UNIT 2 THE MATHEMATICS OF FINANCE

2.1 INTRODUCTION

In this unit, you will get familiar with the basics of simple and compound interest, the concept of time , value and money and various compounding and discounting techniques. Time value of money is the core of all the financial calculations involving values. To understand the meaning of Annuity and Perpetuity and to get idea about certain important terms like loan amortization.

2.2 OBJECTIVES

After studying this unit, you will be able to understand the following:

- Basis for time preference for money.
- Concept of simple and compound interest.
- Understand what gives money its time value.
- Get familiar with methodology of calculating present and future value.
- Learn the concept of doubling period, annuity and perpetuity.

2.3 CONCEPT

The task of the present unit is to explore the basic factors that affect all investment values. The Basic Valuation Model are based on the idea that money has a “time value”. This is the most basic concept of financial management and is the stepping stone for further study. By this concept , it simply means that a 100 rupee note received at present (now) worth more than that a 100 rupee note received in future (or let’s say two years later) and ,this summarize the principle that value of money is dependent on time and “a money received today is worth more than the money received tomorrow”.

2.3.1 BASIS OF TIME PREFERENCE FOR MONEY

There is an individual's time preference for money . Individuals prefer to receive the money now rather than say after two or more years . Therefore , if you have a choice of Rs100 today or Rs100 a year from now , first option is always better , since Rs100 can be reinvested today lets say @ 6% interest which will fetch you Rs106 in a year’s time . So , even if the future payment is same (Rs100) , but the difference in their timing in which they are received can create a great difference in the value of the money. Time value of theory states that “ A rupee tomorrow is worth less than A rupee today”.

Reasons / Justification for Time Preference of Money by Individual's :

- Monetary Inflation: Under inflationary conditions, the purchasing power of money declines over the period of time. Greater the inflation greater will be the decline in the future value of money as compared to its value today.
- Preference of present consumption over future consumption: Individual prefer for immediate consumption over future consumption because of its importance at present or due to uncertainty to enjoy future consumption. Example: A glass of water holds greater significance today to a thirsty man , but counts little to him, if given the next day.
- Cash Flow Uncertainty : the amount which is to be received today is certain , but the amount likely to be received in future holds uncertainty or risk associated to it.
- Opportunities for Re-investment : It is preferable to take the amount today rather than a year latter (if given the choice), as reinvestment opportunity is always there and the interest can be earned in it, which will provide additional cash.

Before moving further , let's understand the basic concept of simple and compound interest :

2.3.2 SIMPLE INTEREST

Simple interest is the interest which is computed only on the principal amount, which is borrowed or deposited. Simple interest is a function of principal amount (borrowed or deposited),rate of interest annually and the number of year for which the principal is borrowed or lent.

Mathematical Formula :

Simple Interest = P x R x N

where 'P' = principal .

'R'= rate of interest per annum.

'N'= number of years.

(If 'X' wants to calculate his total future value at the end of nth year, then F.V is the sum total of principal amount + interest and ascertained as :

Amount at the end of nth year = principal + interest

= P (1+ r n).

Illustration 1 :

Calculate the simple interest and amount of Rs 5000 for a period of 5 years at 12% p.a.

Solution : Interest = 5000 x 0.12 x 5 = 3000.

Amount = P(1+ r n)

= 5000(1+ 0.12x 5) = 8000.

Illustration 2 :

Mr 'X' has deposited 50,000 in the bank which pays 5 % simple interest. Mr 'X' wants to know what is going to be the future value of his money at the end of 7 years period.

Solution : Future Value = P(1 + r n)

= 50,000 (1+ 0.05 x 7)

= 67,500.

Illustration 3 :

Calculate the rate of interest which will Rs 25,000 amount to Rs 31,000 in a period of 5 years ?

Solution : Amount = P(1+ r n)

$31000 = 25000(1+ 5r)$

Rate 'r' = 4.8%.

2.3.3 COMPOUND INTEREST

Compound interest is the interest which is received on the principal amount and on the interest as well . The Interest for one period gets added back to the original principal to get the principal for the next year. Compound interest is also regarded as 'interest on interest' . Simple interest is the interest calculated on the original principal and no compounding of interest takes place unlike compound interest.

