil is more or less than voltage across primary coil depends on the turn ratio of the primary and secondary coils, i.e.

$$V_p = N_p \frac{d\emptyset}{dt} \text{ and } V_s = N_s \frac{d\emptyset}{dt}$$

$$\frac{V_p}{V_s} = \frac{N_p}{N_s} = n, called\ transformation\ ratio$$

13.11.2 Current relationship

If relative permeability of the transformer core $\mu_r = \infty$ then the resistance of the core material $R_{core} = 0$

$$i_p N_p = i_s N_s$$

$$\frac{i_p}{i_s} = \frac{N_s}{N_p} = \frac{1}{n}$$

It is obvious from above relations that a transformer which is step up for voltage is step down for current. If voltage is increased by 'n' times current reduces by a factor 1/n, because output power is equal to the input power for ideal transformer.It means that current ratio isreciprocal of voltage ratio.

13.11.3 Impedance relationship

Every transformer winding has its own resistance, inductive reactance hence impedance

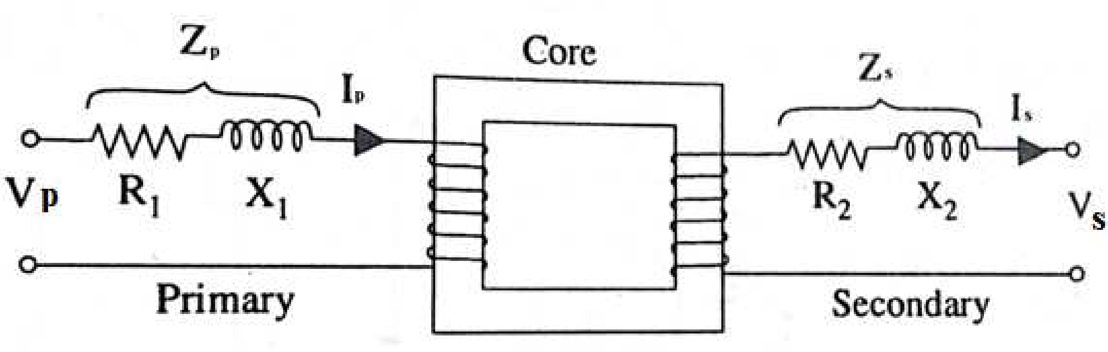


Figure 16

$$Z_{in} = \frac{V_p}{i_p} \text{ and } Z_{load} = \frac{V_s}{i_s}$$

hence $\frac{Z_{in}}{Z_{load}} = \frac{V_p}{V_s}\frac{i_s}{i_p} = \left(\frac{N_p}{N_s}\right)^2 = n^2$

13.11.4 Power relationship

The ideal transformer does not generate, dissipate, or store energy. Therefore the instantaneous power leaving the transformer is the same as that entering. This could be said in other words by saying that if one were to draw a box around an ideal transformer and sum the power flows into (or out of) the box, the answer is zero at every moment in time.

$$P_p = V_p i_p, P_s = V_s i_s$$

hence $\frac{P_p}{P_s} = \frac{V_p}{V_s}\frac{i_p}{i_s} = 1$

13.12 SUMMARY

In this chapter we studied the nature of alternating current in different electronic components (resistor, inductor and capacitor) and in their combinational circuits(LR, RC, LC and LCR). The relation between current and alternating potential shows that current is not always in phase with applied potential difference. Sometimes phase angle is positive and sometimes negative. The current in LCR series circuit is maximum when the natural frequency of the circuit is equal to the frequency of AC source and this condition is called resonance. While in parallel LCR circuit current is minimum at resonance. Hence the series LCR circuit is called acceptor circuit and parallel circuit is called rejecter circuit.

13.13 GLOSSARY

1. Impedance:net resistance produced by all the components in circuit
2. Inductive reactance: resistance produced by induction coil
3. Capacitive reactance: resistance produced by capacitor
4. Resonance: the condition when maximum current flows in the circuit

13.14 REFERENCES

1. Electricity and Magnetism, Berkeley Physics Course Volume-2 by Edward M. Purcell

2. University Physics, Young & Freedman, Pearson, Addition Wesley

13.15 SUGGESTED READINGS

1. Basic Electronics by B.L. Theraja, S. Chand and company Ltd
2. Electricity and Magnetism by Satya Prakash, Pragati Prakashan, Meerut

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Course Title and Code	**: Electricity and Magnetism (BSCPH 102)**
ISBN	**:**
Copyright	**: Uttarakhand Open University, Haldwani, Nainital- 263139**
Edition	**: 2017**
Published By	**: Uttarakhand Open University, Haldwani, Nainital- 263139**
Printed By	**:**

BSCPH-102

Electricity and Magnetism

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Contents

Course 2: Electricity and Magnetism
Course code: BSCPH102
Credit: 3

	LCR circuits Resonance in a series LCR circuit, Q-factor, principle of ideal Transformer	