Compound interest can be compounded 'annually' (the interest is given only once i.e at the end of the year), 'semi-annually'(interest paid twice per year), 'quarterly' (interest paid in 4 equal installment), 'monthly'(interest paid in 12 installments), or 'daily'(interest is paid daily i.e all 365 days).

Symbolically ,

$CA = P (1+ r/n)^{n \times t}$

where, CA = amount at the end of 't' period.

P = principal amount at the beginning of the 't' period.

r = rate of interest

t = time period in years

n = number of compounding in a year.

- If interest is compounded half yearly : $CA = P (1+ r/2)^{2 \times t}$
- If interest is compounded quarterly: t $CA = P (1+ r/4)^{4 \times t}$
- If interest is compounded monthly : $CA = P (1 + r/12)^{12 \times t}$
- If interest is compounded daily : $CA = P (1+ r/365)^{365 \times t}$

Illustration 4 :

Calculate the compound interest for Rs 2000 for 15 month @ 5% compounded quarterly .

Solution : $CA = P (1+r/n)^{4n}$

Where, P = 2000 , rate 'r' = 5%, no of compounding in a year 'n' = 4(i.e quarterly), time period 't'= 15/12 (in years)

$$CA = 2000(1 + 0.05/4)^{4 \times (15/12)}$$

CA = 2000x 1.064 = 2128.

CI = Amount – Principal = 2128 – 2000 = Rs 128.

Note : Here the interest is given in four installments, therefore we divide 5% into four equal parts of 1.25% each .

Illustration 5:

Find the present value of Rs 1000 due in 6 years @ 5 % if the money is compounded semiannually.

Solution : $CA = P(1 + r/2)^{2 \times t}$

where , CA or Amount = 1000, r = 5%, n = 2 (compounded semi-annually), t= 6 yrs, we have to calculate the present value i.e principal 'P' .

$$1000 = P(1 + 0.05/2)^{2 \times 6}$$

1000 = P x 1.34488

P = 1000/1.34488 = RS 743.56

Therefore, the present value of Rs 1000 due in 6 years is Rs 743.5.

Illustration 6 :

Your mother gave you Rs 1000 as a reward for your good performance in examination. You deposited it in a saving bank a/c @ 6% rate of return for 2 years. How much is the future value , you will receive after 2 years from now.

Solution : Future value = Principal + interest

FV = 1000+ (0.06x1000) = 1060.

At the end of 1st year, you get Rs 60 as interest along with 1st year principal amount of Rs 1000, but we will not withdraw this amount at the end of 1st year and reinvest this amount (1060) which act as the principal for the next period. Since there occur compounding of interest (that is, interest is earning interest) , now we reinvest our principal (1060) and calculate interest on it :

FV = Principal + Interest

= 1060 + (0.06 x 1060)

= Rs 1123.6

Therefore , Rs 1123.6 is the future value we will receive after 2 years from now.

Time value of money is the core of most of the financial decisions . Suppose, if you had been given a choice of Rs 1000 today or Rs 1130 two years from now . What will you do ?

You have two options , the second option will pay you more (Rs 130) which is good, but after two years in future which is bad. So, overall , is the second option better or worse ? This is what we are going to understand further in this unit and how big corporate and business make these types of comparison .

2.4 DISCOUNTED CASH FLOW ANALYSIS

It involves calculations and financial decision making by considering at the cash flow from a business activity, where the principal behind the concept is that "the money received in future is less valuable than the money received today".

Here, we will concentrate on the mathematics behind the time value of money and the discounted cash flow problems solving techniques. Various examples which involves discounted cash flow analysis (where, we need to calculated present value 'today's value' of a cash or a series of cash flow which is to be received in future) from our day to day experience are :

- ✓ Suppose, we deposit Rs100 in a savings bank account . How much we will have in after 10 years?
- ✓ We need to take loan of Rs100000 and will have to repay it in 5year's time. How much do I need to pay as EMI per month?
- ✓ What if , we have Rs2500000 as savings for retirement and we need to survive by it for the next 25 years , so how much amount can we withdraw each month out of it ?

All the above examples are discounted cash flow problems .

2.4.1 PRESENT VALUE (PV)

The method of calculating the present value (PV) of a series of Future cash flow is called 'discounting' techniques.

Calculation of PV of a Single Amount: The formula to calculate the P.V of a certain amount to be received after some future periods is :

$$\textbf{Present Value} = [\ \textbf{Future Value}\ (1/(1+r))^n\ ]$$

- The present value 'PV' of a future value 'F.V' due at the due at the end of 'n' conversion period at the rate 'r' , is given by the above formula, where 'r' is referred to as the rate of return, discount rate or the cost of capital or opportunity cost.
- The term $(1/1+r)^n$ is referred to as discount factor or PVF (present value factor) and is always less than 1, indicating that a future amount has a smaller P.V.

P.V can also be represented by this formula for calculations :

$$\mathbf{PV = FV_n\ X\ (\ PVIF_{r,\,n}\ )}$$

where, FV_n = Future value receivable at the end of 'n' years, ($PVIF_{r,n}$) is the present value interest factor for 'n' periods at 'r' rate of interest . We can use a PVIF table which can provide you with the pre- calculated value of PVIF for 'n' years at 'r' rate of interest and we can multiply that value with the FV to arrive at PV.

Illustration 7

Let's say we need 15,00000 in 6 years. If the interest rate is 6%, how much do we need to deposit in the bank now?

Solution : This kind of problem is called the PV problem, as we need to find out the today's value of a certain amount that we are going to receive in future. It is called discounting problems. The problem can be solved in a timeline as follows :

$\mathbf{PV = FV\ \ X\ \ (\ PVIF_{r,\,n}\ )}$

PV = 1,50,0000 x PVIF (6%,6)

= 1500000 x 0.705 = 10,57,500.

So, the present value of Rs1500000 paid 6 years from now at an interest rate of 6% is Rs10,57,500

Note : Use PVIF table and find the value for (6%,6 years)

Another formula can also be used as :

$\textbf{Present Value} = [\ \textbf{Future Value}\ (1/(1+r))^n\ ]$

$PV = 1500000\ (1/(1+0.06))^6$

= 1500000 X 0.70496 = 1,057,440 .

Therefore the present value of Rs15,000,00 paid 6 years from now at an interest rate of 6% is Rs10,57,440.

2.4.2 PRESENT VALUE OF A SERIES OF CASH FLOW

Most of the times we need to calculate the present value of a series of cash flow. Example , in financial analysis and in capital budgeting problems for decision making we need to convert the future value of a series of cash inflow (which can be even or uneven cash flow stream) into its present value. The cash flow can be even and uneven .

A) **Present value of uneven cash flow :**

Investment made my firms sometimes receive an uneven cash flow streams like the cash inflow from the capital investment made to a project or the dividend distribution of equity shares is uneven and growing. The present value of cash flow over a period can be calculated with following formula:

$$\mathbf{PV_n} = \frac{CIF1}{(1+r)^1} + \frac{CIF2}{(1+r)^2} + \frac{CIF3}{(1+r)^3} + \ldots + \frac{CIFn}{(1+r)^n} = \Sigma \frac{CIFt}{(1+r)^t}$$

where , *CIFn* = Cash Inflow occurring at the end of Nth year.

'r' = Discount rate

'n'= duration of the cash flow streams

't'= year in which cash flows are receivable

PV_n = Present value of a cash flow stream

Present value of uneven cash flow can also be resolved using the formula :

$$\mathbf{PV = CIF1x\ PVIF_{(1,r)} + CIF2\ x\ PVIF_{(2,r)} + CIF3\ x\ PVIF_{(3,r)} + \ldots\ldots + CIF_n\ x\ PVIF_{(n,r)}}$$

Illustration 8 :

Calculate the present value @ 10% interest rate from the following information:

Year Cash Inflow (Rs)

0	2000
1	3000
2	4000
3	4500
4	5000
5	3500

Solution :

The Present value calculation is shown below :

$$PV_n = \frac{CIF1}{(1+r)^1} + \frac{CIF2}{(1+r)^2} + \frac{CIF3}{(1+r)^3} + \ldots + \frac{CIFn}{(1+r)^n} = \Sigma \frac{CIFt}{(1+r)^t}$$

PV = $(2000/(1+.10)^0)$ + $(3000/(1+0.10)^1)$ + $(4000/(1+0.10)^2)$ + $(4500/(1+0.10)^3)$ + $(5000/(1+0.10)^4)$ +

$(3500/(1+0.10)^5)$

= 2000 + 2727 + 3305 + 3380 + 3415 + 2173 = 17000

Illustration 9

An investor has an opportunity of receiving a cash flow of 1000, 2000, 500, 1500, 800 respectively at the end of one through five years at 8% rate of interest. Find out the PV of this stream of uneven cash flow.

Solution : The Present Value Calculation is shown below :

PV = CIF1x $PVIF_{(1,r)}$ + CIF2 x $PVIF_{(2,r)}$ + CIF3 x $PVIF_{(3,r)}$ +…... + CIF_n x $PVIF_{(n,r)}$

The problem can be calculated from the above formula as well :

PV = 1000x $PVIF_{(1, 0.08)}$+ 2000 x $PVIF_{(2 ,0.08)}$ + 500 x $PVIF_{(3,0.08)}$ + 1500 x $PVIF_{(4,0.08)}$ + 800 x $PVIF_{(5, 0.08)}$

= 1000x 0.926 + 2000x 0.857 + 500x 0.794 + 1500x 0.735 + 800 x 0.689

= 926 +1714 + 397 + 1102.5 + 551.2 = 4690.7

B) **Present value of a series of even cash flow:**

The Present value of series of even cash flow is also referred to as annuity, which is discussed latter in this section .

2.5 FUTURE VALUE

- If we put Rs100 in a bank account today, how much will we get after two years ?
- Suppose we invest Rs5000 for five years in a savings account that pays 10%interest per year. If we reinvest the interest income, then how much our investment will grow after five years?

Above problem is called the future value problem. We want to know the value in the future of an amount today. It is also called the compounding problem. Compounding is the process of finding the future value of cash inflow or outflow by using the concept of compound interest.

2.5.1 FUTURE VALUE OF A SINGLE CASH FLOW :

The process of investing money and also re-investing the interest earned is called compounding. The future value of an investment after 'n' years when the interest rate is 'r' percentage is :

$$\mathbf{FV_n = PV\ (1+r)^n}$$

OR

$$\mathbf{FV_n = PV \times FVIF_{(r,n)}}$$

where,

FV= future value

PV= present value

'r'= annual rate of interest

'n'= number of periods

$(1+r)^n$ = is future value interest factor(FVIF) or future value factor. The value of FVIF can be calculated by using a FVIF table or by using calculator as well .

Illustration 10 :

Suppose , you deposit Rs 1000 today in a bank that pays 10 percent interest compounded annually, how much will the deposit grow to after 6 years and 12 years.

Solution : the above problem can be solved using the following formula :

$$\mathbf{FV_n = PV\ (1+r)^n}$$

a) Deposit after 6 years :

$FV=1000x\ FVIF_{(6\ ,0\ .10)}$, now look for the value of FVIF for 6 years and 10% from the FVIF table

FV = 1000x 1.772 = 1772 .

b) Deposit after 12 years :

$FV = 1000x\ FVIF_{(12\ ,\ 0.10)}$

FV = 1000x 3.138 = 3138.

2.5.2 FUTURE VALUE OF A SERIES OF UNEVEN CASH FLOW

Some financial instrument generates cash flow which are not constant and vary from period to period. Example : dividend on stock , cash flow generated from business activity are irregular flows .

Compound value of a series of uneven cash inflow or outflow can be calculated by the following

formula : $\mathbf{FV = CF_0\ (1+r)^n + CF_1\ (1+r)^{n-1} + CF_2\ (1+r)^{n-2} + \ ...+ CF_n}$.

OR

$$\mathbf{FV= CF_0\ X\ FVIF_{(r,n)}+CF_1\ X\ FVIF_{(r,\ n-1)}+ \ .....+CF_n}\ .$$

Where, CF_n = Cash flow Compounded for 0 periods .

CF_0 = Cash flow Compounded for the whole 'n' period.

'n' = number of periods from time 0 to the reference date given .

We first have to calculate the future value of each individual cash flow and then sum up all the cash flow to arrive at the total future value of an uneven cash flow stream.

Illustration 11 :

Mr. X deposit Rs1000 today into a bank account that pays a 10% interest rate per year , and follow it up with 4 more deposits to the bank at the end of each year for the next four years, which are given as below:

Year	Cash flow
1^{st}	1500

2^{nd} 2000

3^{rd} 500

4^{th} 1500

How much money will Mr. X accumulate in his bank account at the end of fourth year.

Solution: First step : We have to calculate the future value of each individual cash flow

Second step : Sum up all the cash flow to arrive at the total future value.

Formula : $FV = CF_0\,(1+r)^n + CF_1\,(1+r)^{n-1} + CF_2\,(1+r)^{n-2} + \ldots + CF_n$.

Year	Cashflow
0	1000
1	1500
2	2000
3	500
4	1500

$FV = 1000\,(1+ .10)^{4} + 1500(1+.10)^{4-1=3} + 2000(1+.10)^{4-2=2} + 500(1+.10)^{4-3=1} + 1500(1+.10)^{4-4=0}$

$FV = 1000x1.4641 + 1500x\ 1.331 + 2000\ x\ 1.21 + 500x1.1 + 1500x1 = 7930.6$ The total future value at the end of fourth year is Rs 7930.6

2.5.3 FUTURE VALUE OF A SERIES OF EVEN CASH FLOW

The future value of series of even cash flow is also referred to as annuity, which is discussed latter in this section .

2.6 DOUBLING PERIOD

Whenever we make an investment, we have questions in mind like, how long would it take to double the amount at a given rate of interest? This is doubling period. Doubling period is that time which is required to double our invested amount at a particular interest rate.

It is calculated by two rules :

a) Rule of 72 : According to this rule of thumb, the doubling period is obtain by dividing 72 by the interest rate.

Formula : DP = 72 / I

Where, DP = Doubling period , I = interest rate.

Illustration 12: If you have deposited Rs1000 at 10% interest rate in the bank , how many years will it take to double the amount?

Solution : DP = 72 / I

DP = 72 / 10 = 7.2 years (Approx.) , therefore , the doubling period is 7.2 years.

b) Rule of 69: This method is considered to be more accurate doubling period method. The formula is given by : **DP = 0.35 + 69/ Interest rate**

Lets take the same Illustration as above and using the rule of 69 :

DP = 0.35+69/10 = 7.25 .

c) Effective Rate of interest (ERI) in case of doubling period :

a) ERI in case of Rule of 72 : It is given by the following formula : ERI = 72 / Doubling Period

b) ERI in case of Rule 69 : It is given by the following formula : ERI = (69/DP) +0.35

2.7 ANNUITIES AND PERPETUITIES

An annuity is a fixed stream of cash inflow and cash outflow occurring at a regular interval of time. Some of the examples of annuities are : a) you rent a flat and promise to make a series of payment over an agreed period , is an annuity . b) payment made to LIC premium c) depositing in a recurring deposit account .

2.7.1 WHAT IS AN ANNUITY?

Annuity is a series of regular cash flow for a specific duration. Example: If you have taken a loan for 30 years, you have to make 12 installment each year for 30 years , i.e, it makes for 360 payments. Now, if we need to find the present value of the payments, we have to calculate the P.V of each of the 360 payment individually, which is time consuming and rigorous as well .

There is an easy way to do it, since each of the payment is of equal amount , we take this advantage for the calculation .

Types of annuity : Cash flow may occur at the end of the year or at the beginning of the year. If the cash flow occurs at the end of the year or a period it is called regular or ordinary or 'deferred annuity' whereas, if the cash flow occurs at the beginning of the year or period, it is called 'Annuity due'.

Check Your Progress-A

Fill in the Blanks

1. Time value of money explains the concept that a unit of money obtained today is ______than a unit of money obtained in future.
2. If the nominal rate of interest is 10% per annum and it is compounded annually , then the ERI will be ____ % per annum .
3. The longer the time period, the smaller the present value, when given Rs100 as future value and having a constant interest rate True/ False.
4. In ______ the first payment is delayed beyond one year .
5. In _______ there is an equal cash flow per period forever.
6. An annuity is a stream of_____ annual flow.
7. ________is repayment of loan over a period of time.
8. Earning interest on interest is called _______

2.7.2 FUTURE VALUE OF A DEFERRED ANNUITY

Suppose, we deposited Rs1000 in a savings bank account at the end of each of the next three years (ie at the end of each year 1, 2 and 3) . Calculate how much amount we will have at the end of year 3, if the rate of interest is compounded at 5% annually.

This is the example of Regular or Deferred Annuity ,where the regular payment are made at the end of successive years. In the above example, we need to find the total value of the payment at a point in future.

Formula : $\mathbf{CV_n = P_1 (1+r)^{n-1} + P_2 (1+r)^{n-2} + \ldots + P_{n-1} (1+r) + P_n}$

or

$$\mathbf{CV_{n\ 1} = P\,[((1+r)^n - 1)/\ r]}$$

Where,

P = Fixed Periodic Cash flow

r = Interest Rate

n = duration of the amount

Note : The term in the bracket $[((1+r)^n - 1)/\ r]$ is called the **compound value factor for an annuity** of Re1 , which we shall refer it as CVFA. The above equation can also be written as :

Future value = Annuity cash flow x compound value factor for annuity of Re 1 .

$$\mathbf{FV = A\ X\ CVFA_{n,r}}$$

Illustration 13 : Mr. Shyam has deposited Rs 500 at the end of every year for 6 years at 6% interest . Determine Shyam's money value at the end of 6 years.

Solution : By using the formula

$$CV_n = P_1 (1+r)^{n-1} + P_2 (1+r)^{n-2} + \ldots + P_{n-1} (1+r) + P_n$$

where, r= 6% , n= 6 , P = 600

CV_n = 600(1.338) + 600(1.262) + 600 (1.191) + 600 (1.124) + 600 (1.060) + 600 (1.00)

CV_n = 802.8 + 757.2 + 714.6 + 674.4 + 636 + 600 = 4185

Alternatively , we can use the other formula also :

Future value = Annuity cash flow x compound value factor for annuity of Re 1

Future value = 600 x CVFA(6,6%)

using the CVFA table for 6 years at 6% rate

Future value = 600 x 6.975 = 4185

2.7.3 FUTURE VALUE OF ANNUITY DUE

The basic difference between the regular annuity and the annuity due is that the payments that need to be made for annuity due start right from the first month itself whereas in regular annuity its starts at the end of each month.

- Suppose Mr X deposited Rs100 in a saving account at the beginning of each year for 4 years to earn 6% interest ? How much will be the future value at the end of 4 years *? Or*
- Suppose you are saving Rs500 per month for next four years at an annual interest rate of 5% compounded monthly, whereas each deposit is made at the start of each month. How much will you have at the end of three years. *Or*
- Suppose you bought a TV on instalment basis, and the dealer ask you to make the first payment immediately (at the beginning of the first month) and the remaining instalment in the beginning of the subsequent month.

Such examples involves the calculation of annuity due . The Future value of Annuity due can be calculated with the help of following formula :

Formula : $$\mathbf{CV_n = P\,[(1+I)^n - 1 / I]\ X\ (1+ I)}$$

Or , Future value of an annuity due = Future value of an annuity x (1+i)

$$\mathbf{FV = A\ \ X\ \ CVFA_{(n\ ,\ I)}\ X\ (1+i\ )}$$

where, I = interest rate

p = fixed periodic cash flow

CVFA = compound value annuity factor

Note : the compound value annuity factor (CVFA) value can be calculated from the CVFA table, and should be multiplied by (1+i) to obtain the relevant factor for an annuity due.

Illustration 14 : Mr. Ramesh has deposited Rs 2000 at the beginning of every year for 5 years in a saving bank account at 6% compound interest. What is the value of the money at the end of 5 years .

Solution : By using the above formula to calculate the annuity due :

$$CV_n = P\,[(1+I)^n - 1 / I]\ X\ (1+ I)$$

$$CV_5 = 2000[\ (1+ 0.06)^{5\,-}\ 1 / 0.06)]\ X\ (\ 1+ 0.06)$$

$$CV_5 = 2000 \text{ x } 5.637 \text{ x } 1.06 = 11950.44$$

Alternatively , it can be calculated using CVFA value from the table :

Future value of an annuity due = Future value of an annuity x (1+i)

$$= A \ \text{ x } \ CVFA_{(n\ ,\ I)} \text{ x } (1+i\ )$$

$$= 2000 \text{ x } CVFA_{(5\text{ yrs}, 6\%)} \text{ x } (1+ 0.06)$$

$$= 2000 \text{ x } 5.637 \text{ x } 1.06 = 11950.44$$

2.7.4 PRESENT VALUE OF EVEN CASH FLOW (ANNUITY)

Suppose an investor gets the opportunity of receiving a fixed payment for a certain fixed number of years (called 'Annuity'). The present value of an annuity can be found by calculating the present value of the annual amount every year and will have to aggregate all the present value to get the total present value of the annuity.

Formula **: $PVA_n = CIF\ [\ ((1+I)^{n-}\ 1)/\ I(1+I)^n\ )]$**

where, PVA = Present Value of Annuity

I = Interest rate

n = duration of annuity

CIF = cash inflow

OR , Alternatively can be calculated by :

Present value = $\frac{A}{1+i} + \frac{A}{1+i^2} + \frac{A}{+i^3} + \ldots + \frac{A}{1+i^n}$

where , A = Annuity, I = interest rate

OR ,

Present value = Annuity x Present value of an annuity factor

= A x $PVFA_{n\ ,\ I}$

Illustration 15 : Suppose I wish to determine the PV of the annuity of the cash flow of Rs50,000 per annum for 6 years. The rate of interest I can earn from my investment is 10% .

Solution : $PVA_n = CIF\ [\ ((1+I)^n\ - 1)/\ I(1+I)^n\ )]$

PVA_n = $50{,}000[((1+0.10)^6-1)/\ 0.10(1+\ 0.10)^6\ )]$

= 50,000 x (0.7715/0.1771)

= 50,000 x 4.3562 = 217814.

2.7.5 PRESENT VALUE OF ANNUITY DUE

Let us consider a 2 year annuity of Rs 100 every year, at the rate of 5% per year. If the payment is made at the beginning of the year, what is the present value of this annuity ? To calculate such problem following formula can be used :

Formula : $$\mathbf{PVA_n = CIF\ [\ (1- (1+I)^N)\ /I]\ (1+I)}$$

OR Present value of an Annuity due = Present value of an Annuity x (1+i)

$$\mathbf{PV = A \times PVFA_{n,i} \times (1+ i)}$$

Illustration 16: Mr Shyam received Rs 1000 at the start of every year for 4 years. Calculate the present value of annuity due assuming 10% rate of interest .

Solution : By using the formula :

Present value of an Annuity due = Present value of an Annuity x (1+i)

$= A \times PVFA_{n,i} \times (1+ i) = 1000 \times PVFA_{(4,\ 10\%)} \times (1+0.10) = 3487.$

2.7.6 PERPETUITY

Perpetuity are a lot similar to annuity , but the major difference is that it occurs indefinitely , that is , the financial payment go on forever . Since the financial payments made under this worth less the farther they are in the future , and this put a limit to the value of a perpetuity. The distant payment of perpetuity becomes worth lesser and lesser until they worth almost nothing . Some examples of perpetuity are 'preference share without maturity' (irredeemable preference share) where the company is expected to make dividend payment as long as the company survives , this can be treated as infinite .

Formula : Present value of perpetuity = **Perpetuity / Interest rate .**

Illustration 17

Mr A , is an investor and expects a perpetual sum of Rs 400 annually from the investments made by him at an interest rate of 8%. Calculate the present value of perpetuity.

Solution : Present value of perpetuity = Perpetuity / Interest rate .

PV of perpetuity = 400 / 0.08 = 5000. Therefore, the present value of his perpetuity is Rs 5000.

2.8 LOAN AMORTIZATION

Loan is an amount raised at an interest and repayable at a specified period. Payment of loan is known as amortization . The gradual writing off of an asset or an account over a period is called 'Amortization'. The borrower of loan is usually interested to know the amount of equal installment to be paid every year to pay back the complete loan along with interest. The installment can be calculated with the following formula :

Principal Amount = Loan Installment x $\mathbf{PVIFA_{N,I}}$

where, N= Loan repayment period at specified interest rate , I = Interest.

Illustration 18 : Suppose I have borrowed a 4 year loan of Rs10,000 at 9% . The lender of the loan requires three equal end of year repayments, calculate the annual Installment .

Solution : By using the formula : Principal Amount = Loan Installment x $PVIFA_{N,I}$

we can calculate the Loan Installment = P.A/$PVIFA_{N,I}$

$= 10{,}000/PVIFA_{4,\ 9\%} = 10{,}000/3.240 = 3{,}086.$

2.9 SINKING FUND

Sinking fund is a fund which is created out of fixed payments which are made in each period to accumulate a future sum after a specified period. Example : generally companies make sinking fund to retire debentures on maturity . The factors used to calculate the annuity for a given amount of future sum is called Sinking fund factor .

The formula for calculation of Sinking fund factor is as follows:

Sinking Fund (Annuity) = Future value x (1 / Compound value factor of annuity of Re 1)

= Future value x Sinking fund factor (SFF)

$= FV\ X\ 1/\ CVFA_{n,i}$

$= FV\ (\ I/(1+i)^n - 1)$

2.10 SUMMARY

In this unit, we have discussed about the time preference for money, which signifies that money has a time value. A detailed understanding of what gives money its time value has been discussed later on . We also learnt about the method of calculating present and future value and discounting techniques in making financial decisions. Lastly, we get familiarity with the concept of simple interest, compound interest, doubling period, annuities and perpetuity, loan amortization and sinking fund .

2.11 GLOSSARY

Discounting-The process of ascertainment of present value is called 'Discounting'

Nominal interest rate - Actual rate of interest paid.

Annuity- It is a fixed amount of cash inflow and outflow for a specified period of time.

Amortization- It is a gradual writing off of assets or liability over a certain time period.

2.12 ANSWERS TO CHECK YOUR PROGRESS

Check Your Progress –A

Fill in the Blanks

1. More
2. 10.38 percent
3. true
4. deferred annuity
5. Perpetuity
6. Equal
7. Amortization
8. Compound interest.

2.13 REFERENCES

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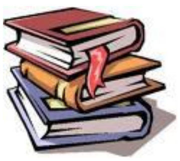

2.14 SUGGESTED READINGS

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2.15 TERMINAL QUESTIONS

Q1. Why is the consideration of time important in financial decision making? How can time value be adjusted? Illustrate your answer .

Q2. How long will it take to double your money if it grows at 12% annually?

Q3. Calculate the Present value of Rs 600 when it is to be a) received one year from now b) received at the end of 5 years c)received at the end of 15 years , given that the time preference rate is 5%.

Q4 Define Annuity? What is the difference between an Ordinary Annuity and an Annuity due?

Q5 Fifteen Annual payments of Rs 5000 are made into deposit account that pays 14% interest per year. What is the Future Value of this annuity at the end of 15 years.

Q6 If the interest rate is 12%, what are the doubling periods as per the rule of 72 and the rule of 69 respectively.

Q7 What is the difference between simple interest and compound interest . Explain with example .

Q8 Define Perpetuity ? What is the formula for the present value of perpetuity?